Comparison of two Integration Methods of Contextual Information in Pixel Fusion

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Abstract: Pixel fusion is used to elaborate a classification method at pixel level and optimize target detection. It must take into account the more accurate as possible information and take advantage of the statistical learning of the previous measurements acquired by sensors. The classical probabilistic fusion methods lack of performance when the previous learning is not representative of the real sensors measurements. The Dempster-Shafer theory is then introduced to face this disadvantage by integrating a further information which is the context of the sensors acquisitions. In this paper, we propose a formalism of the sensor reliability modelization that leads to two methods of integration when all the hypotheses, associated to objects of the scene acquired by sensors, are previously learnt. Afterwards, we are interested in the evolvement of these two methods in the case where the previous learning is unavailable for an object of the scene and a global method of contextual information integration can be deduced.

Keywords: pixel fusion, Dempster-Shafer theory, contextual information, degree of trust, mass set.

1. Introduction

During these last years, the number of image sensors has drastically increased. Then a large set of images simultaneously acquired on the same landscape but in different spectral bands is often available. As the information associated to an object depends on the spectral band, the multi-sensors data fusion aims at combining the information from the different spectral bands in order to significantly increase scene perception. Our pixel fusion method is used to elaborate a new classification method at pixel level and also to optimize target detection. It must take into account the more accurate as possible information and take advantage of the statistical learning of previous measurements acquired by sensors. The classical probabilistic fusion methods lack of performance when the previous learning is not representative of the real sensors measurements due to varying environmental conditions for example.

Consequently, we propose a fusion method based on the Dempster-Shafer theory which allows to easily integrate the context of the sensors measurements in order to take the more accurate as possible information.

The fusion methods need the determination of an “a priori” database made up of probability density laws defined for a given context [1][2]. The acquired measurements are related to the surface properties and the context. Furthermore the probability density laws of the acquired measurements can be different from the laws that are previously learnt to construct the database. Some disturbing parameters must be considered in order to justify this difference. These parameters are either atmospheric disturbances or surface variations (as temporal evolution) [3][4]. These disturbing contributions define the context and are called contextual variables.

The sensor reliability to the context depends on the contextual variables values and must be considered by fusion method [5][6]. We propose a formalism modelization of sensor reliability to the context that leads to two methods of integration when all the hypotheses, associated to objects of the scene acquired by sensors, are previously learnt: the first one amounts to integrate this further information in the fusion rule as degrees of trust and the second models it as mass set (§ 2). These two methods are based on the theory of fuzzy events.

Afterwards, we are interested in the evolvement of these two methods in the case where the previous learning is unavailable for a hypothesis associated to an object of the scene and compare these two methods in order to deduce a global method of contextual information integration in the fusion process (§ 3).

2. Fusion and contextual information modelization methods

The multi-sensors system is composed of $M$ sensors $S_j$ ($j=1,...,M$) that provide measurements $L_j$. This system is used to recognize an object among $N$ ones. An exclusive hypothesis $H_i$ is associated to every object $i$. 
The frame of discernment $E$ is defined by the hypotheses $H_i : E = \{ H_1, H_2, ..., H_N \}$. $H_i$ is the complement of $H_i$ in $E$.

Some disturbing parameters must be considered in order to evaluate the sensor reliability. These parameters define the context and are called contextual variables. A particular context $z = \{ z_1, z_2, ..., z_P \}$ is then defined by $P$ contextual variable values. Moreover, the vector $z^m$ represents the context measurements : $z^m = \{ z_1^m, z_2^m, ..., z_P^m \}$.

The fusion method needs the construction of an “a priori” database made up of the probability density laws defined for a given context. The “a priori” probability density $p(L_j / H_i, z^j)$ of every sensor $S_j$ under hypothesis $H_i$ is previously modeled for the standard contextual variable values $z^j = \{ z_1^j, z_2^j, ..., z_P^j \}$. These variables allow us to consider all reliable sensors measurements. We suppose that the “a priori” probability densities are known for all the hypotheses of the frame of discernment.

The fusion method is based on the Dempster-Shafer theory which allows an easier integration of the context in the decision rule formalism (§ 2.3.2 and 2.3.3). The construction of the basis mass sets $m_{ij}(\cdot)$, each representative of the evidence assigned to the frame discernment $E$, thanks to the measurements provided by the sensor $S_j$ and the learning on $H_i$, is based on the works led by Appriou [1][2] (§ 2.3.1). These mass sets will be considered afterwards as elementary sources.

The contextual information is modeled in the form of mass sets representative of the sensors availability for the considered context and can be introduced at two levels:

- At the elementary level of each sources : the mass set, noticed $m_{ij}(\cdot)$, is equivalent to degrees of trust $d_{ij}$ introduced as weakening factors [1][2][7]. It takes into account the validity of each separate sources.
- At the global level of the association of many sources : the mass set $m_{C}(\cdot)$ amounts to introduce the competitively validity of all the possible associations of sources.

The estimation of these mass sets is based on the fuzzy sets theory.

We propose an original combination rule called CC (Contextual Combination) rule that allows to combine the contextual information with the “a priori” information (§ 2.2). It can be applied to two different levels : before or after fusion operation.

Therefore two different methods of fusion and contextual information integration are proposed and each one uses an unlike representation of the contextual information. The first one uses the set $m_{C}(\cdot)$ and is called CDT (Contextual Degree of Trust) method (§ 2.4). It introduces the CC rule before the fusion operation. The second method uses the set $m_{C}(\cdot)$ and consists in applying the CC rule after the fusion process (§ 2.5). This method is called CMS (Contextual Mass Set) method.

### 2.1. Notations and definitions

The notations and definitions, used in the next paragraphs, are the following:

- The $P$-dimensional space where the context is represented is called $Z$.
- $C_{ij}$ represents the inclusive validity domain or fuzzy subset of contexts for which the assessment of the hypothesis $H_i$ provided by the sensor $S_j$ is valid ($C_{ij} \subset Z$), without knowledge on the validity of any other sensor for any hypothesis and the validity of the sensor $S_j$ for all the hypotheses $H_k$ different from $H_i$ (Figure 1).
- The index $V$ represents a subset of indexes $\{ ij \}$ included in the set stemming from the Cartesian product $\{ 1, ..., N \} \times \{ 1, ..., M \}$ : $V \subseteq \{ 1, ..., N \} \times \{ 1, ..., M \}$
- $c_V$ is the exclusive validity domain or subset of contexts for which every sensor $S_j$ of the association set represented by $V$ is valid for the discrimination of the hypothesis $H_i$ ($\{ ij \} \in V$), and all the others associations of sensor and hypothesis no represented in $V$ are excluded :

$$c_V = \bigcap_{ij \in V} \bigcap_{ij \notin V} C_{ij} \quad \text{ (2.1)}$$

$$\text{with } V \subseteq \{ 1, ..., N \} \times \{ 1, ..., M \}$$

and

$$c_{\bar{V}} = \bigcap_{ij \notin V} \bigcap_{ij \in V} C_{ij} \quad \text{ (2.2)}$$

The Figure 1 illustrates the notions of exclusive and inclusive validity domains for the case of two sensors and two hypotheses. The validity domain $C_{11}$ is the subset of contexts for which the sensor $S_1$ allows to discriminate the hypothesis $H_1$, without knowledge on the validity of any other sensor for any hypothesis : $S_2$ and $H_2$, $S_2$ and $H_2$, and the validity of $S_1$ for $H_2$. The validity domain $C_{11}$ is the subset of contexts where the only valid association corresponds to the hypothesis $H_i$ and the sensor $S_j$.

According to the equation (2.1), this domain is expressed as :

$$c_{\{ 1 \}} = C_{11} \cap \overline{C}_{12} \cap \overline{C}_{21} \cap \overline{C}_{22}$$
2.2. CC rule

We aim at finding a global mass set \( m(.) \) on the frame of discernment \( E \), set of the hypotheses of discrimination \( H_i \) \((i=1,\ldots, N)\). This global set \( m(.) \) is obtained according to the mass sets \( m_{ij}(.) \) provided on \( E \) by sources or combination of sources represented by \( V \) and a bayesian mass set \( m_C(.) \) on \( E_{SH} = \{ c \} \) by using the suitable operations of conditioning, coarsening and refining [7][9].

The term \( c \) represents :
- The inclusive validity domain, with \( V = \{ ij \} \) and \( \{ ij \} \in \{1,\ldots, N\} \times \{1,\ldots, M\} \),
- The exclusive validity domain, with \( V \subseteq \{1,\ldots, N\} \times \{1,\ldots, M\} \).

According to the demonstration led by Fabre [7], the global mass set \( m(.) \) is explained as :

\[
m(A) = m_c(\emptyset \emptyset) \cdot m_{\emptyset \emptyset}(A) + \sum_{V} m_c(\cap c) \cdot m_{V}(A) \tag{2.3}
\]

2.3. Fusion method and decision rule

2.3.1. Expression of the basic mass set

Appriou suggests an approach that consists in introducing each "a priori" probability density \( p(L_j / H_i, z^S) \) among an appropriate mass set \( m_{ij}^b(.) \) [1][2]. Two models of mass set are then defined by an axiomatic approach on the frame of discernment \( \{ H_i, \bar{H}_i, E \} \). We select among these models the less specific one. This mass set, called "basic mass set", is explained by considering all the degrees of trust \( d_{ij} \) equal to 1 [1][2] :

\[
m_{ij}^b(H_i) = 0 \tag{2.4}
m_{ij}^b(\bar{H}_i) = 1 - R_j \cdot p(L_j / H_i, z^S) \tag{2.5}
m_{ij}^b(E) = R_j \cdot p(L_j / H_i, z^S)
\]

where the term \( R_j \) represents a normalization factor defined as :

\[
R_j \in \left[ 0, \max_{i \in [1,N]} \left[ \sup_{L_j} \left( p(L_j / H_i, z^S) \right) \right]^{-1} \right] \tag{2.5}
\]

2.3.2. Fusion rule

The global mass set \( m(.) \) results from the combination of the \( M \) mass sets \( m_j(.) \) associated to the sensor \( S_j \). The combination rule is the orthogonal Dempster-Shafer’s rule [9] :

\[
m(.) = \bigoplus_{j=1}^{M} m_j(.) \tag{2.6}
\]

The mass set \( m(.) \) is obtained by the combination of the \( N \) elementary mass sets \( m_{ij}(.) \):

\[
m_j(.) = \bigoplus_{i=1}^{N} m_{ij}(.) \tag{2.7}
\]

The mass set \( m(.) \) represents the "a priori" information, modeled by the basis mass set \( m_{ij}^b(.) \), and the contextual information at once. When the contextual information is unavailable, the mass set \( m_{ij}(.) \) is similar to the set \( m_{ij}^b(.) \). The focal elements associated to \( m_{ij}(.) \) are \( H_i, \bar{H}_i \) and \( E \).

2.3.3. Decision rule

The objective is to choose one decision \( d_i \) among a finite set \( D \) of \( Q \) possible decisions owing to the assessment provided by the mass set \( m(.) \) on \( E \). The decision \( d_i \) corresponds to the assignment of the observation \( L \) to the set \( F_i \), made up by one or many hypotheses of \( E \). The choice of taking a decision \( d_i \), when the observation \( L \) belongs to \( H_k \), generates a cost \( \lambda(d_i / H_k) \).

The more consensual decision is provided by minimizing the risk function \( R(d_i / L) \) on the set of all the possible decisions [8] :

\[
R(d_i / L) = \sum_{B \in E} m(B) \cdot \min_{H_k \in B} \left\{ \lambda(d_i / H_k) \right\} \tag{2.8}
\]
The costs \( \lambda(d_i / H_k) \) are defined for \( i \in [1, Q] \) and \( k \in [1, N] \) by considering the two following propositions:

- The cost to declare that the observation \( \bar{L} \) is assigned to the set \( F_i \) of hypotheses associated to \( d_i \) when it really belongs to the class \( H_k \), is maximum:
  \[
  \lambda(d_i / H_k) = 1 \quad \text{if} \quad H_k \notin F_i \quad (2.9)
  \]
- The cost to make a good decision is such that:
  \[
  \lambda(d_i / H_k) = \lambda_i \quad \text{if} \quad H_k \in F_i \quad (2.10)
  \]

By integrating the expressions (2.9), (2.10) in the relation (2.8) and using the definition of the plausibility [9], the risk function becomes:

\[
R(d_i / L) = 1 - \left[1 - \lambda_i \right] \cdot \text{PL}(H_k)_{H_k \in F_i} \quad (2.11)
\]

Consequently, the decision rule is obtained by minimizing the risk function (Equation (2.11)) and can be explained as follows:

\[
\max_{i \in [1, Q]} \left\{ \left[1 - \lambda_i \right] \cdot \text{PL}(H_k)_{H_k \in F_i} \right\} \quad (2.12)
\]

In this particular approach, only singletons of hypothesis are taken into account. Therefore, the set \( D \) is composed of as many decisions \( d_i \) as hypotheses \( H_i \) in the frame of discernment \( E \). Moreover, we consider that the cost \( \lambda_i \) for a good classification is equal to 0. Consequently the most likely hypothesis has to justify a maximum plausibility criterion. The decision rule (Equation (2.12)) is rewritten as:

\[
\max_{i \in [1, Q]} \{ \text{PL}(H_i) \} \quad (2.13)
\]

This decision rule is coherent with the criterion introduced in the works led by Appriou [1][2].

As the hypotheses \( H_i \) are singletons of \( E \), the expressions of plausibility and communality are the same [1][2][9]. Consequently, the plausibility is explained as follows:

\[
\text{PL}(H_i) = \prod_{j \in [1, M]} \{ \text{PL}_j(H_i) \} \quad (2.14)
\]

By using the plausibility definition [9], the mass sets \( m_{ij}(.) \) (Equation (2.7)) and \( m_{ij}(.) \) (Equation (2.4)), the plausibility \( \text{PL}_j(.) \) can be explained as follows:

\[
\text{PL}_j(H_i) = K_{ij} \cdot \frac{m_{ij}(H_i) + m_{ij}(E)}{m_{ij}(H_i) + m_{ij}(E)} \quad (2.15)
\]

The term \( K_{ij} \) is a normalization factor independent of the hypothesis \( H_i \).

### 2.4. CDT Method

In this case, the sensor reliability is represented by the mass sets \( m_{ij}(.) \) (§ 2.4.1). These mass sets are combined by the CC rule with the mass sets, associated to the previous learning and obtained owing to the basis mass set \( m_{ij}(.) \), in order to obtain the elementary mass set \( m_{ij}(.) \) (§ 2.4.3). The operation of fusion is then applied on these elementary mass sets in order to obtain a global mass set \( m(.) \) introduced in the decision rule (§ 2.4.4). The architecture of the CDT method is described on Figure 2.

![Figure 2: CDT method architecture](image)

#### 2.4.1. Contextual information representation

The reliability of the source \( \{ij\} \), defined by the association of the sensor \( S_j \) and the hypothesis \( H_i \), to the context is represented by a bayesian mass set \( m_{ij}(.) \) established on the frame of discernment \( E_{Cij} = \{C_{ij}, \bar{C}_{ij}\} \). The set \( C_{ij} \) is defined in § 2.1.

The estimation of the mass set \( m_{ij}(.) \) is performed in several stages:

- **Stage 1**: The probability of the context

Let \( z = \{z_1, \ldots, z_P\} \) be as a random vector of probability density \( p(z|z^m) \) where \( z^m = \{z_1^m, \ldots, z_P^m\} \) is the vector
associated to the variable \( z \) measurements. We have to model this probability density law \( p(z|z^n) \).

- **Stage 2**: Validity domain of every sensor
  The fuzzy sets theory is used to define the validity domain of every sensor.
  \( \mu_{ij} (z_a) \) is the elementary fuzzy membership function associated to the contextual variable \( z_a \), the sensors \( S_j \) and the hypothesis \( H_i \).
  The fuzzy membership function \( \mu_i (z) \) characterizes the sensor availability for the context \( z \) and is expressed with the elementary fuzzy functions:
  \[
  \mu_{ij} (z) = \bigwedge_{u=1}^{P} \mu_{iju} (z_u)
  \]
  \[
  = \mu_{ij1} (z_1) \mu_{ij2} (z_2) ... \mu_{ijP} (z_P)
  \]
  The operator \( \bigwedge \) represents the operator of minimum conjunction [5][6].

- **Stage 3**: Probability of sensor validity
  \( P(S_j / H_i, z^n) \) is the probability that the sensor \( S_j \) is reliable according to the context value \( z^n \) and the hypothesis \( H_i \).
  This probability is explained by using the definition of the probability measures of fuzzy events [10][11]:
  \[
  P(S_j / H_i, z^n) = \int \mu_{ij} (z) \cdot p(z|z^n) \cdot dz
  \]
  When the variable value is certain, the probability density \( p(z|z^n) \) is replaced by the Dirac function \( \delta(z-z^n) \) and the equation (2.17) becomes:
  \[
  P(S_j / H_i, z^n) = \mu_{ij} (z^n)
  \]

- **Stage 4**: Mass set \( m_{Cij} (.) \)
  The probability (2.17) can be explained as a bayesian mass set \( m_{Cij} (.) \) such that:
  \[
  m_{Cij} (C_{ij}) = P(S_j / H_i, z^n)
  \]
  \[
  m_{Cij} (C_{ij}) = 1 - P(S_j / H_i, z^n)
  \]
  \[
  m_{Cij} (C_{ij} \cup \overline{C}_{ij}) = 0
  \]

2.4.2. “A priori” information representation

Two mass sets \( m_{ij}^w(.) \) \((w = 1,2)\) are introduced to model the “a priori” information: one mass set uses the measurements as if there were completely reliable \((w = 1)\) and the other is representative of the total uncertainty \((w = 2)\). These mass sets are defined \( \{H_i, \overline{H}_i, E\} \) owing to the basis mass set \( m_{ij}^b(.) \) (Equation (2.4)):

\[
 m_{ij}^1(.) = m_{ij}^b(.)
\]
\[
 m_{ij}^2(E) = 1
\]

2.4.3. Combination of the mass sets

The expression of the elementary mass sets \( m_{ij}(.) \) is provided by applying the CC rule (Equation (2.3)) on the mass sets \( m_{ij}^w(.) \) \((w = 1,2)\) and on the bayesian mass set \( m_{ij}^b(.) = m_{ij} (.) \).

This expression \( m_{ij}(.) \) is the similar to the one resulting from a weakening operation applied on the basis mass set \( m_{ij}^b(.) \) in the case where the weakening factor \( d_{ij} \) is such that:

\[
 d_{ij} = P(S_j / H_i, z^n)
\]

The CDT method is then the same as the method improved by Appriou based on the introduction of degrees of trust as weakening terms [11][2][9]. Consequently, the elementary mass set \( m_{ij}(.) \) can be explained as:

\[
 m_{ij} (H_i) = 0
\]
\[
 m_{ij} (\overline{H}_i) = d_{ij} \left[ 1 - R_j \cdot p(L_j / H_i, z^n) \right]
\]
\[
 m_{ij} (E) = 1 - d_{ij} \cdot R_j \cdot p(L_j / H_i, z^n)
\]

Notes: When the degrees of trust \( d_{ij} \) are equal to 1, the Dempster-Shafer theory represents the probabilistic approach of the maximum likelihood which supposes that the probability density \( p(L_j / H_i, z^n) \) is perfectly representative of real probability density.

2.4.4. Fusion and decision rule

The elementary mass sets \( m_{ij}(.) \) are fused according to the fusion rule (Equations (2.6) and (2.7)) in order to deduce a global mass set \( m(.) \).

By using the equations (2.14), (2.15) and (2.22), the decision rule (2.13) becomes:

\[
 \max_{i \in [1, N]} \left\{ \prod_{j=1}^{M} [1 - d_{ij} + d_{ij} \cdot R_j \cdot p(L_j / H_i, z^n)] \right\}
\]

2.5. CMS Method

In a first time, the expression of the mass set \( m_{ij}(.) \) related to the contextual information is established. In a second time, the mass set \( m(.) \), representative of the weight of evidence assigned to an association of sources and obtained by the fusion of these sources, is explained. Lastly, the combination of these mass sets
Stage 3 : Expression of the mass set \( m_c(.) \)
The probability of validity of one or many sources associations (Equation (2.24)) is used to explain the exclusive probability of validity of one or many sources associations \( P(c_V) \) [5][6]. \( P(c_V) \) can be explained as a bayesian mass set \( m_c(.) \) constructed on the set \( \{c_V\} (V \subseteq \{1, \ldots, N\} \times \{1, \ldots, M\}) \).

Then, the mass set expression \( m_c(.) \) may be defined as follows [5][6] :

\[
m_C(c_V) = P(c_V) = \sum_{W \subseteq V} \prod_{ij \in W} C_{ij} \cdot P(z^{m_i})
\]

\( W-V \) represents the cardinal of the subset \( W-V \).

2.5.2. "A priori" information representation

A mass set \( m(.) \) is constructed on the frame of discernment \( E \) and supposed that all the associations of sources, represented by the subset \( V \) of indexes \( \{ij\} \), are valid. These mass sets result from the orthogonal sum of the basis mass \( m_{b_{ij}}(.) \) (Equation (2.4)) where \( \{ij\} \in V \):

\[
m_V(.) = \bigoplus_{V \subseteq \{1, \ldots, N\} \times \{1, \ldots, M\}} m_{b_{ij}}(.)
\]

2.5.3. Expression of the global mass set

In this case, the CC rule (Equation (2.3)) is applied on the mass sets \( m_V(.) \) on \( E \) and the bayesian mass set \( m_c(.) \) on \( \{c_V\} \) in order to obtain the global mass set \( m(.) \) on \( E \). The global mass set \( m(.) \) is explained as :

\[
m(A) = m_c(c_V) \cdot m_{\emptyset}(A) + \sum_{V \subseteq \{1, \ldots, N\} \times \{1, \ldots, M\}} m_{c_V}(c_V) \cdot m_V(A)
\]

It is to note that when only one association of sensor \( S_j \) and hypothesis \( H_i \) is considered, the probability \( P(C_j / z^m) \) is then explained as the probability \( P(S_j / H_i, z^m) \) (Equation (2.17)).
3. Evolution of the CDT and CMS methods for a non-learnt class

The principle is the same as the one described in the § 2. The only change arises from the classes used to construct the frame of discernment. In fact, we admit the existence of a further hypothesis for which the previous learning is not available. Consequently, the "a priori" probability associated to this further hypothesis is unknown. The introduction of this further class is quite legitimate. In fact, a class like backgrounds can be made up of several objects and the previous learning of one or many objects of this class can be unavailable. Consequently, the initial class is divided into two classes: the first one amalgamates the objects for which the previous learning is known and the second one regroups all the objects for which the "a priori" probability is unavailable.

The new frame of discernment $E^i$ consists then of the $N$ hypotheses of the frame $E$ and a further hypothesis $H_{N+1}$ allocated to the non-learnt class: $E^i = \{H_1, ..., H_{N+1}\}$. The frame $E^i$ is then refined in comparison with the frame $E$.

The expression of the basis mass sets related to the "a priori" information is established according to the work led in the § 2.3.1. For the hypothesis $H_{N+1}$ associated to the non-learnt class, the corresponding mass sets are constructed by considering the fact that no information is available. Consequently, all the mass of evidence is assigned to $E^i$.

The fusion rule is based on the orthogonal combination rule of Dempster-Shafer introduced in the § 2.3.2. Moreover, the decision rule is the same as the one introduced in the § 2.3.3 (Equation (2.12)). In this case, it is benefit to choose the costs $\lambda_i$ associated to a good decision different from 0, contrary to the values introduced in the § 2.3.2, in order to integrate the fact that no information is available on $H_{N+1}$ and only on it.

In the case where a non-learnt class is added to the frame of discernment, the CDT and CMS methods become respectively the refined CDT and CMS methods. We have shown that the refined CDT and CMS methods may be considered as two implementations of the same global method called "Global Refined Method" [7].

In this general method, the contextual information is taken into account as a mass set in order to realize a fusion process based on the CC rule. This mass can be explained by two different ways:

- The mass set $m_c^d(.)$ depends on the probabilities of validity of each source and is explained owing to degrees of trust on the frame of discernment $\{c_v\}$ [7]:

$$m_c^d(c_V) = \prod_{\{i\} \in V} d_{ij} \cdot \prod_{i \in V} (1 - d_{ij}) \quad (3.1)$$

with $V \subseteq \{1, ..., M\} \times \{1, ..., N+1\}$

$$m_c^d(c_G) = \prod_{\{i\} \not\in \{1, ..., M\}} (1 - d_{ij})$$

- The mass set $m_c(.)$ is directly explained by the probabilities of validity of the different combinations of sources. The construction of this set is inspired by the process described in the § 2.5.1.

These mass sets are combined with the mass sets $m_i(.)$ stemming from the fusion of the basis mass sets $m_i(.)$, according to the CC rule (Equation (2.3)). The global mass set deduced from this operation is introduced in the expression of the plausibility in order to explain the decision rule.

We deal with the problem of one non-learnt class only. However the generalization to the case where several classes are not previously learnt is evident.

4. Conclusions

Pixel fusion aims at combining the images from several sensors in order to increase scene perception. The Dempster-Shafer’s theory is used to realize pixel fusion and needs a minimum of “a priori” knowledge like previous learning of measurements provided by sensors.

Moreover, the Dempster-Shafer’s theory allows to integrate further information such as the sensors reliability to the context. Their reliability depends on the context modeled by the contextual variables.

In the case where all the hypotheses introduced in the frame of discernment are learnt, two methods, that integrate the sensors reliability at different levels, are developed: the CDT and CMS methods.

In the CDT method, the mass sets stemming from contextual information and previous learning are combined before the fusion operation. Practically, this leads to elaborate a degree of trust assigned to each source corresponding to the association of a sensor and a hypothesis.

In the CMS method, a mass set integrates the validity of the different associations of these sources. Consequently in the later, the sensor combination uses all the possible associations of one (or many) sensor(s) and hypothesis. The combination of mass sets representative of "a priori" and contextual information is realized after the fusion operation. For these two methods, the ponderation terms are constructed owing to the theory of fuzzy events. These methods are...
described in the case where the validity domain of each sensor depends on the hypothesis. However, these methods are still valid in the case where the validity domain of each sensor is not related to the hypothesis and depends only on the sensor properties since that the combination is applied to the mass sets associated to sensors.

In the case where a non-learnt hypothesis is added to the frame of discernment, the two methods of contextual information integration and fusion evolve. By comparison of these two refined methods, we deduce a global fusion method based on the integration of sensors reliability as a mass set where this one can be explained by two different ways. For the first method, the mass set related to the sensors reliability is expressed as a function of degrees of trust. The global refined method can be only used in the case where the validity domain of each sensor depend on the hypothesis.

The CDT and CMS methods have been successfully implemented for several typical cases and have provided encouraging results. In the next future, we will work in order to compare the CDT and CMS methods and define their respective validity domains. Moreover, we will compare the two ways of mass set expression, used by the global refined method, in order to obtain their respective validity domains. This global method can be extended to other expressions of the mass set that is representative of sensors reliability. In particular, the degrees of trust can be computed by different ways.

5. References