Daylight estimation in a faulty light sensor system for lighting control

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Abstract—We consider the problem of daylight estimation in a daylight-adaptive lighting system. In such a lighting system, the illuminance levels of light sources are adapted to changing daylight such that a desired illuminance distribution is achieved at the workspace plane. Each light source has an associated light sensor at the ceiling that measures the net illuminance due to contributing light sources and daylight, for a given sensor opening angle. We present a daylight estimation method under the scenario of a few light sensor failures, based on fusion of illuminance measurements at functioning light sensors. The proposed fusion method is used to estimate daylight illuminance values at the zones corresponding to failed light sensors and is a solution of a quadratic programming optimization problem. The efficacy of the solution is shown using photometric numerical data.

Index Terms—Illuminance estimation, light sensor fusion, interpolation, lighting control

I. INTRODUCTION

Design of daylight-adaptive lighting systems has received recent attention, given their potential energy savings [2], [4], [6], [7]. In such systems, the amount of artificial light is controlled in accordance with daylight illuminance so as to provide the desired illuminance. Feedback on the achieved illuminance is provided by light sensors [2]. In this paper, we consider a fusion method for daylight estimation in a daylight-adaptive lighting control system given that some light sensors are faulty.

We consider a centralized lighting control system with a light sensor embedded in each light source. The control objective of the lighting system is to provide a desired illuminance spatial distribution at the workspace plane, while adapting to daylight. The workspace plane is a horizontal plane at a certain height from the ceiling plane in which the light sources and light sensors are located. Each light sensor measures the total illuminance value, which is the net illuminance contribution due to daylight and artificial light, within the region covered by its opening angle. The desired illuminance distribution at the workspace plane is specified in terms of target illuminance values at the light sensors. This is done using a dark-calibration step in absence of daylight. The light sources are driven to a reference dimming level that results in a reference illuminance distribution at the workspace plane and reference illuminance values at the light sensors. Also, in this step, the mapping relating light source output at light sensor positions is obtained. Note that the mapping of light source output to corresponding average illuminance values in zones is not known; also unknown is the mapping from illuminance values in zones to light sensor positions.

Each light sensor sends its illuminance measurements to the central controller. Based on light sensor measurements and target illuminance values, the controller computes the dimming levels of each light source such that the desired illuminance distribution is attained under specific illuminance rendering constraints.

An approach to adapting the lighting system to daylight is that light sensors are used to estimate daylight contribution at the workspace plane. The central controller then adapts dimming values based on this estimated information. In this paper, we consider the case that some light sensors fail. We propose an interpolation method to estimate daylight illuminance values at zones corresponding to such sensor/light source positions using illuminance values at functional light sensors. The estimated illuminance values are determined as a solution of a quadratic programming problem. We evaluate the proposed method using numerical data from a lighting system in an open office space. The proposed method is shown to result in daylight illuminance values closer to actual levels compared to a reference method that interpolates by taking an average of neighborhood illuminance values.

The problem considered herein may be seen as a field estimation problem [5], [8], [9], [10], [11], [12]. The problem of light field sampling using a distributed sensor system was considered in [3]. In [5], the accuracy of field estimation using a wireless network of energy-constrained sensors was studied. A mobile sensing strategy was advocated in [8] for sampling and reconstruction of spatial fields. The field estimation problem was dealt with in [9] using a finite element method based physical model and sensor measurements. Kalman filtering methods for decentralized acoustic field estimation were presented in [10]. Graph-theoretic techniques were used to model spatial correlations of a field and Kalman-Bucy filtering was used to estimate values at certain positions of interest [12]. These methods require knowledge of the field spatial and temporal correlations. In the problem considered herein, the spatial and temporal correlations of the illuminance field are not readily available or simple to obtain. Furthermore, the illuminance measurements are at the light sensor in the ceiling and not over the workspace plane of interest.
II. LIGHTING SYSTEM DESCRIPTION AND PROBLEM SETUP

We consider a lighting system in an indoor office as depicted in Fig. 1, with \( P \) light sources each embedded with a light sensor. The light sensor has a limited sensing area defined by its opening angle. Parallel to the ceiling is the workspace plane where the spatial illuminance distribution is of interest. The workspace plane is assumed to be divided into \( P \) logical zones, as shown in Fig. 1. Denote \( \mathbf{d} = [d_1, d_2, \ldots, d_P] \) to be the dimming vector containing dimming levels, \( d_n (0 \leq d_n \leq 1) \), of the \( n \)-th light source.

A. Illuminance at workspace plane

The average net illuminance \( w_p \) at the \( p \)-th zone in the workspace plane, given that the lighting system is at dimming vector \( \mathbf{d} \), may be written as

\[
w_p = v_p(d) + u_p
\]

where \( v_p(d) = \sum_{n=1}^{P} H_{p,n} d_n \) and \( u_p \) are the illuminance contributions due to lighting system and daylight at the \( p \)-th zone, respectively. Here, \( H_{p,n} \geq 0 \) is the (unknown) illuminance contribution to the average on the \( p \)-th zone when the \( n \)-th light source is at maximum intensity. Denote \( \mathbf{H} \) to be a matrix whose \((p,n)\)-th element is \( H_{p,n} \). We consider a centralized lighting control algorithm that renders a desired illuminance distribution using knowledge of the daylight contribution at each zone, \( u_p, p = 1, \ldots, P \) [6]. Illuminance values at the workspace place however cannot be measured; only illuminance measurements at light sensors are available.

B. Illuminance at light sensor

The measured illuminance at a light sensor at the ceiling is the net illuminance due to contributing light sources and daylight reflected from the objects (e.g. furniture) in the office. Denote \( E_{p,n}(d_n) \) to be the measured illuminance at the \( p \)-th light sensor when the \( n \)-th luminaire is at dimming level \( d_n \), in the absence of daylight. We assume that the illuminance scales linearly with the dimming level,

\[
E_{p,n}(d_n) = d_n E_{p,n}(1).
\]

This assumption holds well for practical light sources, e.g. LED luminaires. For notational convenience, we use \( E_{p,n} \) instead of \( E_{p,n}(1) \) hereon.

The net illuminance at the \( p \)-th sensor at the ceiling, given that the lighting system is at dimming vector \( \mathbf{d} \) and under daylight, can then be written as

\[
i_p(d) = l_p(d) + s_p,
\]

where \( l_p(d) = \sum_{n=1}^{P} d_n E_{p,n} \) is the illuminance due to the lighting system and \( s_p \) is the illuminance due to daylight measured at the \( p \)-th sensor. In practice, the mappings \( E_{p,n} \) may be computed a priori in a calibration phase by turning on the light sources one at a time and measuring illuminance values at the light sensors.

Hence, we can obtain the current daylight contribution at the \( p \)-th light sensor as

\[
s_p = i_p(d) - \sum_{n=1}^{P} d_n E_{p,n}
\]

where \( d_n \) is the current dimming level at the \( n \)-th light source and \( i_p(d) \) is the current illuminance measurement at the \( p \)-th light sensor.

We know the illuminance measurements at the light sensors but not the corresponding average illuminance at the workspace plane. We rewrite (3) in vector form, and in terms of illuminance values at the workspace plane, as

\[
i(d) = l(d) + s
\]

\[
= Gv(d) + Gu
\]

where

\[
s = Gu
\]
and

$$
G = \begin{bmatrix}
G_{1,1} & \cdots & G_{1,p} \\
\vdots & \ddots & \vdots \\
G_{p,1} & \cdots & G_{p,p}
\end{bmatrix},
$$

$$
i(d) = [i_1(d), \ldots, i_p(d)]^T,
$$

$$
l(d) = [l_1(d), \ldots, l_p(d)]^T,
$$

$$
s = [s_1, \ldots, s_p]^T,
$$

$$
v(d) = [v_1(d), \ldots, v_p(d)]^T,
$$

$$
u = [u_1, \ldots, u_p]^T.
$$

Here, $G_{m,p} \geq 0$ is the illumination contribution at the $m$-th light sensor when the average illumination at the $p$-th zone due to the lighting system is at the maximum. Note that in (7), we assume that the relation between the average illumination value over the $p$-th zone and the illumination value at the $m$-th light sensor is the same for daylight and artificial lighting. This, in practice, is an approximation. If a sophisticated daylight illumination model is available, then we would have $s = G \mathbf{u}$, where $G$ is the matrix relating the average illumination over zones to the illumination values at the light sensors due to daylight. For simplicity, in this paper, we assume $G = G$ and use the relation (7).

C. Calibration phase

A calibration phase is performed when there is no daylight and the lighting system is set to reference dimming vector $d^* = [d_1^*, \ldots, d_p^*]^T$. Hence, we know that during the calibration phase,

$$
l(d^*) = Gv(d^*).
$$

From (8), we have that when the illumination measurement vector at the light sensors is $l(d^*)$, then the vector of average illumination values in the workspace plane is $v(d^*)$.

The transfer matrix $E = GH$, with $(m,n)$-th element $E_{m,n}$, is also determined in this phase, as discussed in Section II-B, by turning on individual light sources one at a time at maximum dimming and making illumination measurements at the light sensors.

III. PROBLEM FORMULATION

We consider the scenario when a set of light sensors stop functioning, i.e. we only have the set of illumination measurements $\{i_m(d), m \in \mathcal{M}\}$ where $\mathcal{M}$ is the set of indices of functioning light sensors. Our objective is to estimate the daylight contribution at those zones with failed light sensors, $\{i_m(d), m \notin \mathcal{M}\}$, given that we have measurements from functioning light sensors, $\{i_m(d), m \in \mathcal{M}\}$.

We know the transfer matrix from luminaires to light sensors $E$, daylight contribution at the light sensors $s$, reference dimming vector $d^*$ and corresponding illumination measurements at light sensors, $l(d^*)$, and at workspace plane, $v(d^*)$. Further, we have neighborhood information for each zone. Let $\mathcal{N}_p$ be a set with indices of neighboring zones of the $p$-th zone, i.e. $m \in \mathcal{N}_p$ if and only if zone $m$ and $p$ are neighbors. Note that $p \in \mathcal{N}_p$.

Hence, our problem is to estimate the daylight contribution over the workspace plane, $\mathbf{u} = [u_1, \ldots, u_p]^T$, such that the difference between estimated illumination (7) and measured illumination (4) at functioning light sensors is minimized, i.e.

$$
\arg\min_{G,H,\mathbf{u}} \sum_{m \in \mathcal{M}} \left| \frac{s_m - \sum_{n \in \mathcal{M}} G_{m,n} u_n}{s_m} \right|^2
$$

subject to

$$
\begin{align*}
Hd^* &= v(d^*) \\
Ed^* &= l(d^*) \\
GH &= E \\
G_{m,n} &\geq 0, \ p = 1, \ldots, P \\
H_{m,n} &\geq 0, \ m = 1, \ldots, P, \ n = 1, \ldots, P.
\end{align*}
$$

IV. INTERPOLATION METHOD

In general, (9) has several solutions depending on the choice of $G$ and $H$. Here, we propose a method to obtain a unique approximated solution to (9) that is independent of $G$ and $H$. Let the difference between the ratio of daylight and reference illumination within a neighborhood be limited and equal to

$$
\frac{u_m}{v_m(d^*)} - \frac{u_p}{v_p(d^*)} = \epsilon_{m,p}, \ p \in \mathcal{N}_m.
$$

In practice, this ratio is small or limited because the reference illumination vector $v(d^*)$ is set such that a uniform illumination over the workspace plane is achieved (e.g. 600 lux), and the daylight distribution does not vary much across neighboring zones.

Using (10), we can bound the illumination values at the $p$-th zone,

$$
\frac{u_m v_p(d^*)}{v_m(d^*)} - \epsilon_{m,p} v_p(d^*) \leq u_p \leq \frac{u_m v_p(d^*)}{v_m(d^*)} + \epsilon_{m,p} v_p(d^*).
$$

Next, using (11), we lower bound the illumination value (7) at the $m$-th functioning light sensor,

$$
s_m = \sum_{p=1}^{P} G_{m,p} u_p
\geq \sum_{p \in \mathcal{N}_m} G_{m,p} u_p
\geq \sum_{p \in \mathcal{N}_m} G_{m,p} \epsilon_{m,p} v_p(d^*) - \sum_{p \in \mathcal{N}_m} G_{m,p} \epsilon_{m,p} v_p(d^*).
$$

(12)
Further, from (12) we have
\begin{align*}
s_m & \geq \sum_{p \in \mathcal{N}_m} G_{m,p} \frac{u_m}{v_m(d^*)} \sum_{n=1}^{P} H_{p,n} d_n^* \\
& \quad - \sum_{p \in \mathcal{N}_m} G_{m,p} \epsilon_{m,p} v_p(d^*) \\
& = \sum_{n=1}^{P} \beta_{m,n} E_{m,n} \frac{u_m}{v_m(d^*)} d_n^* - \sum_{p \in \mathcal{N}_m} G_{m,p} \epsilon_{m,p} v_p(d^*) \\
& \geq \sum_{n \in \mathcal{N}_m} \beta_{m,n} E_{m,n} \frac{u_n}{v_n(d^*)} d_n^* - \sum_{n \in \mathcal{N}_m} \beta_{m,n} E_{m,n} \epsilon_{m,n} d_n^* \\
& \quad - \sum_{p \in \mathcal{N}_m} G_{m,p} \epsilon_{m,p} v_p(d^*) \\
& \triangleq L_m \\
\end{align*}
(13)

where
\begin{align*}
\sum_{p \in \mathcal{N}_m} G_{m,p} H_{p,n} &= \beta_{m,n} E_{m,n} \\
\frac{u_m}{v_m(d^*)} &\geq \frac{u_n}{v_n(d^*)} - \epsilon_{m,n}, \ n \in \mathcal{N}_m
\end{align*}
and $0 \leq \beta_{m,n} \leq 1$. Note that the lower bound depends on terms $\{\beta_{m,n} E_{m,n} \epsilon_{m,n}\}$ and $\{G_{m,p} \epsilon_{m,p}\}$.

Then, we proceed to seek a vector $z = [z_1, \ z_2, \ldots \ z_P]^T$, such that the estimated illuminance at the $m$-th light sensor using transfer matrix $E$ is within the bounds given by (13), i.e.
\begin{equation}
L_m \leq \sum_{n=1}^{P} E_{m,n} z_n \leq s_m
\end{equation}
(14)

where $z_n \geq 0$, $n = 1, \ldots, P$. Note that the lower bound in (14) is always satisfied if we choose $z_n$ such that
\begin{equation}
u_n = \frac{z_n}{d_n^*} v_n(d^*).\end{equation}
(15)

Using (15), we can rewrite the ratio in (10) as
\begin{equation}
\left| \frac{z_m}{d_m^*} - \frac{z_p}{d_p^*} \right| = \epsilon_{m,p}, \ p \in \mathcal{N}_m.
\end{equation}
(16)

Next, we proceed to upper bound vector $z$. Let consider the case when $\epsilon_{m,n} = 0$, $\forall m, n$, i.e.
\begin{equation}
\frac{z_n}{d_n^*} = \frac{z_m}{d_m^*}, \ \forall m, \forall n.
\end{equation}

Hence, the upper bound in (14) can be rewritten as
\begin{align*}
\sum_{n=1}^{P} E_{m,n} z_n &\leq s_m \\
\sum_{n=1}^{P} E_{m,n} d_n^* &\leq s_m \\
\sum_{n=1}^{P} E_{m,n} d_n^* &\leq s_m \\
\frac{z_m}{d_m^*} &\leq \frac{s_m}{\sum_{m,n} E_{m,n} d_n^*}.
\end{align*}
(17)

Finally, we obtain an approximated solution to (9) as
\begin{equation}
\bar{z}^*_m = \frac{z_m}{d_m^*} v_n(d^*), \ n = 1, \ldots, P
\end{equation}
where $z^* = [z^*_1, \ z^*_2, \ldots \ z^*_P]^T$, is the vector with non-negative entries that (i) reduces the error between estimated illuminance values, given transfer matrix $E$, and measured illuminance values at functioning light sensors; (ii) tightens the lower bound in (14) (i.e. reduces the weighted sum of absolute differences in (16) for all zones, where the weights are proportional to $\{E_{k,m}\}$; and (iii) satisfies upper bounds in (14) and (17). This problem can be formulated as
\begin{align*}
\bar{z}^* & = \arg \min_{z} \lambda_1 \sum_{m \in \mathcal{M}} \left| \frac{s_m - \sum_{p=1}^{P} E_{m,n} z_p}{s_m} \right| \\
& \quad + \lambda_2 \sum_{k=1}^{P} \sum_{n \in \mathcal{N}_k} \left( \frac{E_{k,m}}{\max E_{k,m}} \left| \frac{z_k}{d_k^*} - \frac{z_m}{d_n^*} \right|^2 \right) \\
& \quad \text{s.t. } \left\{ \begin{array}{l}
z_p \geq 0, \ p = 1, \ldots, P, \\
\frac{z_m}{d_m^*} \leq \frac{s_m}{\sum_{m,n} E_{m,n} d_n^*}, \ m \in \mathcal{M}, \\
\sum_{n=1}^{P} E_{m,n} z_n \leq s_m, \ m \in \mathcal{M} \end{array} \right\}
\end{align*}
(18)

where $\lambda_1 + \lambda_2 = 1$, $\lambda_1$ and $\lambda_2$ are factors that control the trade-off between achieving the measured daylight illuminance vector, $\bar{s}$, and satisfying condition (16), respectively. The problem in (18) is a quadratic programming problem with linear constraints, which can be solved efficiently [1].

V. NUMERICAL RESULTS

In this section, we present numerical results to show the performance of the proposed interpolation method, which is a solution of (18). The office has length 14.4 m and width 7.2 m with height of the ceiling of 2.7 m. The distance of the ceiling to the workspace plane is 1.94 m. There are $P = 24$ light sources, with co-located light sensors, arranged in a grid of 3 by 8 as shown in Fig. 1. The light source indexing in Fig. 1 also corresponds to that of the zones. The office has windows on one side of the room for daylight. We consider a light sensor with a half opening angle of 60 degrees.

The average reference illuminance at the workspace is 600 lux, with variations within 20%. This may be realized by
setting the lighting system to the reference dimming vector \( d^* = 0.9 \times 1 \), where \( 1 \) is a vector of ones of size \( 24 \times 1 \). Respectively, the reference illuminance at the \( m \)-th light sensors is given by

\[
l_m(d^*) = 0.9 \sum_{n=1}^{P} E_{m,n}, \quad m = 1, \ldots, P.
\]

In Table I, we show the reference illuminance (under dimming vector \( d^* \)) and the daylight illuminance (under a clear sky model) values at each zone in the workspace plane. Note that the variation in daylight distribution at the workspace between neighboring zones is small or limited. The neighborhood of the \( m \)-th light source corresponds to the closest light sources to it.

We compare our method with a simple estimation method as reference that interpolates daylight contribution from neighborhood measurements, i.e.

\[
\tilde{u}_p^* = \begin{cases} 
    c_p s_p \frac{s_p}{|M \cap N_p|} \sum_{m \in M \cap N_p} c_m s_m & \text{if } p \in M \\
    0 & \text{otherwise}
\end{cases}
\]

where

\[
c_p = \frac{u_p(d^*)}{l_p(d^*)}
\]

is a calibration factor and \(|\cdot|\) is the cardinality of the set. For our method, we choose the parameters \( \lambda_1 = 0.9 \) and \( \lambda_2 = 0.1 \).

First, we simulate \( K = 10^3 \) instances with different ratio, \( |M|/P \), of functioning light sensors. We compare both methods by using the mean of the absolute ratio error defined by

\[
\text{ERR} = \frac{1}{K |M|} \sum_{K} \sum_{m \in M} \left| \frac{\tilde{u}_m^{(k)} - u_m}{u_m} \right|
\]

for different values of \(|M|\) where \( \tilde{u}_m^{(k)} \) is the estimation of daylight at the \( m \)-th zone during the \( k \)-th simulation.

In Fig. 3, we plot the results of the mean and the standard deviation of the simulations. We can see that in general our proposed method achieves a smaller error in the estimated daylight contributions as compared to the reference method.

Further, the performance of our proposed method remains the same for ratios of functioning light sensors higher that 60%. This means that the daylight estimate is robust to a reasonably high number of failed light sensors. When the ratio of functioning light sensors is low (below 20%), the reference method performs slightly better than the proposed method as seen by the lower variance of ERR in Fig. 3. This is because when most of the light sensors are not functioning, it is better to not extrapolate and set the estimated daylight contribution to zero.

Next, we evaluate the performance of the proposed and reference methods under noisy light sensor measurements. For this, we added white Gaussian noise to the sensor illuminance measurements in (3) with zero mean and a variance equal to 10% of illumination value. In Fig. 4, we show the mean and standard deviation of ERR for different ratio, \( |M|/P \), of functioning sensors for \( K = 10^3 \) realizations. We can see that the performance of both methods degrades, but our proposed method still provides a better daylight estimate compared to the reference method. Further, our method achieves a smaller ERR when all sensors are functioning. This is because of the additional constraint in the ratios between daylight and reference illuminance distributions (10).

In Fig. 5, we show the result for a given realization. Here, we have 8 functioning light sensors, and \( M = \{3, 5, 6, 9, 11, 16, 23, 24\} \). The measured daylight values at functioning light sensors are shown as green diamonds in Fig. 5. In Fig. 5, we plot the actual daylight and estimated daylight under our proposed method and the reference method for each zone. Note that our method achieves a closer daylight distribution when compared to the reference method. Furthermore, we obtain an estimation of daylight contribution over zone 18. The reference method fails to obtain

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<th>( u_m )</th>
<th>( v_m(d^*) )</th>
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**TABLE I**

REFERENCE ILLUMINANCE AND ACTUAL DAYLIGHT CONTRIBUTION PER ZONE
Fig. 4. Comparison between proposed and reference method under sensor noise for different ratio of functioning light sensors

an estimation because this zone has no functional light sensors in its neighborhood.

Fig. 5. Comparison between proposed and reference method for $M = \{3, 5, 6, 9, 11, 16, 23, 24\}$

VI. CONCLUSIONS

We presented a fusion method for estimating daylight measurements in a system where a set of light sensors stop functioning. The daylight is interpolated using illuminance measurements from the remaining functioning light sensors. The performance of the algorithm was evaluated using numerical results, under different ratio of functioning light sensors. The results showed an improvement in the estimated daylight contribution when compared to a simple neighborhood averaging method.

REFERENCES