Adaptive Filter for Linear Systems with Generalized Unknown Disturbance in Measurements

Yanbo Yang, Yuemei Qin, Yan Liang, Quan Pan and Feng Yang
School of Automation
Northwestern Polytechnical University, Xi’an, China
Email: qinym1990@126.com

Abstract—The paper presents the problem of state estimation of linear stochastic time-varying system with generalized unknown disturbance (GUD) existing in the measurements. Such GUD can reflect the effects of sensor bias, deception jamming, navigation bias and so on. An upper-bound filter (UBF) is designed for such systems, and its optimal parameters are derived so that the minimum upper-bounds filter (MUBF) is obtained. The simulation about tracking a target via a biased sensor shows the effectiveness of the proposed filter.

Keywords—Adaptive filtering, discrete time systems, generalized unknown disturbance.

I. INTRODUCTION

The Kalman Filter (KF), as an optimal linear minimum mean square error estimator, is widely used in many actual application fields, such as target tracking. However, its performance will degrade greatly if the modeling errors caused by modeling uncertainty, system faults, parameter variations and external disturbance cannot be well represented. Such modeling errors can be treated as unknown disturbance (UD) to the nominal model [1]–[3]. Many studies have been made on the filter design of the stochastic systems with UD.

The UD was modeled as a randomly switching parameter obeying a Markov chain, and the corresponding estimation problem was hence transformed into the simultaneous implementation of model-based filtering and model identification [4]. The UD was characterized by a vector of unknown and constant parameters and a modular design technique was proposed to estimate the system states and the unknown parameters in [5]. A robust hybrid estimation algorithm was proposed to estimate the continuous state and the discrete state of a stochastic linear hybrid system with unknown fault inputs [6]. Recently, we proposed a minimum upper-bound filter (MUBF) for linear systems with generalized disturbance input (GDI) - a class of statistically constrained disturbance inputs which can represent an arbitrary linear combination of dynamic disturbance inputs, random disturbance inputs and deterministic disturbance inputs [7]. All above filters are only applicable for linear stochastic systems with the GDI existing in the dynamic model.

If the structured UD exists in both the dynamic model and measurements, then the common idea is to decouple UD with states for obtaining the UD-free dynamic model and measurement model, and then design the optimal filter based on the resultant models. State estimators with global optimality in the sense of unbiased minimum variance were proposed for the linear system with UD [8], [9]. A robust parametrized minimum variance filter was derived for uncertain systems with UD to achieve an optimal compromise between the optimal unbiased minimum-variance filter and a robust Kalman filter [10]. A full order observer was designed and a more general approach for the observer’s design was given [11], and a high-order sliding-mode observer was further designed [12].

By the fact that the UD without any a priori information in measurements was seldomly considered in filter design, we present the state estimation problem under the generalized UD (GUD). An upper-bound filter (UBF) is designed for linear systems and its optimal parameters are derived so that the minimum upper-bounds filter (MUBF) is obtained. The simulation about tracking a target with a biased sensor shows the effectiveness of the proposed method.

This paper is organized as follows. The problem formulation is given in Section II. Section III presents the design of an UBF for linear time-varying systems with the GUD appeared in the measurement equation, and its optimal parameters are also derived for proposing the MUBF. A simulation is given to evaluate and analyze the proposed filter in Section IV. Section V supplies the conclusions.

Throughout this paper, the superscripts “-1” and “T” represent the inverse and transpose operations of a matrix, respectively. $E$ is mathematical expectation. $I$ and $O$ denote the identity matrix and the zero matrix with proper dimensions, respectively. “col” denotes the column vector. For any two square matrices $A$ and $B$, $A - B \geq 0$ and $A - B > 0$ mean that $A - B$ is positive semi-definite and positive definite, respectively. The symbol “$\Delta$” means definition and the notation $\otimes$ refers to the Kronecher product.

II. PROBLEM FORMULATION

Consider a new discrete-time linear stochastic system

\[
\begin{align*}
    x_{k+1} &= F_k x_k + B_k u_k + \Gamma_k q_k \\
    z_{k+1} &= H_{k+1} x_{k+1} + A_{k+1} \delta_{k+1} + v_{k+1}
\end{align*}
\]

where $x_k \in R^n$ is the state vector, $z_k \in R^m$ is the measurement vector, $u_k \in R^l$ is the control input, and $\delta_k \in R^\delta$ is the unknown stochastic disturbance. Matrices $F_k$, $B_k$ and $\Gamma_k$ are known with appropriate dimensions. $H_{k+1}$ and $A_{k+1}$ are the measurement matrix and the disturbance

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...
coefficient matrix with proper dimension, respectively. The process noise $q_k \in \mathbb{R}^q$ and measurement noise $v_k \in \mathbb{R}^m$ are zero mean white noise with known covariance $Q^k \geq 0$ and $R^k \geq 0$, respectively. Noises $q_k$, $v_k$ and the initial state $x_0$ are independent mutually.

**Remark 2.1:** In engineering application, there exist many types of GUDs simultaneously. Hence we consider $\delta_{k+1}$ as the same as that in [7]

$$
E \{ \delta_k q_{k-1}^T \} = O_{s \times p} \quad (\forall j \geq k) \quad (2)
$$

where $O_{s \times p}$ and $O_{s \times m}$ are zero matrices with dimension $s \times p$ and $s \times m$, respectively. As shown in [7], the introduced vector $\delta_{k+1} \in \mathbb{R}^s$ represents the GUD by the fact that it can represent an arbitrary linear weighted sum of $f_1 k, f_2 k$ and $\omega_k$, where $f_1 k$ is an linear time-varying function of $Q^{k-1}, V^k,$ and $\Delta^k$ with $Q^{k-1} \triangleq \begin{pmatrix} q_0^T & \cdots & q_{k-1}^T \end{pmatrix}^T, \ V^k \triangleq \begin{pmatrix} v_1^T & \cdots & v_k^T \end{pmatrix}^T,$ and $\Delta^k \triangleq \begin{pmatrix} \delta_1^T & \cdots & \delta_k^T \end{pmatrix}^T$. $f_{2k}$ is an arbitrary deterministic time-varying function; and $\omega_k$ is white noise independent of $Q^{k-1}, V^k,$ and $\Delta^k$. It is obvious that $f_1 k, f_2 k$ and $\omega_k$ represent a class of UD with dynamic property, deterministic UD and random UD, respectively, and hence the new term $\delta_{k+1}$ in (1) can represent the more general uncertainty.

The aim in this paper is to construct the upper bound of estimation errors and then minimize such bounds in the pursuit of the desirable (optimal) filtering parameters by the following considerations. First, to directly decouple the estimation errors with GUD or estimate covariance of estimation errors always needs the condition about enough measurements, which is hardly satisfied actually. In contrast, constructing the upper bound always needs mild condition much looser than the former. Second, in target tracking, track lose always happens in the case of the significant peak of estimation errors and hence decreasing the peak or its upper bound is actually more desirable.

### III. UPPER-BOUND FILTER DESIGN FOR LINEAR TIME-VARYING SYSTEMS

**Definition 3.1:** A linear filter (3)-(6) for system (1)-(2) is called an upper-bound (UBF) if

- state prediction $\hat{x}_{k+1|k} = F_k \hat{x}_k|k + B_k u_k \quad (3)$
- measurement prediction $\tilde{z}_{k+1|k} = H_{k+1} \hat{x}_{k+1|k} \quad (4)$
- filtering residual $\gamma_{k+1} = z_{k+1} - \tilde{z}_{k+1|k} \quad (5)$
- state estimation $\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \gamma_{k+1} \quad (6)$

if there exists a sequence of positive-definite matrix $P^*_{k+1|k}, S^*_{k+1}$ and $P^*_{k+1|k+1}$ that satisfy

$$
P^*_{k+1|k} \geq P_{k+1|k} \quad (7)
$$

$$
S^*_{k+1} \geq S_{k+1} \quad (8)
$$

$$
P^*_{k+1|k+1} \geq P_{k+1|k+1} \quad (9)
$$

where

- state prediction error $\hat{x}_{k+1|k} \triangleq x_{k+1} - \hat{x}_{k+1|k} \quad (10)$
- state estimation error $\hat{x}_{k+1|k+1} \triangleq x_{k+1} - \hat{x}_{k+1|k+1} \quad (11)$

and the filter gain $K_{k+1}$ is a function of $P^*_{k+1|k}$ and $S^*_{k+1}$.

The GUD $\delta_{k+1}$ in (1) is the barrier to filter implementation because it is impossible to directly estimate the state prediction covariance, innovation covariance and state estimation covariance due to the existence of $\delta_{k+1}$. Therefore, the idea of UBF design is motivated by the fact that determining the upper bounds of covariance requires fewer parameters to be estimated than determining the error covariances, and hence has the looser condition to guarantee the existence.

**Remark 3.1:** After putting (1) and (4) into (5), we have the filtering residual

$$
\gamma_{k+1} = H_{k+1} \hat{x}_{k+1|k} + A_{k+1} \delta_{k+1} + v_{k+1} \quad (12)
$$

By the fact that $v_{k+1}$ is independent of $x_{k+1}$ and $\hat{x}_{k+1|k}$, we obtain the independence between $v_{k+1}$ and $\hat{x}_{k+1|k}$. Further using the independence of $\delta_{k+1}$ and $v_{k+1}$ in (2), then (8) can be rewritten as

$$
S_{k+1} = E \{ \Theta \Theta^T \} = E \{ \Theta \Theta^T \} + R_{k+1} \quad (13)
$$

where $\Theta = H_{k+1} \hat{x}_{k+1|k} + A_{k+1} \delta_{k+1} + v_{k+1}$ and $\widetilde{\Theta} = H_{k+1} \hat{x}_{k+1|k} + A_{k+1} \delta_{k+1}$. As shown in (13), the presence of $\delta_{k+1}$ represents the uncertainty. Thus, we have

$$
\begin{align*}
S_{k+1} & \geq E \left\{ \left( H_{k+1} \hat{x}_{k+1|k} + A_{k+1} \delta_{k+1} + v_{k+1} \right) \cdot \left( H_{k+1} \hat{x}_{k+1|k} + A_{k+1} \delta_{k+1} + v_{k+1} \right)^T \right\} + R_{k+1} \\
& = E \left\{ H_{k+1} \hat{x}_{k+1|k} \cdot \hat{x}_{k+1|k}^T + H_{k+1} \hat{x}_{k+1|k}^T \cdot H_{k+1} \hat{x}_{k+1|k} \right\} + R_{k+1} \\
& = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} + K_{k+1} R_{k+1} K_{k+1}^T.
\end{align*}
$$

The following recursive upper-bound structure is considered:

$$
P^*_{k+1|k} = H_k P^*_{k|k} H_k^T + \Gamma_k Q_k \Gamma_k^T \quad (15)
$$

$$
S^*_{k+1} = H_{k+1} P^*_{k+1|k} H_{k+1}^T + A_{k+1} \left( \tau_{k+1} \Sigma \right) A_{k+1}^T + R_{k+1} \quad (16)
$$

$$
P^*_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P^*_{k+1|k} (I - K_{k+1} H_{k+1})^T + K_{k+1} R_{k+1} K_{k+1}^T \quad (17)
$$

where the adjust factor $\tau_{k+1} \geq 0$ is a parameter to be estimated. Here, $\Sigma > 0$ is the setting matrix. $\Sigma$ can be positive-definite matrix that are given. For example, you can chose it as identity matrix.

After above discussion, it is needed to know whether there exists an UBF based on (15)-(17). If an UBF exists, it is further needed to determine the optimal filter parameters through minimizing the upper-bounds of $P^*_{k+1|k}, S^*_{k+1}$ and $P^*_{k+1|k+1}$ so that the minimum UBF (MUBF) can be obtained. The following theorem provides the solution.
Theorem 3.1: If the following two conditions are satisfied:

i) \( P_{\tau k + 1} \geq P_{\tau k \mid k} \) 

ii) \( S_{\tau k + 1}^* \geq S_{\tau k + 1} \)

then there exists a UBF with structure (15)-(17) and optimal parameters \( \tau_{\tau k + 1} \) and \( K_{\tau k + 1}^{opt} \). For any \( \tau_{\tau k + 1} \) satisfying \( S_{\tau k + 1} \leq S_{\tau k + 1}^* \) and any filter gain \( K_{\tau k + 1} \), there exists

\[
P_{\tau k + 1 \mid k + 1} \bigg| K_{\tau k + 1} \leq P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt}, K_{\tau k + 1}^{opt} \leq P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt}, K_{\tau k + 1}^{opt}
\]

(20)

\[
S_{\tau k + 1} \leq S_{\tau k + 1}^* \bigg| \tau_{\tau k + 1}^{opt} \leq S_{\tau k + 1} \bigg| \tau_{\tau k + 1}
\]

(21)

and the optimal filter parameters are

\[
\tau_{\tau k + 1}^{opt} = min \{ \tau_{\tau k + 1} \mid \tau_{\tau k + 1} \in \Omega_{\tau k + 1} \}
\]

(22)

\[
K_{\tau k + 1}^{opt} = P_{\tau k + 1 \mid k}^{*} H_{k + 1}^{T} S_{\tau k + 1}^{-1}
\]

(23)

where the set \( \Omega_{\tau k + 1} \Delta \{ \tau_{\tau k + 1} \mid \tau_{\tau k + 1} \geq 0, S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \geq S_{\tau k + 1} \} \).

Proof:

(1) Existence of UBF.

As the first condition of Theorem 3.1, and we assume

\[
P_{\tau k + 1}^* \geq P_{\tau k \mid k}
\]

Using mathematical induction and \( P_{\tau k + 1 \mid k} = F_k P_{\tau k \mid k} F_k^{T} + \Gamma_k Q_k T_k \), we can have

\[
P_{\tau k + 1 \mid k} - P_{\tau k + 1 \mid k} = F_k \bigg( P_{\tau k \mid k} - P_{\tau k \mid k} \bigg) F_k^{T} \geq 0
\]

(24)

As \( F_k \) is a full rank square matrix, so (25) can get as following

\[
P_{\tau k + 1 \mid k} - P_{\tau k + 1 \mid k} \geq 0
\]

(25)

Left multiply \( F_k \) and right multiply \( F_k^{T} \) in (25), then leads to

\[
P_{\tau k + 1 \mid k} - P_{\tau k + 1 \mid k} \geq 0
\]

(26)

So we can obtain (7). Putting (7) into (17) yields

\[
P_{\tau k + 1 \mid k + 1} = (I - K_{\tau k + 1} H_{k + 1}) P_{\tau k + 1 \mid k} (I - K_{\tau k + 1} H_{k + 1})^{T} + K_{\tau k + 1} R_{\tau k + 1} + K_{\tau k + 1}^{T}
\]

(27)

As \( P_{\tau k + 1 \mid k + 1} = (I - K_{\tau k + 1} H_{k + 1}) P_{\tau k + 1 \mid k} (I - K_{\tau k + 1} H_{k + 1})^{T} + K_{\tau k + 1} R_{\tau k + 1} + K_{\tau k + 1}^{T} \), then (9) can be obtained.

(2) Optimal Filter Parameters.

According to Definition 3.1, the set \( \{ \tau_{\tau k + 1} \mid S_{\tau k + 1}^* \geq S_{\tau k + 1} \} \) will not be empty if an UBF exists. Because \( S_{\tau k + 1}^* \mid \tau_{\tau k + 1} > S_{\tau k + 1} \mid \tau_{\tau k + 1} \) when \( \tau_{\tau k + 1} \leq \tau_{\tau k + 1} \), so if \( \tau_{\tau k + 1} \in \Omega_{\tau k + 1} \), then must have \( \tau_{\tau k + 1} \geq 0 \). Therefore \( \Omega_{\tau k + 1} = \{ \tau_{\tau k + 1} \mid \tau_{\tau k + 1} \geq 0, S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \geq S_{\tau k + 1} \} \cap \{ \tau_{\tau k + 1} \mid S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \geq S_{\tau k + 1} \} \) is not empty. Hence there exists

\[
\tau_{\tau k + 1}^{opt} = min \{ \tau_{\tau k + 1} \mid \tau_{\tau k + 1} \in \Omega_{\tau k + 1} \}
\]

Then, it is only needed to testify that \( F_{\tau k + 1}^{opt} \) and \( K_{\tau k + 1}^{opt} \) guarantee (20)-(21). It is easy to know \( \tau_{\tau k + 1}^{opt} \) to any \( \tau_{\tau k + 1} \in \Omega_{\tau k + 1} \) according to the definition of \( \tau_{\tau k + 1}^{opt} \). Thus

\[
\Delta S_{\tau k + 1} = S_{\tau k + 1}^* - S_{\tau k + 1} \leq S_{\tau k + 1}^* - S_{\tau k + 1} \bigg| \tau_{\tau k + 1}^{opt} \geq 0
\]

(28)

\[
\Delta P_{\tau k + 1 \mid k + 1} = P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt}, K_{\tau k + 1}^{opt} - P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt}, K_{\tau k + 1}^{opt}
\]

(29)

In deriving (28), both the expression of \( S_{\tau k + 1}^* \) in (16) and the equality (30) are used. The equation (30) holds because the factor \( R_{\tau k + 1} \) is not included in (15). To obtain (29), the equality (30), the covariance update expression \( P_{\tau k + 1 \mid k + 1}^* \) in (31) and \( S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \leq S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \) are used. The last inequality holds because \( S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \leq S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \) according to (28),

\[
P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt} - P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt}
\]

(30)

Through (16) and \( R_{\tau k + 1} > 0 \), \( S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \geq R_{\tau k + 1} > 0 \) exists. So the symmetric and positive definite matrix \( S_{\tau k + 1}^* \mid \tau_{\tau k + 1} \) can be represented as \( G_{\tau k + 1}^{T} G_{\tau k + 1} \), where \( G_{\tau k + 1} \) is of full rank. Suppose the optimal gain \( K_{\tau k + 1}^{opt} \) exists, and then its existence should be testified. After transformation, there is

\[
P_{\tau k + 1 \mid k + 1}^* \bigg| \tau_{\tau k + 1}^{opt}, K_{\tau k + 1}^{opt} = P_{\tau k + 1 \mid k}^{*} \bigg| \tau_{\tau k + 1}^{opt} - B_k + B_k T_{k + 1}^{*}
\]

(32)

where \( B_k + 1 = P_{\tau k + 1 \mid k}^{*} \bigg| \tau_{\tau k + 1}^{opt}, H_{k + 1}^{T} G_{\tau k + 1}^{T} \)

The minimum value of (32) is \( P_{\tau k + 1 \mid k}^{*} \bigg| \tau_{\tau k + 1}^{opt} - B_k + B_k T_k^{*} \) for \( K_{\tau k + 1} G_{\tau k + 1} - B_k + (K_{\tau k + 1} G_{\tau k + 1} - B_k) T \geq 0 \). Thus, \( P_{\tau k + 1 \mid k + 1}^{*} \bigg| \tau_{\tau k + 1}^{opt}, K_{\tau k + 1}^{opt} \) reaches its optimal solution value if and
Step 1: Initialization. Give the initial state $\hat{x}_{0|0}$ and covariance $P_{0|0}^*$. 

Step 2: Prediction. Compute $\hat{x}_{k+1|k}$, $\hat{z}_{k+1|k}$ and $P_{k+1|k}^*$ by (3), (4) and (15), respectively.

Step 3: Filtering Residual. Compute the filtering residual by (5).

Step 4: Innovation covariance. Substitute $S_{k+1}$ by its unbiased estimate $\hat{S}_{k+1} \triangleq \gamma_{k+1} G_{k+1}^T G_{k+1} \gamma_{k+1}^T$.

Step 5: Convex Optimization.

(a) Inequality Constraint. Substitute corresponding terms into (35), then solve out $\tau_{k+1}$.

(b) Innovation Correct. Recompute the innovation covariance by (16).

(c) Filter Gain. Compute filter gain by (23).

Step 6: Estimation. Update the state estimation $\hat{x}_{k+1|k+1}$ by (6) and the covariance $P_{k+1|k+1}^*$ by (17).

### TABLE I: The UBF Algorithm

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initialization.</td>
</tr>
<tr>
<td>2</td>
<td>Prediction.</td>
</tr>
<tr>
<td>3</td>
<td>Filtering Residual.</td>
</tr>
<tr>
<td>4</td>
<td>Innovation covariance.</td>
</tr>
<tr>
<td>5</td>
<td>Convex Optimization.</td>
</tr>
<tr>
<td>6</td>
<td>Estimation.</td>
</tr>
</tbody>
</table>

IV. SIMULATION

In this part, we consider an example of tracking a target with the biased sensor to evaluate the performance of the UBF. Assume Radar sensor is located at the origin of the common Cartesian coordinate system. The target does constant-velocity movement with time. The state vector is the target state vector, which is $x(t_k) = \{x_T(t_k), \dot{x}_T(t_k), y_T(t_k), \dot{y}_T(t_k)\}$. Considering the reality measurement of Radar sensor, then the observation is given as following.

$$ z_k = \begin{bmatrix} r(t_k) \\ \theta(t_k) \end{bmatrix} = \begin{bmatrix} \sqrt{x_T^2(t_k) + y_T^2(t_k)} \\ \arctan\left(\frac{y_T(t_k)}{x_T(t_k)}\right) \end{bmatrix} + A_k [\Delta r_k, \Delta \theta_k]^T + v_k $$

The parameters of the target state are given by initial position: (40km, 50km), initial velocity: (0.5km/s, 0.6km/s), standard deviation of process noise is 0.002km in both $x$ and $y$ directions. The corresponding matrices are $F_k = I_2 \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $\Gamma_k = I_2 \otimes \begin{bmatrix} T^2/2 & 0 \\ 0 & T \end{bmatrix}$ and $A_k = 3I_2$, respectively.

The parameter of measurement equation in (1) is

$$ H_k = \begin{bmatrix} \frac{\partial \sqrt{x_T^2(t_k) + y_T^2(t_k)}}{\partial x_T(t_k)} & \frac{\partial \sqrt{x_T^2(t_k) + y_T^2(t_k)}}{\partial y_T(t_k)} \\ \frac{\partial \arctan\left(\frac{y_T(t_k)}{x_T(t_k)}\right)}{\partial x_T(t_k)} & \frac{\partial \arctan\left(\frac{y_T(t_k)}{x_T(t_k)}\right)}{\partial y_T(t_k)} \end{bmatrix} |_{x(t_k) = \hat{x}_{k|k-1}} $$

as that in the extended Kalman Filter (EKF). The measurement noise variances of the sensor is $R_{k+1} = \text{diag}\{0.2km^2, 0.002rad^2\}$. The unknown stochastic system biases vector is given as $\delta(t_k) = \{\Delta r_k, \Delta \theta_k\}$ with $\Delta r_k$ and $\Delta \theta_k$ representing the measurement bias of the sensor in range and angle direction, respectively. Assume the system...
biases vary as follows.

\[
\Delta r_k = \begin{cases} 
0, & 1 \leq k \leq 30 \\
0.5 \text{ km}, & 31 \leq k \leq 70 \\
-0.5 \text{ km}, & 71 \leq k \leq 90 \\
0.6 \text{ km}, & 91 \leq k \leq 100 
\end{cases}
\]

\[
\Delta \theta_k = \begin{cases} 
0, & 1 \leq k \leq 25 \\
0.005 \text{ rad}, & 26 \leq k \leq 60 \\
-0.004 \text{ rad}, & 61 \leq k \leq 85 \\
0.007 \text{ rad}, & 86 \leq k \leq 100 
\end{cases}
\]

where \( k \) is the simulation step number. The matrix \( \Sigma \) is set as \( \text{diag}\{1^2, 0.01^2\} \). The observation interval \( T \) is 1s and 100 steps are run in the simulation.

In the simulation, we choose the KF as the compared algorithm by modeling the system biases as a part of the state vector with zero initialization, assuming the bias dynamics are time-invariant and substituting the measurement marix as \( [H_{k+1}^T; A_{k+1}^T]^T \). By this way, the result figures obtained by MUBF and KF via 100 Monte Carlo simulations are shown in Figs. 2-5.

Figs. 2-3 show the RMSEs (root of mean square errors) of target positions along \( x \) and \( y \) directions. From these two figures, it is obvious that the RMSEs obtained by the MUBF are very small and convergent while the RMSEs obtained by the KF are divergent. From Figs. 4-5, it is shown that the RMSEs obtained by the MUBF of target velocities along both \( x \) and \( y \) directions are well convergent, but they obtained by the KF are divergent. Therefore, the proposed MUBF is efficient in dealing with the situation with GUD existing in measurements.

V. CONCLUSION

This paper considers a discrete-time linear stochastic system with GUD in the sensor measurements. An UBF is proposed, and the optimal filter parameters are derived so that the MUBF is obtained. The simulations show that the proposed MUBF is effective, compared with the classical KF method.

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REFERENCES


