Hybrid Approach to Multiattribute Decision Making under Uncertainty

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Abstract - The research described in this paper addresses issues of designing a computationally effective decision support system which can assist a decision maker in making an optimal choice between several discrete alternatives. A new hybrid approach to multi-attribute decision making under uncertainty incorporating Neural Networks and the Dempster-Shafer theory of evidence is introduced. A neural network is employed for representation and quantification of a decision maker’s pairwise preferences of one alternative over the other. The Dempster-Shafer theory of evidence is used for combining the evidences representing these preferences for modeling the choice of the most preferable alternative. The designed method also includes consideration of subjective judgments about attributes representing aggregated concepts along with quantitative attributes. The case study considered in the research has demonstrated the feasibility of the application of this approach to Fusion 2/3 Level problems, namely to threat assessment.

Key words: decision support, subjective judgment, multiattribute decision making, neural networks, the Dempster-Shafer theory of evidence

1. Introduction

In today’s world, decision makers face continually increased amounts of data coming from multiple sensors, communication systems, and large databases. They also have to respond more quickly. At the same time, human decision making capabilities remain limited: short-term memory, the base for perception and processing, is limited to four chunks of information [1]. These factors require the development of computerized decision aids that model the decision making process and help to overcome human limitations.

The research described in this paper addresses issues of designing a computationally effective decision support system based on the multi-attribute decision theory. The multi-attribute decision theory is used to model subjective judgment of an expert who has to make optimal choices between several discrete alternatives. The judgment modeling is based on the notion of the underlying multi-attribute expected utility (cost) or value of the expected future outcome associated with each alternative that reflects how well the alternative is rated against a chosen goal. The optimal choice corresponds to the maximal expected utility or to the minimal cost associated with the alternatives. In many cases the decision situation is very complex, making it almost impossible to evaluate existing alternatives. However, it is often possible to represent each alternative with a set of features (attributes) and evaluate each alternative based on the value of the attributes associated with it.

Generally, the multi-attribute decision making process comprises two phases: the interpretation phase and the reasoning phase. The interpretation phase includes:

• construction of decision alternatives
• choice of attributes (qualitative and quantitative)
prediction of expected values of each attribute for each alternative.

The reasoning phase includes preference based evaluation of alternatives and selection of the alternative corresponding to the optimal choice.

There have been several methods developed for modeling the alternative selection process of decision makers in many applications such as manufacturing, market research, transportation, etc. (see, e.g. [2,3]). Most of these methods are based on explicit modeling of the underlying utility (cost) as a function of the attributes characterizing the alternatives. However, the methods that use explicit utility functions have to make an assumption about the form of this function [4-6]. These assumptions constitute constraints that may lead to decreased adequacy of the model. Other methods used in modeling multi-attribute decision making do not require explicit construction of a utility function [7,8], and use heuristic search to find the most attractive alternative. However, this type of method demands considerable input from decision makers during the knowledge acquisition stage of the development of these methods, and in most cases, the burden put on experts is substantial [7]. Another drawback of these methods is that they may not work efficiently in the case when the number of alternatives changes.

This paper presents a computationally efficient, connectionist decision-support system which simplifies the knowledge acquisition process without putting any constraints on the form of the utility function. This method also incorporates uncertain and incomplete quantitative as well as qualitative representations of attributes. The system is also capable of adapting to any potential change of decision makers’ preferences and/or changes in the decision situation.

The introduced hybrid method utilizes a connectionist approach in order to represent qualitative expert preference of one alternative over the other in numeric form. Then, the Dempster-Shafer Theory of Evidence [9] is used to combine these preferences and make a decision about the most preferable alternative.

The Dempster-Shafer Theory of Evidence is a tool for representing and combining measures of evidence. This theory is a generalization of Bayesian reasoning and it is more flexible than the Bayesian one when our knowledge is incomplete, and we have to deal with uncertainty, ignorance, and conflicting information.

The Neural Networks possess many computational and representational capabilities which make them especially suitable for representing qualitative expert preferences [10-12]:

- ability to learn from available data and to construct, verify, and validate themselves
- ability to cope with the brittleness problem;
- ability to easily adapt themselves to changes in decision environment and decision makers preferences

The detail description of the introduced hybrid approach is presented in the next sections. In Section 2, we give detailed description of our multi-attribute decision making system. Section 3 describes the process of quantification of the qualitative attributes, Section 4 describes the NN architecture for expert knowledge representation, Section 5 presents the evidential decision making process, Section 6 shows the applicability of the designed method to threat prediction and describes experiments and results.

2. Hybrid system for multi-attribute decision making

We consider here the problem of modeling subjective judgment of a single decision maker who may have imperfect knowledge about the decision situation. As it was mentioned above, the multi-attribute decision making process consists of interpretation and reasoning steps. We assume here that the interpretation step is completed and we have already chosen a set of decision alternatives and a set of attributes and defined the expected values of the quantitative attributes and evaluation grades reflecting subjective judgment about the qualitative attributes. Our effort will be concentrated on developing an approach to a computationally effective reasoning process of automated
selection of the most attractive alternative. In our approach, we neither make any assumption about the form of the utility function or explicitly model it. Instead, similar to [10], at the knowledge acquisition stage we model pairwise preferences of the expert with the neural networks. The expert is asked to compare pairs of alternatives and to order them according to his preference. Attributes of these alternatives along with the expert preferences are used to train the neural network. Utilization of pairwise comparisons only instead of comparison of all the alternatives simultaneously reduces the burden put on the human expert during the knowledge acquisition stage since it is easier for him to chose between only two alternatives. It also reduces the number of input nodes for the Neural Network and, therefore, the number of patterns and amount of time required for training of the system. This is especially important in real-life applications when construction of a large training set may be very expensive or impractical. The results of pairwise comparisons are used to compute a belief in the level of preference for each alternative considered for decision making. Utilization of the Dempster-Shafer theory of evidence instead of the heuristic search usually following the result of pairwise comparison of alternatives, allows us to deal with conflicting information that inevitably follows the results of pairwise comparison of alternatives with the NN. The conflict appears due to uncertainty related to imperfect expert knowledge about the value of numeric attributes, subjective judgment characterizing non-numeric attributes, occasional inconsistent judgments of the human expert, and the neural network errors. Instead of choosing the alternative corresponding to the maximum utility, we make our decision based on maximum belief in the level of preference computed with the Dempster combination rule as a function of quantified pairwise preferences.

Since we consider both numeric and non-numeric attributes represented by the expert subjective judgments often evaluated through a number of related factors, a quantification preprocessing for these qualitative attributes may be required.

The designed decision support system consists of the following components:
- a process for quantification of the qualitative attributes
- an NN-based pairwise comparison model
- an evidential decision making process

All the components of the system will be described in detail in next sections. The information flow of the system is presented in Figure 1.

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Informational database for training and testing
Subjective judgments about qualitative attributes
Quantification of qualitative attributes
Quantitative Attributes
Neural Network-based expert pairwise preference modeling
Quantified pairwise preferences
Evidential Reasoning Model

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![Figure 1. Hybrid Decision Support System](image)

### 3. Quantification of the qualitative attributes.

As was mentioned in the previous sections, in our system we consider a situation where a set of attributes $Y$ defining the alternatives comprises two subsets, $Y = Y_1 \cup Y_2$, where the attributes $y_k \in Y_1, \ k = 1, \ldots, K_1$ are numeric and attributes $y_n \in Y_2, \ n = K_1 + 1, \ldots, K$ are non-numeric. One way of incorporating numeric and non-numeric attributes is to quantify the non-numeric attributes. In the simplest situation, the states of the attributes for each particular alternative often
can be represented by evaluation grades assigned by the decision maker. These evaluation grades reflect the decision maker’s subjective judgment about the quality of the state of the attributes, and define the preference degree. It is possible to employ these evaluation grades for numerical representation of the qualitative attributes. In decision making under uncertainty, the expert can assign more then one evaluation grade to the attribute. For example, the attribute can be good with a certain degree of confidence and, at the same time, excellent with a certain degree of confidence. In more realistic cases, the qualitative attributes present aggregated concepts and can be only evaluated through a number of related factors. The expert can assign single or multiple evaluation grades with some level of confidence only to the factors and it may be necessary to combine these levels of confidence in order to numerically assess the qualitative attributes.

The quantification process adopted here utilizes the Dempster Shafer theory of evidence for combination of multiple evaluation grades for multiple factors characterizing qualitative attributes and is similar to the method presented in [13].

Let a set of possible evaluation grades $G = \{g_1, \ldots, g_m\}$ define a frame of discernment $\Theta = \{\theta_1, \ldots, \theta_n\}$ with hypothesis $\theta_i$: the quality of attribute $n$ corresponds to evaluation grade $g_i$. Let $\Phi \in 2^\Theta$, where $2^\Theta$ is a set of all subsets of $\Theta$. Evaluation grades $G$ are sorted in non-decreasing order and $g_1$ and $g_m$ are the worst and the best grades, respectively. We assume also that the number of evaluation grades is the same for all qualitative attributes and that, due to uncertainty, the quality of attributes can correspond to more than one evaluation grade. The objective is to define confidence levels for evaluation grades for the qualitative attribute $y_n$ through subjective judgment of the decision maker about factors $F_n = \{f_n^j\}, j = 1, \ldots, J$ influencing the evaluation of $y_n$ in each alternative. We consider levels of confidence assigned to evaluation grades as weights of evidence in support of hypotheses $\phi_\theta \subset \Theta$. For alternative $A_i$, let $m_{\phi_\theta}(f_n^j(A_i))$ be a basic probability assignment in support of hypothesis $\theta_i$ based on the quality of $f_n^j$ and $c_{\phi_\theta}(f_n^j(A_i))$, a confidence level that the decision maker assigns to hypothesis $\theta_i$. Then we can write: $m_{\phi_\theta}(f_n^j(A_i)) = \alpha_j c_{\phi_\theta}(f_n^j(A_i))$, where $\alpha_j$ is a coefficient defined by the relative importance of the factor $f_n^j$ in evaluation of attribute $y_n$ of alternative $A_i$. Confidence level is defined during the expert knowledge acquisition phase such that $\sum c_{\phi_\theta}(A_i) \leq 1$. Combining basic probability assignment defined for all the factors with the Dempster rule of combination [9], we obtain the evaluation grades for qualitative attribute $A_i$:

$$m_{\phi}(y_n(A_i)) = \bigoplus_{j, \theta_i} m_{\phi_\theta}(f_n^j(A_i)) \text{ for any } \phi \in \Theta.$$  

For simplification of calculations needed for implementation of the Dempster Rule of combination we adopt the “rationality assumptions” [13] that assume that a decision maker supports no more than two consequent grades, for example, bad and very bad or fair and good. Since the basic probability assignments participating in the combinations are not zero only on singletons, the result of the combinations $m_{\phi}(y_n(A_i)) \neq 0$ only if $\phi = \theta_i$ or $\phi = \Theta$.

The existing solution methods for the multi-attribute decision making require a single value assigned for each attribute. In [13], for example, the confidence levels supporting the evaluation grades are converted into a single preference degree as follows:

$$p(y_n(A_i)) = \sum_i (m_{\phi_\theta}(y_n(A_i)) p(\phi_i)$$

$$+ m_{\phi_\theta}(y_n(A_i)) p(\Phi)),$$

where $p(\phi_i)$ is the scale of $\phi_i$ and assumed to be an increasing function defined on $[-1,1]$ with $p(\phi_i) = -1$ and $p(\phi_m) = 1$. However, the form of function $p(\phi_i)$ is arbitrary which may contribute to the overall uncertainty existing in the problem. Utilization of the NN for modeling
pairwise preference of an expert introduced in
our system does not require a single preference
degree assigned to a qualitative attribute and
allows us to avoid additional uncertainty related
to the unknown function $p(\theta)$.  

4. Neural network for representation of pairwise
decision maker preferences

In our decision support system, a NN serves as a
tool for transforming the qualitative preferences
of a decision maker for a pair of alternatives into
numerical values. At the training stage, we
present examples of alternative pairs from the
training set as inputs along with corresponding
expert preferences as outputs. During the training
process the system adjusts itself to respond
correctly to this training set. After completion of
the self-adjusting process, the system will
respond correctly to an unknown input. A
supervised fully connected NN is used for our
system.

Each pair of alternatives used for the NN training
is represented by a $2N$-tuple $T_A, A_i, A_j$, where
$T_A = (T_1, ..., T_N, T_1', ..., T_N')$, $T_i, T_j'$
are attribute vectors of alternatives $A_i$ and $A_j$, respectively. $N$ is the
number of elements in each tuple:

$$ N = K_1 + m \sum_{i=1}^{K_2} J_i $$

where $K_1$ is the number of
quantitative attributes, $K_2$ is the number of factors
characterizing quantitative attribute $i$, and $m$ is the number of evaluation grades.

A set of target outputs for the NN comprises $2$-dimensional binary vectors: $(1,0)$ if alternative
$A_i$ is more preferable for the expert then
alternative $A_j$ and $(0,1)$ otherwise. As the result
of training, the NN weights will adjust
themselves in such way that the NN outputs will
be as close as possible to the respective targets.
Therefore in our decision support system, the
NN represents a transformation function $R(A_i, A_j)$ of qualitative expert preferences on a
pair of alternatives into a two-dimensional vector
$(O_1, O_2)$ with the following decision rule:

- $A_i \succ A_j$ if $O_i > O_j$ and $A_i \succ A_j$, if
- $O_i = O_j$.

Here $A_i \succ A_j$ denotes that alternative $A_i$ is
more preferable than alternative $A_j$ and
$A_i \succ A_j$ denotes no preference.

During the decision making phase, attributes of
each pair of alternatives from a set under
investigation will be presented to the trained NN.
The output vectors $\{(O_1, O_2)\}$ will be
employed to represent a measure of confidence in
the choice of more preferable alternatives. These
measures of confidence will be then combined in
the framework of the Dempster-Shafer theory of
evidence.

5. Evidential decision making process

This section describes a decision making process
that evaluates preference relationships within
pairs of alternatives represented by the NN
outputs and chooses the most preferable
alternative. Generally, if we have the preference
relationships for each pair of alternatives and
these relationships are non-conflicting, we are
able to find the most preferable alternative using
one of the existing methods, such as
mathematical programming or heuristic search.
In our case, when the information presented to
the expert is noisy, incomplete, and contains
qualitative attributes, the set of obtained
preference relationships might be conflicting. In
practice, we also should not expect the expert to
supply consistent preference relationships on
pairs of alternatives during the knowledge
acquisition stage, especially when the number of
attributes and/or the number of required
evaluation grades is high. The NN that produces
these outputs is only a model of these preferences
and it can also incur this conflict. In order to
combine this conflicting information and be able
to make the decision about the best alternative, we employ the Dempster-Shafer theory of evidence which is very efficient when we need to combine conflicting information coming from several sources.

Let us consider a set of \( M \) alternatives \( \mathcal{A} = \{ A_i \} \) and an NN trained to model pairwise expert preferences. At the decision making stage, for all different pairs of alternatives \( \{(A_i, A_j)\} \) we obtain a set of NN output vectors \( \mathcal{O} = \{(O_i, O_j)\} \) with \( |\mathcal{O}| = M(M - 1) / 2 \). Let a set \( \mathcal{O}^k \subset \mathcal{O} \) be a set of outputs containing \( O_k \), where \( |\mathcal{O}^k| = M \).

Let \( \mathcal{X} = \{\xi_1, \ldots, \xi_M\} \) be the frame of discernment, where \( \xi \) represents a hypothesis that the most preferable alternative is \( A_i \). We can also consider an NN output \( (O_i, O_j) \) for a pair of alternatives \( A_i \) and \( A_j \) as independent evidences for hypothesis \( \xi_i \) and \( \xi_j \), where \( O_i \) and \( O_j \) are the values that reflect the measure of belief in hypothesis \( \xi_i \) and \( \xi_j \). For each \( (O_i, O_j) \in \mathcal{O}^k, j = 1, \ldots, M, j \neq k \), we can consider a simple support function \( m_{A_k} = O_k, m_{A_k}(\mathcal{X}) = 1 - O_k \) with focal elements \( \xi_k \). Then a separable support function \( m_{A_k} \) representing a combined belief in \( \xi_k \) based on all pairs from \( \mathcal{O}^k \) is a combination of \( m_{A_k} \):

\[
m_{A_k}(\xi_k) = 1 - \prod_{j=1}^{M} (1 - m_{A_j}(\xi_k)), \text{ if } i = k,
\]

\[
m_{A_j}(\xi_k) = 0, \text{ if } i \neq k,
\]

\[
m_{A_k}(\mathcal{X}) = \prod_{j=1}^{M} (1 - m_{A_j}(\xi_k)).
\]

Combining all the \( m_{A_k} (k = 1, \ldots, M) \), according to the Dempster rule of combination, we can obtain now

\[
m(\xi_k) = \frac{m_{A_k} \prod_{j \neq k} (1 - m_{A_j})}{\sum_{k=1}^{M} m_{A_k} \prod_{j \neq k} (1 - m_{A_j}) + \prod_{j=1}^{M} (1 - m_{A_j})}.
\]

Since \( \xi_k \) is an atomic hypothesis, \( Bel(\xi_k) = m(\xi_k) \) and the most preferable alternative \( A_m \) corresponds to the highest combined belief: \( m(\xi_m) = \max_{1 \leq m \leq M} m(\xi_m) \).

\section{6. The hybrid system for threat prediction: experiments and results}

The introduced hybrid approach to multi-attribute decision making is problem independent and can be used for designing a decision-aid tool in various applications such as study of consumers’ attitudes and preferences, analysis of investment alternatives, situation assessment and prediction, etc. In order to demonstrate viability of the introduced method and its applicability to the Level 2/3 Fusion problems \cite{14}, we conducted a case study where we applied this approach to modeling threat prediction.

Specifically, we model a procedure of selection of the most likely threat direction (decision alternative) by considering the relative level of danger from force aggregates in different sectors of an unclassified North Korean Tactical Scenario developed for the study. For designing a case study, North Korea was divided into three zones with each zone divided into six equal geographical sectors representing a possible threat direction. For training and evaluation of the designed hybrid system we built 17 scenarios for each zone.

The level of danger was based on the analyst’s implicit awareness of Combat Compound Value (CCV) represented an “underlying value of threat” for any of the defined sectors. The CCV for each sector included subjective judgments about terrain and quantitative information regarding each type of intelligence data used in the assessment. Terrain within the CCV represented an aggregated concept and was evaluated through relevant factors (mobility and detectability) that were qualified as POOR, AVERAGE or GOOD with some confidence levels. Qualitative judgments of mobility was based on difficulty in Cross
Country Movement (CCM) over the terrain while qualitative judgments of detectability were made based upon the concealment potential of terrain. Quantitative information was represented by lethality that measured force projection capability based on a total ordering of the in-garrison power projection capability of each type of unit (Armor, Infantry, Artillery, Anti-Air), and the number of units found in a sector.

The informational database of 51 scenarios was used for evaluation of the designed hybrid system. The objective of this evaluation was twofold. First, we intended to demonstrate the ability of the NN to model expert pairwise preferences with both quantitative and qualitative attributes. Second, we intended to show that the combination of the NN outputs with the Dempster rule of combination allows us to model the decision maker choice of the most preferable alternative (the most likely direction of treat in the threat prediction problem).

A fully connected three layer NN with 27 hidden nodes trained with the back propagation algorithm [15] was employed for modeling an expert’s pairwise preferences. The training and testing were performed with the “one-scenario-taken-out” method. For training we used 750 pairs of directions as input patterns along with corresponding expert preferences as outputs, while a test set at each cycle contained 15 directions. Each direction is represented by the number of units to reflect lethality and the subjective judgment of the analyst about terrain characteristics. The training and testing results for modeling the decision maker preferences between two alternatives are shown in Table 1.

The NN output vectors corresponding to the results of pairwise comparison of directions from one scenario were combined following the evidential routine introduced in Section 5. The result of combination was tested against the analyst choice of the most likely direction of threat. The accuracy of prediction of the most likely direction of threat by the decision support system is shown in Table 2.

<table>
<thead>
<tr>
<th>Prediction accuracy in modeling the most likely direction of threat</th>
<th>First choice</th>
<th>First &amp; second choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy of pairwise preferences prediction</td>
<td>84.1%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The results of our experiments with simulated data demonstrated a high degree of agreement between the system and the decision maker. Consideration of two choices (the best and the second best) allows us to further improve the system accuracy while introduction of the second choice does not degrade the utility of the system since it is still easier for experts to make a choice between two alternatives. More experiments with larger databases and more realistic scenarios are required in order to make final conclusions about performance of the system.

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8. References


