Bayesian Techniques for Edge Detection on Polarimetric SAR Images

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Abstract—We propose a Bayesian edge detector to be fed by polarimetric, possibly multifrequency, SAR data. It can be used to detect dark spots on the ocean surface and, hence, as the first stage of a system for identification and monitoring of oil spills. The proposed detector does not require secondary data (namely pixels from a slick-free area), but for a certain a priori knowledge. The performance assessment, carried out using both synthetic and real SAR recordings, shows that it has better capabilities in terms of detection and false alarms control than previously-proposed classical (i.e., non-Bayesian) detectors.

Index Terms—Bayesian detection, Oil slicks, Polarimetric SAR data.

I. INTRODUCTION

Synthetic Aperture Radars (SARs) represent a powerful tool for monitoring oil spills on the sea surface. In practice, oil floating on the sea surface reduces the energy backscattered by the illuminated area since oil slicks produce an increased viscosity of the top layer of the sea surface which damps out the short gravity and capillary waves responsible for the amount of backscattered energy measured by the SAR. The quantitative assessment has demonstrated the potential of (ERS-1/2) SAR images to detect even very thin pollution layers at low wind speeds (3-4 m/s) or thick oil emulsions at wind speeds up to 12 m/s [1].

Nevertheless, many other phenomena can produce the same effect on SAR images (the so-called look-alikes); for instance, dark spots can be caused by low wind, grease ice, internal waves, rain cells, and biogenic films. For this reason, monitoring oil spills on the sea surface is a very challenging task and requires advanced functionalities in order to discriminate between real oil spills and look-alikes [2]-[4]. In order to increase the detectability of slicks, it has been proposed to use both multifrequency and multipolarization data [5]-[11]. In many cases, proposed algorithms assume that a set of secondary data, i.e., a slick-free area, is available. However, the availability of such data might be difficult to meet in many situations of practical interest. In [7]-[11] edge detectors (EDs) based upon the generalized likelihood ratio test (GLRT) to be fed by polarimetric and/or multifrequency data have been proposed. The design of the GLRTs relies on the assumption that pixels of either a slick-free or a slick-covered area, within the region under test, can be modeled in terms of Gaussian vectors with a common (unknown) covariance matrix. Remarkably, such schemes guarantee the constant false alarm rate (CFAR) property and, above all, have been proven effective on real data recordings.

In this paper, we extend the edge detection idea to a Bayesian framework. Bayesian statistics has been successfully used to attack adaptive radar detection in non-homogeneous environments, see, for instance, [12], [13]. In fact, Bayesian tools are typically effective to handle heterogeneities and to include a priori information. The idea herein pursued is to assume that the covariance matrix of each pixel is a random quantity with some preassigned a priori distribution. This framework allows to obtain a general and flexible, yet simple, model of non-homogeneous environments, without very restrictive assumptions. The adaptive detector can be obtained averaging out the random covariance matrices from the likelihood functions. Actually, we design a GLRT edge detector to be fed by polarimetric, possibly multifrequency, data. As in [10], we assume that the region under test can be partitioned into two sets containing either slick-free or slick-covered pixels. The GLRT is constructed assuming that each pixel is ruled by a complex normal model conditioned on the covariance matrix of the data that, in turn, is described (for both slick-free and slick-covered data) by a complex inverse Wishart distribution. Moreover, pixel data are conditionally independent

 Again, implementation of the edge detector requires maximizing the likelihood ratio with respect to a suitable collection of partitions of the data under test. Such set of partitions models the possible unknown shape of the slick. It is important to stress that the algorithm herein proposed is aimed at providing candidate oil spills to be fed to a classifier.

The remaining of the paper is organized as follows: Section II is devoted to the problem formulation and to the design of the edge detector while in Section III we carry out a performance assessment based upon both simulated and real SAR data. Finally, some concluding remarks are reported in Section IV.

1Based on the above assumptions, if \( K \) pixel data are drawn from a slick-free or a slick-covered area, \( K \) times the sample covariance matrix of such data is ruled by a complex Wishart distribution, conditioned on the covariance matrix of the data.
II. Problem Formulation and Detector Design

Denote by $r_{i,j}$ the $N$-dimensional vector whose $n$-th entry, $r_{i,j}(n)$ say, is the complex reflectivity of the $(i,j)$-th pixel of the SAR image in the $n$-th polarimetric channel, $n = 1, \ldots, N$, i.e., $r_{i,j} = [r_{i,j}(1) \cdots r_{i,j}(N)]^T$ where $T$ denotes transpose. The aim is to conceive a detector to discriminate between the $H_0$ hypothesis that $M$ adjacent pixels belong to either a slick-free or a slick-covered area and the alternative $H_1$ that they contain the edge of a slick. To this end, denote by $S$ the set of all pairs of indices corresponding to the $M$ pixels under test; the idea is that, if those $M$ adjacent pixels contain the edge, the slick does not cover the entire region under test and we can split returns into two groups according to their statistical characterization. Consequently, from a statistical point of view, we suppose that, conditionally to the covariance matrix, $R_{i,j}$ say, polarimetric return $r_{i,j}$ is a zero-mean, complex normal random vector $[14], [15]$; in addition, conditionally to the $R_{i,j}$’s, the $r_{i,j}$’s are independent random vectors. In symbols, we write $r_{i,j} \sim CN(0, R_{i,j})$. Moreover, we assume that $R_{i,j}$ is drawn from a complex inverse Wishart distribution, with $\mu(\mu > N)$ degrees of freedom, i.e.,

$$f(R_{i,j}) = \frac{|(\nu - N)\overline{R}|^\nu}{\Gamma_N(\nu)|R_{i,j}|^{\nu+N}}\text{etr}\{-((\nu - N)R_{i,j}^{-1}\overline{R})\}$$ \hspace{1cm} (1)

where $| \cdot |$ is the determinant of the matrix argument, etr(\cdot) stands for the exponential of the trace of the matrix argument, and $\Gamma_N(\nu)$ is given by

$$\Gamma_N(\nu) = \pi^{N(N-1)/2} \prod_{n=1}^N \Gamma(\nu - n + 1)$$

with $\Gamma(x)$ being, in turn, the Eulerian Gamma function. We denote this distribution as $R_{i,j} \sim CW^{-1}((\nu - N)\overline{R}, \nu)$. It is worthwhile pointing the role of the parameters of the distribution. In fact, $\overline{R}$ represents the expected value of $R_{i,j}$ while $\nu$ sets the “distance” between $\overline{R}$ and $R_{i,j}$; as $\nu$ increases $R_{i,j}$ is closer to $\overline{R}$ (in the sense that the variance of $R_{i,j}$ decreases). Such a model allows to take into account a certain statistical variability of polarimetric returns on a pixel-by-pixel basis. In fact, we could even suppose that the $R_{i,j}$’s are independent random matrices. However, for the case at hand, it is reasonable to assume that pixels in spatial proximity of either a slick-free or a slick-covered area possess one and the same value of the covariance matrix. In other words, we assume that $R_{i,j} = M_j$ for pixels that belong to the slick and $R_{i,j} = M_0$ for the remaining pixels of the slick-free area. Accordingly, we set $M_j \sim CW^{-1}((\nu_j - N)\overline{M_j}, \nu_j)$, $M_0 \sim CW^{-1}((\nu_0 - N)\overline{M_0}, \nu_0)$, and suppose that $M_j$ and $M_0$ are independent random matrices. $\overline{M_j}$, $\nu_j$, $\overline{M_0}$, and $\nu_0$ represent the a priori knowledge on the problem at hand. In Section III we discuss how to set them. It follows that the detection problem to be solved can be restated in terms of the following binary hypothesis test

$$H_0:\begin{cases}
r_{i,j}|M_h \sim CN_N(0, M_h), & (i,j) \in S \\
M_h \sim CW^{-1}((\nu_h - N)\overline{M_h}, \nu_h), & h = 0 \text{ or } 1
\end{cases}$$

$$H_1:\begin{cases}
r_{i,j}|M_0 \sim CN_N(0, M_0), & (i,j) \in S_0 \\
M_1 \sim CW^{-1}((\nu_1 - N)\overline{M_1}, \nu_1), & (i,j) \in S_1 \\
r_{i,j}|M_0 \sim CN_N(0, M_0), & (i,j) \in S_0 \\
M_0 \sim CW^{-1}((\nu_0 - N)\overline{M_0}, \nu_0), & (i,j) \in S_0
\end{cases}$$ \hspace{1cm} (2)

where $h = 1$ ($h = 0$) if the region under test is completely covered by the slick (the sea), while the pair $(S_0, S_1)$ is an unknown partition of the set $S$, denoted in the following as $S_0; S_1$, in particular, $S_0$ is a proper subset of $S$ and represents the region covered by the slick (under $H_1$), while $S_0$ is the complement of $S_1$ with respect to $S$. For future convenience, let us denote by $M_0$ and $M_1$ the cardinalities of $S_0$ and $S_1$, respectively (it is then apparent that $M = M_0 + M_1, M_1 \neq M$). Thus, $h$, under $H_0$, and the pair $(S_0, S_1)$, under $H_1$, are unknown quantities. Summarizing, we are modeling the pixels of the region under test in terms of the conditionally, given the covariance matrices, complex normal model and testing whether the region can be partitioned into two (unknown) subsets corresponding to two (random) covariance matrices (for slick-free and slick-covered areas), as an alternative to the same (random) covariance matrix. According to the Neyman-Pearson criterion, the optimum solution to the hypotheses testing problem (2) is the likelihood ratio test, but, for the case at hand, it cannot be implemented since total ignorance of the parameter $h$, under $H_0$, and the pair $(S_0, S_1)$, under $H_1$, is assumed. We thus switch to a GLRT-based decision scheme. The GLRT is tantamount to replace the unknown parameters with their maximum likelihood estimates under each hypothesis based on the entirety of data, namely to implement the following decision rule

$$\Lambda(r) = \max_{p(r) \in \mathcal{P}_S, H_1} \max_{p(r) \in H_0} \frac{p(r|H_1)}{p(r|H_0)} \geq \gamma$$ \hspace{1cm} (3)

where $r$ denotes the vector obtained by stacking up the vectors $r_{i,j}$, $(i,j) \in S$, $\mathcal{G}(S)$ is a collection of partitions of $S$, $p(r|\mathcal{P}_S, H_1)$ and $p(r|h, H_0)$ are the probability density functions (pdfs) of $r$ under $H_1$ and $H_0$, respectively, and $\gamma$ is the threshold value set in order to ensure the desired probability of false alarm ($P_{fa}$). Subsequent developments require specifying the pdf of $r$ under both hypotheses. Previous assumptions imply that the pdf of $r$, given $M_h$ under $H_0$ and given $M_0$ and $M_1$ under $H_1$, can be written as

$$p(r|M_h, H_0) = \left[\frac{1}{\pi^N |M_h|}\right]^M \text{etr}\left[-M_h^{-1} \sum_{(i,j) \in S} r_{i,j}r_{i,j}^T\right]$$
under the $H_0$ hypothesis and
\[
p(r|\mathcal{P}_S, M_0, M_1, H_0) = \left(\frac{1}{\pi^N |M_0|}\right)^{M_0} \text{etr} \left[-M_0^{-1} \sum_{(i,j) \in \mathcal{S}_0} r_{i,j} r_{i,j}^\dagger\right] \times \left(\frac{1}{\pi^N |M_1|}\right)^{M_1} \text{etr} \left[-M_1^{-1} \sum_{(i,j) \in \mathcal{S}_1} r_{i,j} r_{i,j}^\dagger\right]
\]
under the $H_1$ hypothesis, where $\dagger$ denotes conjugate transpose. Moreover, the pdf of $r$, under $H_0$, can be computed averaging $M_h$ out of the $p(r|\cdot)$
\[
p(r|h, H_0) = \int p(r|M_h, H_0)p(M_h)dM_h.
\]
Similarly, we have that
\[
p(r|\mathcal{P}_S, H_1) = \int \frac{p(r|M_h, H_0)p(M_h)dM_h}{p(M_0)} \times p(M_1)p(M_0)dM_1dM_0.
\]
Using the fact that the inverse Wishart distribution is a conjugate prior [16], the above integrals can be easily computed (see also [12]) and, after some algebra, the GLRT can be re-written (up to an irrelevant positive factor) as
\[
\min_{h \in \{0,1\}} \Gamma_N(\nu_h) \left|\sum_{(i,j) \in \mathcal{S}} r_{i,j} r_{i,j}^\dagger + \left(\nu_h - N\right) M_h\right|^{\nu_h+M} \times \max_{\mathcal{P}_S \in \mathcal{G}(S)} \left\{ \Gamma_N(\nu_0 + M_0) \left|\sum_{(i,j) \in \mathcal{S}_0} r_{i,j} r_{i,j}^\dagger + \left(\nu_0 - N\right) M_0\right|^{\nu_0+M_0} \middle| \frac{\Gamma_N(\nu_1 + M_1) \left|\sum_{(i,j) \in \mathcal{S}_1} r_{i,j} r_{i,j}^\dagger + \left(\nu_1 - N\right) M_1\right|^{\nu_1+M_1}}{\Gamma_N(\nu_0 + M_0) \left|\sum_{(i,j) \in \mathcal{S}_0} r_{i,j} r_{i,j}^\dagger + \left(\nu_0 - N\right) M_0\right|^{\nu_0+M_0}} \right\} \frac{H_1}{H_0} > \gamma.
\]
(4)

Now it only remains to perform maximization with respect to the unknown partition $\mathcal{P}_S$ of the set $S$ under the $H_1$ hypothesis. As in [10], [17], we propose to maximize over a finite subset of partitions $\mathcal{G}(S)$ of the scene under test. A possible choice for the elements of the set $\mathcal{G}(S)$ is given in Fig. 1: the dark zone represents the set $S_1$, while the white one indicates the set $S_0$; a more detailed discussion on the choice of such templates is reported in Sect. III. The above derivation can be straightforwardly extended to the case that returns from more than one frequency band are available. To this end, we denote by $r^{(B)}$ the polarimetric vector obtained by stacking up the vectors associated with returns from pixels of the region under test in the $B$ frequency band, $r^{(B)}_{i,j}, (i,j) \in \mathcal{S},$ say. The GLRT on two frequency bands can be re-written as
\[
\min_{h \in \{0,1\}} \max_{\mathcal{P}_S \in \mathcal{G}(S)} \Lambda^{(B_1)}(r^{(B_1)}) \Lambda^{(B_2)}(r^{(B_2)}) \frac{H_1}{H_0} > \gamma
\]
(5)

where
\[
\Lambda^{(B)}(r^{(B)}) = \frac{\Gamma_N(\nu^{(B)}_h)}{\Gamma_N(\nu^{(B)}_h + M)} \times \frac{\left|\sum_{(i,j) \in \mathcal{S}_0} r^{(B)}_{i,j} r^{(B)}_{i,j} + \left(\nu^{(B)}_h - N\right) M^{(B)}_h\right|^{\nu^{(B)}_h + M}}{\Gamma_N(\nu^{(B)}_0 + M_0) \left|\sum_{(i,j) \in \mathcal{S}_0} r^{(B)}_{i,j} r^{(B)}_{i,j} + \left(\nu^{(B)}_0 - N\right) M^{(B)}_0\right|^{\nu^{(B)}_0 + M_0}} \times \frac{\Gamma_N(\nu^{(B)}_1 + M_1)}{\Gamma_N(\nu^{(B)}_0 + M_0) \left|\sum_{(i,j) \in \mathcal{S}_1} r^{(B)}_{i,j} r^{(B)}_{i,j} + \left(\nu^{(B)} - N\right) M^{(B)}_1\right|^{\nu^{(B)} + M_1}}.
\]
(6)

GLRTs (4) and (5) will be referred to in the following as Bayesian edge detectors (B-EDs).

III. PERFORMANCE ASSESSMENT

In this section, we assess the performance of the proposed algorithm using both simulated and real data. For comparison purposes we also consider a previously proposed, classical (i.e., non-Bayesian) ED, referred to in the following as deterministic ED (D-ED) [7], [8], [10], [11]. All of the needed parameters have been set based on a polarimetric and multifrequency SAR image (L and C frequency bands) collected during the SIR-C/X-SAR mission. More specifically, we have processed non-calibrated, single-look complex data of the North Sea (Germany) collected at 54°58′N 7°45′E, October 6, 1994, with resolution of 22 m in ground range and 6.2 m in azimuth. In particular, both EDs assume $N = 3$ and use VV, VH, and VH+HV polarimetric channels. An image of the sensed scene is displayed in Fig. 2: it contains several slicks, deployed as part of an experimental campaign aimed at assessing to what extent polarimetric and multifrequency SAR data could be exploited for detecting surface films with different viscoelastic properties, see [5], [11] and references therein for more details. Remember that the B-ED requires knowledge of the matrices $M^{(B)}_0$ and $M^{(B)}_1$. To this end, we have estimated $M^{(B)}_0$, $B = L, C$, by the sample covariance matrix based on the overall data of the available SIR-C/X-SAR image in the $B$ band. As to $M^{(B)}_1$, we have used the following matrices
\[
M^{(C)}_1 = \sigma^{2(C)} \text{diag}(6, 3, 2)
\]
and
\[
M^{(L)}_1 = \sigma^{2(L)} \text{diag}(13, 4, 1)
\]
with $\sigma^{2(C)} = \sigma^{2(L)} = 10^{-3}$. Again, the values on the diagonal of the above matrices roughly fit those obtainable from real data (over pixels of the slicks). Moreover, the B-ED assumes $\nu^{(B)}_h = 5$, $h = 0, 1$, corresponding to a high variability of the $R_{i,j}$. The $P_{fa}$ is set to $10^{-5}$ and the corresponding thresholds
have been evaluated over $10^5$ Monte Carlo runs using data simulated according to the models described in Section II with $h = 0$ (slick-free area).

We have thus applied the B-ED and the D-ED to the simulated image of Fig. 3 and to the real image shown in Fig. 2. The tests have been applied over overlapping windows (of $M$ pixels) with the current one obtained by shifting the previous window of 1 pixel in range or in azimuth. In Figs. 4-5 and 6-7 we report the output of the EDs, fed by the synthetic image of Fig. 3, for the cases $M = 16$ and $M = 64$, respectively; for the case $M = 16$, the superiority of the B-ED, compared to the D-ED, is apparent, while for $M = 64$ the two EDs provide a similar performance. At least for this simulated scenario, this means that the B-ED allows to maintain lower levels of computational complexity (i.e., lower values of $M$) while ensuring a good performance. Furthermore, note that the choice made for $G(S)$ in Fig. 1 takes into account only straight edges (horizontal, vertical, and diagonal), while the possible shapes of the actual edges can be different. However, from our experience, mainly matured on real images, the inclusion of additional edge patterns in $G(S)$ does not seem to improve the final performance. Among other, this has the obvious advantage of maintaining a lower computational complexity.

In Figs. 8-9 we report, instead, results obtained applying tests to the real data shown in Fig. 2. The original image has been decimated by a factor 2 in range and 3 in azimuth in order to come up with independent adjacent pixels [18] (given the VV or HH polarimetric channel, and given the $R_{i,j}$). We set $M = 100$ in order to obtain satisfactory performance on real data ($M = 64$ provides practically the same performance). Inspection of the figures highlights that five out of the seven deployed slicks are detected. Moreover, it is apparent that

- the B-ED guarantees an actual $P_{fa}$ closer to the nominal one (i.e., $10^{-3}$) than that of the D-ED;
- the detectors have almost the same power; actually, the B-ED is slightly superior to the D-ED.

We have also modified both detectors replacing the templates of Fig. 1 with those used in [7], namely introducing a set of filters characterized by a space guard. Now the EDs compare two well-separated rectangular areas. This choice has been made because, usually, a real SAR image of a slick does not present very sharp edges, but soft transitions from one region to another. By the inclusion of a space guard between $S_0$ and $S_1$, we expect to select more homogeneous sub-regions and possibly increase the detection performance. In particular, we set the distance between the two areas equal to two pixels along the $x$ or $y$ direction, depending on the actual orientation of the filter. The angular increment between “consecutive orientations” is $\pi/4$ radians corresponding to eight different
Fig. 5. Output of the D-ED for the image of Fig. 3, $M = 16$, $P_{fa} = 10^{-3}$.

Fig. 6. Output of the B-ED for the image of Fig. 3, $M = 64$, $P_{fa} = 10^{-3}$.

Fig. 7. Output of the D-ED for the image of Fig. 3, $M = 64$, $P_{fa} = 10^{-3}$.

filters. Notice that the modified detectors are no longer the true GLRTs since test statistics are computed on sets of pixels that depend on the filter: different filters correspond to different sets of pixels. Observe also that threshold values necessary to guarantee a preassigned $P_{fa}$ must be re-calculated. Figs. 10 and 11 show that these modified EDs guarantee slightly better performance than those obtainable using the templates of Fig. 1.

We have also computed the performance of the B-ED with $\sigma^2(C) = \sigma^2(L) = 10^{-4}$. For this case the nominal threshold value corresponding to $P_{fa} = 10^{-3}$ returns $P_{fa} \approx 3 \times 10^{-4}$. However, empirically setting the threshold to guarantee $P_{fa} = 10^{-3}$ provides the output of Figure 12 where six out of the seven deployed slicks are detected.

IV. CONCLUSION

This paper has derived and assessed a Bayesian edge detector to be fed by polarimetric, possibly multifrequency, SAR data. It can be used to detect dark spots on the ocean surface and, hence, as the first stage of a system for identification and monitoring of oil spills. It does not require secondary data (namely pixels from a slick-free area), but for the average covariance matrices of polarimetric returns. The performance assessment, carried out using both synthetic and real data, has shown that it can guarantee better performance than a previously-proposed classical (i.e., non-Bayesian) detector. As a final comment, it is instructive to note that the B-EDs are not so different from their classical counterparts; however, in the proposed schemes, we have introduced the a priori knowledge,
Fig. 10. Output of the B-ED fed by a SAR image collected in C and L frequency bands with $M = 100$ and the modified templates; results are based on the real image of Fig. 2 and the nominal threshold to guarantee $P_{fa} = 10^{-3}$.

Fig. 11. Output of the D-ED fed by a SAR image collected in C and L frequency bands with $M = 100$ and the modified templates; results are based on the real image of Fig. 2 but the B-ED uses $\sigma^2(C) = \sigma^2(L) = 10^{-4}$ and a threshold empirically set to guarantee $P_{fa} = 10^{-3}$.

Fig. 12. Output of the B-ED fed by a SAR image collected in C and L frequency bands with $M = 100$ and the modified templates; results are based on the real image of Fig. 2 and the nominal threshold to guarantee $P_{fa} = 10^{-3}$.

namely the $\overline{M}^{(B)}_h$ s, that counterbalances, in a way depending on the $\nu^{(B)}_h$ s, the influence of the sample covariance matrices estimated on the data.

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