Fuzzy Inference-Based Dynamic Determination of IMM Mode Transition Probability for Multi-Radar Tracking

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Abstract—A new method of determining IMM mode transition probability for multi-radar tracking in air traffic control is presented. In order to improve the accuracy of maneuvering target tracking under a multi-radar environment, dynamic determination of the IMM mode transition probability is studied. The aperiodic measurement inputs from asynchronous multi-radar are also taken into account. In addition to dynamic determination with aperiodic measurement inputs, fuzzy inference is used to adjust the mode transition probability from one mode to another. A simulation study shows that this method for dynamic determination of IMM mode transition probability gives better tracking performance than conventional methods in terms of position accuracy.

Keywords—IMM; mode transition probability; fuzzy inference; multi-radar tracking; air traffic control

I. INTRODUCTION

Target tracking in air traffic control has a number of technical problems to be solved, such as variously maneuvering multi-targets, asynchronous sensor input data from multi-sensors, nonlinearity of target maneuvers, limited measurements by radar sensing, and so on. For the various and nonlinear maneuvers of targets, an estimation method is developed based on a hybrid dynamic system that includes both continuous and discrete system characteristics. As a representative method of hybrid system estimation, Interacting Multiple Model (IMM) was suggested by H. A. P. Blom [1] in 1984. Since Rong Li and Bar-Shalom [2] applied IMM to the aircraft tracking problem, it has been widely used in the surveillance data processing of air traffic control. IMM has improved tracking accuracy performance by providing data on multiple dynamics of various flight modes, such as constant velocity (CV), constant turn (CT), and constant acceleration (CA)[3].

The performance requirements of target tracking for air traffic control include small estimation error for constant-velocity maneuvers, a low estimation error peak compared to the measurement, and accurate indication of flight mode derived from the detection of maneuvers. For these required performance indices, the followings are needed to be optimized: the dynamic model of aircraft considering every flight mode; model parameters such as system process noise and measurement noise; mode transition probabilities [4].

In their research concerning IMM performance improvement through optimization of the sub-model set and model parameters, Rong Li and Bar-Shalom [2] studied flight modes and system process noise, and R. W. Osborne [5] studied measurement noise. For optimization of the mode transition probability of IMM, a dynamic determination using sojourn time is introduced in [6]. However, the average sojourn time for each dynamic model should be determined empirically, and it is known that constant mode transition probabilities are used in most surveillance data processing systems of real-world air traffic control system.

In this study, based on the observation that the measurement inputs from multi-radar is aperiodic and a small mode change probability is acceptable for the small time interval of measurement inputs, dynamic determination of mode transition probability is investigated. Fuzzy inference is suggested for the adjustment of mode transition probability to utilize the knowledge of experts about flight mode transitions in air traffic control.

II. ALGORITHM DEVELOPMENT

A. Determination of mode transition probability considering the erratic time intervals of input data.

The Markov mode transition probability, \( \pi_{ij} \), is the probability of mode change from i-th mode at time \( k-1 \) to j-th mode at time \( k \), and expressed as in Eq. (1).

\[
\pi_{ij} = \{ PM_j(k) | M_i(k-1) \} = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1j} \\ P_{21} & P_{22} & \cdots & P_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ P_{i1} & P_{i2} & \cdots & P_{ij} \end{pmatrix}
\] (1)

Since the effect of mode transition probability on tracking performance does not overshadow design parameters such as
flight modes or system processing noise [4], constant mode transition probability is used in many practical applications. However, in the asynchronous multi-radar environment of air traffic surveillance data processing, the sampling time intervals of measurements change depending on the number of radars, rotation period of each radar, relative positions of a target and the radars, and the detection probability of each radar. Therefore, the mode transition probability is more important in the True Multi-Radar Tracking method, in which each measurement is used at the moment of sensing. Intuitively, the probability of the flight mode changing should be small when the sampling time interval is small, and it would be reasonable to change the mode transition probability in accordance with the time interval of sensing.

As a preliminary study about the relationship between the mode transition probability and the time interval of sensing, a simulation study was used to check the position accuracy of tracking with different values of mode transition probability and the time interval of sensing. Referring to real recorded tracking result data of maneuvering aircraft, an aircraft's trajectory is modeled as a sequence of 10 segments of flight modes, as shown in Table 1. The transition from one mode to the next is modeled according to first order lag dynamics.

\[
x_{CV}(k) = F_{CV} x_{CV}(k-1) + \Gamma v_{CV}(k-1)
\]

\[
\begin{bmatrix}
1 & T & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix}
x_{CV}(k-1) + 
\begin{bmatrix}
\frac{1}{2} T^2 & 0 \\
T & 0 \\
0 & 1 & T^2 \\
0 & 0 & 1
\end{bmatrix}
v_{CV}(k-1)
\]

\[
x_{CT}(k) = F_{CT} x_{CT}(k-1) + \Gamma v_{CT}(k-1)
\]

\[
\begin{bmatrix}
\sin \omega T & 0 & -1 - \cos \omega T \\
\omega & 0 & 0 \\
-\frac{1}{2} \sin 1 \cos 1 & 0 & 1 \cos 1 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
x_{CT}(k-1)
\]

\[
\begin{bmatrix}
T & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
v_{CT}(k-1)
\]

where the state vector \( x_{CV} \) is the position and the velocity in a local 2D plane, \( [x \ y \ \dot{x} \ \dot{y}] \). \( x_{CT} \) includes the turn rate: \( x_{CT} = [\xi \ \dot{\xi} \ \eta \ \dot{\eta}] \). \( T \) is the sampling period. \( v_{CT} \) and \( \dot{v}_{CT} \) are the white noise processes with zero mean for noise modeling, and they have error covariance of \( Q_{CV} = \Gamma \sigma_{vcv}^2 \Gamma \) and \( Q_{CT} = \Gamma \sigma_{vcv}^2 \Gamma \) with typical values of \( \sigma_{vcv} = 0.1 m/s^2 \) and \( \sigma_{vcv} = 0.5 m/s^2 \) [4]. The measurement equation is defined as Eq. (4), where \( w(k) \) denotes measurement noise.

\[
z(k) = H x(k) + w(k)
\]

\[
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} x(k) + w(k)
\]

Mode transition probability is given as Eq. (5), where subscript 1 denotes CT mode and 2 denotes CT mode.

\[
\pi_y = \begin{pmatrix}
p_{11} & 1 - p_{11} \\
1 - p_{22} & p_{22}
\end{pmatrix}
\]

The variables \( p_{11} \) and \( p_{22} \), which measure the probability of maintaining the mode, are given as 0.95 in a practical air traffic control application. In this study, the various values are tested to check the relationship between mode transition probability and sampling rates.

Fig. 4 shows the simulation results of the Root Mean Square (RMS) tracking position error (for 500 iterations of a Monte

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>Start Time (sec)</th>
<th>Duration (sec)</th>
<th>Flight Mode</th>
<th>Turn Rate (deg/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>330</td>
<td>CV</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>330</td>
<td>63</td>
<td>CT</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>393</td>
<td>115</td>
<td>CV</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>508</td>
<td>56</td>
<td>CT</td>
<td>-0.25</td>
</tr>
<tr>
<td>5</td>
<td>564</td>
<td>105</td>
<td>CV</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>669</td>
<td>46</td>
<td>CT</td>
<td>-2.0</td>
</tr>
<tr>
<td>7</td>
<td>715</td>
<td>122</td>
<td>CV</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>837</td>
<td>26</td>
<td>CT</td>
<td>-0.25</td>
</tr>
<tr>
<td>9</td>
<td>863</td>
<td>68</td>
<td>CT</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>931</td>
<td>86</td>
<td>CV</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The sensing performance of the radar is modeled as \( \sigma_p \) (standard deviation of range error) = 70m and \( \sigma_p \) (standard deviation of azimuth error) = 0.08°. The radar is assumed to be located at \( x = -100 km \) and \( y = 0 km \) in the local frame of the initial target position. IMM with CV and CT modes is used for target tracking, and EKF is used for CT mode. The CV and CT modes included in IMM are the Nearly Constant Velocity Model and Nearly Constant Turn Model based on White Noise Acceleration (WNA), and the state equations are given as Eqs. (2) and (3).
Carlo simulation) for various mode transition probabilities and constant sampling rates. The results show that the RMS position error decreases with high-frequency sampling, but the mode transition probability for the minimum RMS error differs depending on the sampling frequency.

From the results shown in Fig. 4, the optimal mode transition probability for each constant sampling frequency can be determined by fitting a curve to the data, as shown in Fig. 5 and given as Eq. (6), where it is assumed \( p_{11} = p_{22} \) for simplicity.

The membership functions for sampling time interval are SM (Small) and BG (Big), as shown in Fig. 3. \( \bar{T} \) denotes the average sampling time for multiple radars, and \( N_{\text{Radar}} \) is the number of radars. The membership functions for turn rate \( \omega \) and change of turn rate \( \Delta \omega \) are NB (Negative Big), NM (Negative Medium), ZO (Zero), PM (Positive Medium), and PB (Positive Big). \( c_1, c_2, c_3, \) and \( c_4 \) are determined as shown in Eqs. (7) and (8), where \( \omega_{st} \) and \( \Delta \omega_{st} \) are the standard values of turn rate and change in turn rate. Both variables can be determined based on statistical studies of recorded radar data or by using expert knowledge in air traffic control.

\[
f(T) = -0.00011T^2 - 0.00205T + 0.997 \quad (6)
\]
For \( \omega \),
\[
(c_1, c_2, c_3, c_4) = (-2 \omega_{st}, -\omega_{st}, \omega_{st}, 2\omega_{st})
\] (7)

For \( \Delta \omega \),
\[
(c_1, c_2, c_3, c_4) = (-2\Delta \omega_{st}, -\Delta \omega_{st}, \Delta \omega_{st}, 2\Delta \omega_{st})
\] (8)

Using the fuzzy inference method suggested here, we can determine whether \( p_{ii} \) is big or small. The fuzzy sets for the output variables, BG (Big), NO (Nominal), and SM (Small), are used as given in Fig. 5.

![Fig. 5. Fuzzy Sets for Output](image)

The fuzzy rules are as follows:

1) If \( T \in SM \), then \( p_{11} \in NO \) and \( p_{22} \in NO \).
2) If \( T \in BG \) and \( \omega \in NM \) and \( \Delta \omega \in ZO \), then \( p_{11} \in BG \) and \( p_{22} \in SM \).
3) If \( T \in BG \) and \( \omega \in PM \) and \( \Delta \omega \in ZO \), then \( p_{11} \in BG \) and \( p_{22} \in SM \).
4) If \( T \in BG \) and \( \omega \in ZO \) and \( \Delta \omega \in ZO \), then \( p_{11} \in SM \) and \( p_{22} \in BG \).
5) If \( T \in BG \) and \( \omega \in NB \) and \( \Delta \omega \in PM \), then \( p_{11} \in SM \) and \( p_{22} \in BG \).
6) If \( T \in BG \) and \( \omega \in PB \) and \( \Delta \omega \in NM \), then \( p_{11} \in SM \) and \( p_{22} \in BG \).
7) If \( T \in BG \) and \( \omega \in NM \) and \( \Delta \omega \in NM \), then \( p_{11} \in BG \) and \( p_{22} \in SM \).
8) If \( T \in BG \) and \( \omega \in PM \) and \( \Delta \omega \in PM \), then \( p_{11} \in BG \) and \( p_{22} \in SM \).
9) If \( T \in BG \) and \( \Delta \omega \in NB \), then \( p_{11} \in NO \) and \( p_{22} \in NO \).
10) If \( T \in BG \) and \( \Delta \omega \in PB \), then \( p_{11} \in NO \) and \( p_{22} \in NO \).
11) If \( T \in BG \) and \( \omega \in ZO \) and \( \Delta \omega \in NM \), then \( p_{11} \in BG \) and \( p_{22} \in SM \).
12) If \( T \in BG \) and \( \omega \in ZO \) and \( \Delta \omega \in PM \), then \( p_{11} \in BG \) and \( p_{22} \in SM \).

For defuzzification, the weighted average method is used. The output after defuzzification is the normalized amount of required adjustment of \( p_{ii} \). If \( \Delta p_{ii} \) indicates this normalized output value from defuzzification, then \( p_{ii} \) can be adjusted as given in Eq. (9).

\[
p_{ii,new} = \min(0.995, p_{ii} - (\Delta p_{ii} - 0.5)\text{max}(\Delta p_{ii}))
\] (9)

In Eq. (6), \( \text{max}(\Delta p_{ii}) \) is the maximum variation of \( p_{ii} \) obtained by fuzzy inference. It is a positive value and a design parameter to be determined by users.

### III. SIMULATION STUDY

The same reference trajectory given in Table 1 is used for the simulation study. Table II presents the assumed conditions of the radars in this study. The periods of rotations and the performances of the radars are assumed to be similar, with the required performance of an en-route radar set forth in the Eurocontrol Standard Document for Radar Surveillance in En-Route Airspace and Major Terminal Areas, SUR.ET1.S T01.1000-STD-01-01 [7]. The reference trajectory and simulated radar data with error modeling are shown in Fig. 6.

**TABLE II. CONDITIONS OF RADARS**

<table>
<thead>
<tr>
<th>Radar</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X (km)</td>
<td>-100</td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>Y (km)</td>
<td>0</td>
<td>50</td>
<td>-90</td>
</tr>
<tr>
<td>Period of Rotation (Sec)</td>
<td>10</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Initial Azimuth of Radar (deg)</td>
<td>0</td>
<td>120</td>
<td>240</td>
</tr>
<tr>
<td>Performance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{\text{Azimuth}}) (deg)</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>(\sigma_{\text{Range}}) (m)</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>(P_{\text{Detect}})</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
</tbody>
</table>

![Fig. 6. Reference Trajectory and Simulated Sensor Data](image)

In this simulation study, \( \omega_{st} \) and \( \Delta \omega_{st} \) of Eqs. (7) and (8) are assumed as \( \omega_{st} = 3\text{deg/sec} \) and \( \Delta \omega_{st} = 1.5\text{deg/sec}^2 \) from expert knowledge, and \( \text{max}(\Delta p_{ii}) \) of Eq. (9) is given as 0.2. As a tracking method for asynchronous multi-radar tracking, multiple plot variable update [8] and IMM are used.

The simulation results are shown through Figs. 7-10, and all of the results in Figs. 7-10 are obtained from a Monte Carlo...
simulation of 500 trials. Fig. 7 shows the RMS position error of
the tracking results. For comparison with the tracking results
associated with dynamic $p_{ii}$, two typical values of $p_{ii}$ are
selected: 0.95 and 0.98. The change histories of the mode
probabilities are shown in Figs. 8 and 9. Compared with the
reference trajectory described in Table 1, the mode
probabilities of Figs. 8 and 9 indicate that the tracker detected
the mode transition correctly.

Figs. 7-9 indicated that the dynamic $p_{ii}$ yields a smaller
RMS position error than the static $p_{ii}$. For a more readily
apparent difference, the comparison of RMS position errors for
various static $p_{ii}$ and dynamic $p_{ii}$ values is shown in Fig. 10.
In Fig. 10, the minimum RMS position error of static $p_{ii}$ is
78.88m, which is obtained using $p_{ii} = 0.99$. The RMS
position error of dynamic $p_{ii}$ is 78.74m, which is smaller than
the minimum RMS position error of static $p_{ii}$.

IV. CONCLUSION

A dynamic determination of IMM mode transition
probabilities was presented, taking into account erratic
intervals of sensor data input from asynchronous multiple
radars and tendencies of state variables. As an empirical
method, preliminary study results about the relationship
between static $p_{ii}$ and RMS position errors were used to
determine $p_{ii}$, and the fuzzy inference method was
incorporated for the adjustment of $p_{ii}$ according to the
tendency of state variable $\omega$. Through a simulation study, it
was shown that the suggested method gives the smallest RMS
position error compared with any other static $p_{ii}$.

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REFERENCES


