Fusion of Possible Biased Local Estimates in Sensor Network Based on Sensor Selection

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Abstract — The paper addresses the problem of estimation fusion in sensor network, in the presence of possible biased local estimates. A sensor selection-based fusion scheme is presented to deal with this problem, which seek to select a subset of sensors to be fused, for the purpose of achieving a better estimation performance. Firstly, we introduce an optimization criterion over any given subset of sensors based on the similarity measure among local estimates. Secondly, we invoke the cross entropy (CE) method to solve the resulting combinatorial optimization problems. We also explore the efficiency and performance of the proposed approach via simulation experiments, compared with other recently proposed techniques.

Keywords—Sensor selection; cross entropy; sensor biases; estimation fusion; fault tolerant.

1. INTRODUCTION

Distributed estimation fusion is an important research topic in the area of data fusion. Lots of valuable research results have been achieved [1–7]. The potential advantage of fusing the redundant and diverse sensor information contributes to increase the reliability and flexibility of the whole multi-sensor system.

However, in some practical tracking applications, some of sensor reports may be corrupted by some undesirable factors, such as sensor biases, etc.. Note that in this case, local trackers cannot produce the bias estimates based on their own local measurements. As a result, local tracks based on biased measurements are also biased. Moreover, the negative impact resulting from sensor biases cannot be ignored. For example, the range bias and azimuth bias are commonly considered for a measurement process implemented in the local Polar coordinate system (LPCS). Among them, the azimuth bias is the critical factor compared with the range bias, which produces more severe influence on the target’s estimate, and will be enlarged with the increasing of the distance from the sensor to the target. For 3 degrees of azimuth bias, the position deviation could reach up to 10 kilometers for a target located 200km from the sensor. Obviously, the direct fusion of all the local estimates (biased and unbiased) will not produce a satisfying result.

In this paper, what we are concerned about is how to implement the efficient fusion in such a challenging case. More explicitly, it is assumed that only some of local estimates are believable (unbiased), and the others are faulty (biased). To implement the fusion in the presence of sensor biases, sensor registration [8,9] is needed in general to remove the sensor biases from the biased sensor reports. In some ideal cases, the sensor biases may be estimated and corrected online. However, in many challenging cases where the sensor biases may be weakly observable, it is impossible to implement the bias compensation in practice. In [10-11], a fault-tolerant mechanism as a combination of CI (Covariance Intersection) and CU (Covariance Union) (CI/CU) was presented, in which the Mahalanobis distance was suggested for detecting the statistical difference between local estimates and a user-defined threshold was proposed to detect the dissimilarity among local estimates. However, determining
the threshold is not an easy task in general. In [12], a probability framework based on Gaussian mixture models (GMMs) for fusing location estimates, which may be biased and insistent, is presented. The exact probability that local sensors are biased is needed to construct the Gaussian component of the mixture model. When the prior information is unknown exactly, the performance of the method may experience an evident degradation. In [13], to fuse incoherent local estimates, a fast and fault-tolerant fusion algorithm (IT-FFGCC) is proposed by introducing an estimate-dependent adaptive parameter, which can obtain robust fusion and the degree of robustness varies with that of incoherency between estimates to be fused.

This paper addresses the problem of estimation fusion in sensor network, in the presence of possible biased local estimates. A sensor selection-based optimization model is presented to deal with this problem. Sensor selection [14-17] is one of the major tasks of sensor resource management (SRM) in target tracking. Generally, the motivation of selecting sensors rather than utilizing all $N_s$ sensors lies in two aspects: computational efficiency of parameter estimation and energy consumption of sensor operations. In this paper, it is employed for another purpose: selecting a subset $\Lambda$ of all $N_s$ sensors with as much unbiased sensor reports as possible.

Two key points need to be elaborated here. One is to construct the objective function to optimize. The other one is to develop an efficient method to solve the optimization model. In view of the fact that unbiased local estimates is affected only by random errors, we assumed that unbiased local estimates gather together into a cluster, compared with biased ones. This inspires us to construct the criterion to optimize based on the similarity measure among local estimates. In addition, sensor selection is a complex combinatorial optimization problem in essence. One of its biggest difficulties is the combinatorial nature. Along with the increasing of the sensor number, the complexity of the problem is challenging. Here, we invoke the cross entropy (CE) method [18-21] to solve the above optimization model. The CE (cross-entropy) method is a generalized Monte-Carlo technique for combinatorial and continuous optimization, and (originally) for estimation of rare-event probabilities. The main idea behind the CEO is representing the solution space with a set of parameters and defining a probability distribution on these parameters. Then, two successive steps are iterated: sampling from the existing distribution, and updating this distribution using a set of elite (better value) samples[18]. They are inherently global search methods and, therefore, may reduce the risk of getting stuck in shallow local maxima.

The rest of the paper is organized as follows. Section II formulates the problem of sensor selection-based fusion. Section III illustrates the procedure of implementing the proposed sensor selection-based fusion model by CE method. Simulation results compared with competitive algorithms are given in Section IV to demonstrate the power of the proposed approach. Finally, concluding remarks are in Section V.

II. PROBLEM FORMULATION

Consider an estimation fusion problem with $N_s$ sensors and one target in the surveillance region. It is assumed that one part of sensor reports is believable (unbiased), and the other part is faulty (biased). Each local track at time $k$ is represented by a two-tuples $\{\hat{x}_k^m, P_k^m\} \ (m = 1, 2, \ldots, N_s)$, where $\hat{x}_k^m$ and $P_k^m$ mean the state estimate and error covariance, respectively. To simplify, the time index $k$ will be omitted later.

Obviously, local tracks produced by biased measurements are also biased. In this case, local estimates from different sensors may differ from each other significantly. For example, one position estimate is far from another by more than a kilometer, but their respective covariance matrixes are in a small level indicating that the estimate is accurate to within a few meters [13]. This is due to that, the measurement model adopted by some local tracker mismatch with the real one (where the covariance information is convincing no longer, which cannot reflect the accuracy of the local estimate well).

The objective of the paper is to select a subset $\Lambda$ of all $N_s$ sensors with as much unbiased sensor reports as possible.
Noted that the statistical distance between unbiased local estimates should be at a small quantity. The statistical distance between a biased estimate and an unbiased estimate is large, since they differ from each other significantly. As a result, the local unbiased local estimates gather together into a cluster. To select unbiased local estimates to be fused, we seek to minimize the sum of all the distances between any pairwise sensors. In this way, the optimization model can be given by

\[
\min J = \sum_{i,j \in \mathcal{R}} s_{ij} \\
\text{s.t. } |\mathcal{R}| = K
\]  

(1)

where \( \mathcal{R} \subseteq \{1, 2, \ldots, N_s\} \) is the optimization variable, and \(|\mathcal{R}|\) denotes the cardinality of set \( \mathcal{R} \). \( s_{ij} \) is the square Mahalanobis distance defined by

\[
s_{ij} = (\hat{x}_i - \hat{x}_j)^T (P + P^T)^{-1} (\hat{x}_i - \hat{x}_j) \]  

(2)

A. Review of the CE method

The CE method has a simplistic model and its resistance to trap in local minima/maxima makes it a suitable candidate for solving complex combinatorial optimization problem. It involves an iterative procedure where each iteration can be broken down into two phases [20].

1. Generate a random data sample according to a specified mechanism.

2. Update the parameters of the random mechanism based on the data to produce ‘better’ sample in the next iteration.

Let \( \mathcal{A} \) be a finite set of elements and \( S() \) be a performance function defined on \( \mathcal{A} \). Consider the following maximization problem:

\[
\max S(x) \\
\text{s.t. } x \in \mathcal{A}
\]  

(4)

In implementing the CE method, one searches for a probability distribution concentrated near the global extremum of the objective function. For this purpose, a parameterized probability density function \( f(x; v) \) on the set \( \mathcal{A} \) is needed. Then, one seeks to construct a sequence of parameter vectors \( v^{(0)}, v^{(1)}, v^{(2)}, \ldots \) such that \( f(x; v^{(t)}) \) becomes concentrated around the global optimum \( x^* \) as \( t \) increases. The essential idea to achieve this purpose is to sample from \( f(x; v^{(t)}) \) and construct the next parameter vector as the maximum likelihood estimate of the distribution parameter based on the elite samples. Namely,

\[
v^{(t+1)} = \arg \max_v \ln f(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_{\Gamma \rho}; v) \]  

(5)

where \( \tilde{X}_i (i = 1, 2, \ldots, \Gamma \rho) \) are the \( \Gamma \rho \) elite samples, \( f(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_{\Gamma \rho}; v) \) is the joint density evaluated at \( \tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_{\Gamma \rho} \). The new parameter \( v^{(t+1)} \) defines a new distribution from which we can sample again and repeat the procedure. In general, CE algorithm proceeds as follows:

1. Set \( v^{(0)} = u \), \( t = 1 \).

2. Generate samples \( X_1, X_2, \ldots, X_\Gamma \) from the

III. SENSOR SELECTION-BASED FUSION BY CE OPTIMIZATION

The cross entropy (CE) method developed recently is the optimization heuristics that have proved useful in many combinatorial optimization problems. In this section, we firstly review the basic CE method, and then show how to solve the optimization problem (3).
density \( f(x; v^{(t-1)}) \), and evaluate the objective function and order them from the smallest to largest: 
\( S_1 \leq S_2 \leq \ldots \leq S_{\Gamma} \), then compute the sample \((1 - \rho)\) -quantile \( \gamma' = F_{[(1-\rho)\Gamma]} \) . where the parameter \( 0 \leq \rho < 1 \), which determines what proportion of samples will be selected to update the parameter \( v \).

3. Based on the set \( \{ \hat{X}_1, \hat{X}_2, \ldots, \hat{X}_{\Gamma} \} \) of elite samples, find the maximum likelihood estimate \( \hat{v}^{(t)} \) of the parameter \( v \). Smooth the estimate via \( v^{(t)} = \alpha \hat{v}^{(t)} + (1 - \alpha) v^{(t-1)}, 0 \leq \alpha \leq 1 \) to prevent the CE from premature convergence. Then, denote the new probability distribution by \( f(x; v^{(t)}) \).

4. If there is no significant improvement happened on the variable \( v^{(t)} \) for several iterations, or convergence to a degenerate distribution, then stop; otherwise set \( t = t + 1 \) and reiterate from step 2.

Assumed that the dimension of the sample \( X_i (i = 1, 2, \ldots, \Gamma) \) is \( n \), and each component assumes a finite number of values \( \{ a_1, \ldots, a_p \} \). The parameter \( v \) of probability density function \( f(x; v) \) is characterized by \( v \triangleq \{ v_{jk} \}_{j=1,\ldots,n; k=1,\ldots,p} \). The important property that makes the CE method easy to implement is that there is a simple component-wise analytical solution to (5)

\[
v_{jk} = \frac{\sum_{l=1}^\Gamma 1\{X_{ij}=a_l\}1\{s_l(X_l) \geq \gamma^{(t)}\}}{\sum_{l=1}^\Gamma 1\{s_l(X_l) \geq \gamma^{(t)}\}}
\]

where \( X_{ij} \) is the \( j \)th element of the \( i \)th sample \( X_i \) drawn from \( f(x; v^{(t-1)}) \), and \( 1\{A\} \) is the indicator function of set \( A \).

Namely, the updated value of each parameter is the relative frequency of the appearance of the corresponding value in the current elite sample.

**B. Sensor Selection-based Fusion by CE Optimization**

Our goal is to find the optimal binary string \( z = [z_1, z_2, \ldots, z_N] \), \( z_i \in \{0,1\} \) to optimize the objective in (3).

The population of samples in an iteration is denoted by \( Z \)

\[
Z = \left[ \begin{array}{c}
Z_1 \\
Z_2 \\
\vdots \\
Z_{\Gamma}
\end{array} \right] = \left[ \begin{array}{cccc}
Z_{11} & Z_{12} & \ldots & Z_{1N_x} \\
Z_{21} & Z_{22} & \ldots & Z_{2N_x} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{\Gamma 1} & Z_{\Gamma 2} & \ldots & Z_{\Gamma N_x}
\end{array} \right]
\]

where \( Z_i = [Z_{i1}, Z_{i2}, \ldots, Z_{iN_x}] \), \( z_{ij} \in \{0,1\} \).

In our problem, the dimension of each sample \( Z_i \) is \( N_s \).

Each component of the sample assumes a finite number of values \( \{a_1, a_2\} = \{1,0\} \). For any given sample \( Z_i \), we consider the following family of probability mass functions

\[
f(Z_i; v) = f(Z_{i1}, \ldots, Z_{iN_x}; v) = \prod_{k=1}^{2} \prod_{j=1}^{N_x} v_{jk}^{z_{ij} \cdot v_{jk}} \cdot (1 - z_{ij})^{1 - v_{jk}}
\]

where \( v_{jk} (j = 1, 2, \ldots, N_x; k = 1, 2) \) means the probability of the \( j \)th component takes the value \( a_k \), and \( A_k = \{ l \mid l \in \{1, 2, \ldots, N_x\}, Z_{jl} = k \} \).

The complete routine for the optimization model (3) as follows.

1. Input local estimates \( \{\hat{x}_m, P_m\} (m = 1, 2, \ldots, N_s) \), and set initial parameters \( v_1^{(0)} = v_2^{(0)} = 0.5 \), let \( t = 1 \).

2. Generate \( \Gamma \) initial samples from the Bernoulli distribution (8) by using the parameter \( v^{(t-1)} \), and sort the \( \Gamma \) samples according to their fitness value, and set \( \gamma' = F_{[(1-\rho)\Gamma]} \).

3. Select the samples whose fitness values are greater than \( \gamma' \). And based on the elite samples, update the parameter \( v^{(t)} \) according to (6).

4. Smooth the estimate via
\[ \dot{v}^{(t)} = \alpha \dot{v}^{(t)} + (1 - \alpha)\dot{v}^{(t-1)}, \quad 0 \leq \alpha \leq 1 \] to prevent the CE from premature convergence. If stopping criterion is not satisfied, set \( t = t + 1 \) and go back to step 2. Otherwise, go to step 6.

5. Output the sample \( \mathbf{Z}_i = [Z_{i,1}, Z_{i,2}, \ldots, Z_{i,N_s}] \) with best fitness value, and obtain the final sensor selection subset \( \mathcal{R}^* = \{l | l = 1, 2, \ldots, N_s; Z_{i,l} = 1\} \). Compute the fusion estimate \( \hat{x}_{\mathcal{R}^*} \) and the corresponding error covariance \( P_{\mathcal{R}^*} \) based on the selected sensor subset \( \mathcal{R}^* \) by the rule of simple convex combination (SCC).

It is noted that the constraint \( \sum_{j=1}^{N_s} z_j = K \) in the optimization model (3) may not be satisfied during the above process. To deal with the problem, an extra routine on Step 2 inspired by [15] is imposed to satisfy the constraint. For a certain sample, if the sum \( K' \) of its components is less than \( K \), then we add a combination of \( (K - K') \) sensors from \( (N_s - K) \) initially unselected sensors with equal probability. On the contrary, if \( K' > K \), select a combination of \( K \) sensors from \( K' \) selected sensors.

C. Discussion

In above work, the cardinality \( K \) of the selected subset is to be fused determined in advanced. However, in many practical applications, the cardinality \( K \) of the selected subset is unknown, and also not predetermined. A new optimization model is needed to adaptively select as many unbiased estimates among \( N_s \) potential sensors as possible, which is one of future research directions.

In addition, we only considered the single-target case here, which provides an simple insight of sensor selection problem in the presence of possible biased local estimates. In multi-target scenario, implementing data association and sensor selection jointly will be discussed in the future work.

IV. SIMULATION RESULTS

In this section, we illustrate some simulations to show the performance of the proposed approach, compared with competitive algorithms. The following simulation results are given by implementing the proposed sensor selection-based fusion methods (named by SS) and competitive algorithms (CU[10], IT-FFGCC[13]).

In this scenario, 30 sensors are distributed uniformly in the region \([-100km 100km] \times [-100km 100km]\). The target is initially located at \((50, 150) km\) with a velocity of \((0.32, -0.17) km/s\). The target follows a constant velocity (CV) model with the covariance of process noise \( Q = 0.0002km/s^2I \), where \( I \) is a \( 2 \times 2 \) identity matrix. The sample interval \( T \) is set to be 1s. Each sensor measures the range and angle information to the target. The random measurement errors for both sensors are modeled as white Gaussian noise with the variance \((0.1km)^2\) and \((0.005rad)^2\). Among them, 5 sensors are corrupted by sensor biases. The sensor biases are represented by a two-tuple, where the first component means the range bias, and the second one means the azimuth bias. The details are as follows.

\begin{align*}
\text{Bias}\{1\} &= [1.1 km; 0.0042 rad]; \\
\text{Bias}\{3\} &= [-0.3 km; 0.0072 rad]; \\
\text{Bias}\{8\} &= [1.5 km; 0.0038 rad]; \\
\text{Bias}\{13\} &= [-0.5 km; -0.0044 rad]; \\
\text{Bias}\{17\} &= [1.5 km; 0.0038 rad];
\end{align*}

Fig.1 illustrates the measurement positions given by all the local sensors at scan \( t=50T \) in one run. Converted measurement Kalman filter (CMKF) is used to produce local estimates. Fig.2 shows that the position RMSE of sensor selection-based (SS) fusion compared with competitive algorithms over 100 Monte-Carlo runs, when given different cardinality \( K \) of selected subset.
From the above figure, it is easily seen that the proposed SS outperforms the IT-FFGCC and CU. Along with the increasing of the cardinality $K$ of selected subset, the Position RMSE decreases. When $K=25$, the proposed approach attains the optimal fusion performance. When $K>25$, biased sensors are included in the selected subset, which results in the performance degradation.

Figs.3-8 show the effect of parameters (sample size $\Gamma$, quantile $p$ and smoothing factor $\alpha$) used in CE method for $K=25$. Among them, Fig.3 shows the best fitness value attained by the CE method according to different sample size. Fig.4 shows the corresponding calculation number required in CE method to achieve the optimal fitness, where the calculation number is evaluated by multiplying the iteration number and sample size. Fig.5 illustrates the performance of the CE method for different quantile $\rho$. Fig.6 shows the corresponding iterations required in CE method to achieve the optimal fitness. Figs.7-8 illustrate the effect of CE method for different smoothing parameter $\alpha$. 
From Figs. 3-4, it is seen that, the optimal fitness value can be achieved when the sample size is larger than 30. Moreover, along with the increasing of the sample size, the calculation number tends to increase. Fig. 5 shows that the optimal fitness value can be achieved when the quantile equals to be 0.2. Along with the increasing of the quantile, the iteration number becomes larger. As a result, a feasible interval for the quantile is $\alpha = [0.2, 0.6]$. Similarly, a feasible interval for the smoothing factor lies is $\rho = [0.3, 0.7]$.

V. CONCLUSIONS

This paper proposed an adaptive fusion mechanism in sensor network based on sensor selection, in the presence of possible biased local estimates. The proposed approach can select a subset of sensors to be fused when given the cardinality of the subset. Two key points are elaborated here. One is the construction of the objective function to optimize. The other one is to develop an efficient method to solve the complex combinatorial optimization problem. Simulation results demonstrate the efficiency and performance of the proposed approach, compared with other recently proposed techniques. Further research includes some extensions of the proposed approach to the multi-sensor multi-target scenario, and the case in the presence of unknown cross-correlation in the errors of local estimates.
REFERENCES