Control of sensor with unknown clutter and detection profile using Multi-Bernoulli filter

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Abstract—This paper builds on the recently developed adaptive multi-Bernoulli filter, proposing a novel sensor control solution within the multi-object filtering scheme. Our sensor control method does not need any prior information on clutter and sensor field-of-view parameters. In addition, our control objective is based on the novel strategy of minimizing the uncertainties (quantified by variance) of the cardinality, and object state estimates as well as the estimated rate of clutter. In terms of computation, our method is efficient, as it does not need to perform Monte Carlo sampling in the space of measurement sets. This method is particularly useful in space situational awareness applications such as detection and tracking of space junk, as currently, there is limited information on the distribution of traceable objects in the space and clutter, and our method can effectively handle such uncertainties.

I. INTRODUCTION

In multi-object systems, apart from the process and measurement noise, clutter measurements and uncertain sensor field of view (FoV) are additional sources of uncertainty. Knowledge of clutter intensity and sensor FoV are presumed, even though they are not available in space object tracking applications. This paper focuses on the sensor control problem in a multi-object system where the clutter rate and detection profile are unknown. The aim of sensor control is to find the most rewarding control command amongst a finite set of admissible commands via a decision making process in such a way that the utility of the received measurement(s) is maximized. Multi-object Bayesian filtering framework, assumes that there are uncertainties associated with both the state and the observation spaces, e.g. non-linearity, noise, clutter and detection uncertainties and, sensor control decisions are affected by these uncertainties. Having the previous decisions and observations up to a given time, the sensor control problem can be cast as a partially observed Markov decision process (POMDP) [1].

A number of different approaches have recently been proposed to solve the sensor control problem. Mahler introduced the Finite Set Statistics (FISST) theory and proposed several approximations to Bayes multi-object filter, such as Probability Hypothesis Density (PHD) and Cardinalized PHD filters. Then he proposed a new approach to the sensor control problem by applying PHD filter in conjunction with Csiszár or f-divergence as the reward function. Later, he introduced a new reward function named posterior expected number of objects (PENT) [2]–[4]. Applications of PENT in space-based surveillance was investigated by Zatezalo et al. [5]. Ristic et al. derived analytical expressions for Rényi divergence between two Poisson and i.i.d. cluster random finite sets [6], [7] to be used as a cost function to solve sensor control problem.

A common strategy in probabilistic sensor control is to use an information theoretic objective function to maximize the existing information embedded in the observations. Such objective functions quantify the difference between two posterior multi-object densities. In this approach, the control command is chosen to provide the maximum difference between the updated multi-object density and its prediction [6], [8]. Using the FISST framework, in two papers, Ristic et al. [6], [7] proposed Rényi divergence as the objective function to compare prediction and update densities divergence. In the first paper, Rényi’s divergence function implementation was investigated for the general form of multi-object filter in FISST context. The Computational intractability of this approach, led to the second paper [7]. In the second paper, the Cardinalized Probability Hypothesis Density (CPHD) filter was used to carry out the filtering task and Rényi divergence function was formulated in the CPHD filtering framework.

Computing Rényi’s divergence function needs Monte Carlo (MC) sampling in both the state and observation spaces and the computational complexity significantly increases with the number of objects. This problem was alleviated by a new approach independently proposed in [9], [10]. Our approach is based on minimizing the statistical expectation of the variance of the multi-Bernoulli cardinality. The proposed approach can be easily computed from the cardinality balanced multi-object multi-Bernoulli filter (CB-MeMBer). In various multi-object state estimation applications of multi-Bernoulli random finite set have lead promising results [11]–[15]. We showed that our approach outperforms Rényi’s divergence implementation of PHD-based filters both in terms of accuracy and computational complexity.

In this paper we proposed a novel sensor control solution within the adaptive multi-Bernoulli filtering scheme developed in [16]. In [17]–[20], Mahler et al. proposed a general method-
ology to address uncertainties caused by unknown clutter intensity and sensor FoV for PHD and CPHD filter. That approach was later extended to the CB-MeMBer filter by Vo et al. [16].

We employ the filter developed in [16] because the CB-MeMBer filter multi-object state estimation does not require clustering in SMC implementation. The contributions of this paper are twofold. Firstly, unlike the existing solutions developed for sensor control within Bayesian multi-object filtering schemes, our method does not rely on knowing the clutter and sensor FoV parameters a priori. Secondly, we propose a new cost function (as an objective function) based on minimizing the uncertainties in the estimated cardinality and states of the objects as well as uncertainties in adaptive estimates of the rate of clutter, to solve the sensor control problem. The use of proposed cost function reduces the computational complexity while increases accuracy of the state estimation.

II. OVERVIEW OF ADAPTIVE MULTI-BERNOULLI FILTERING

The (FISST) framework provides a rigorous foundation for multi-object tracking via the Bayes multi-object filter. The FISST approach to multi-object filtering has significantly impacted on a wide range of research areas such as autonomous vehicles and robotics [21], radar tracking [22] and image processing [23].

A. Multi-object Bayes Filter

In FISST formulation, state and observation spaces are represented by random finite sets (RFSs) whose numbers of elements (cardinality) and elements values randomly vary for each time step $k$. The state and observation RFSs are denoted as follows:

$$
X_k = \{ x_k^i : i = 1 \ldots M(k) \} \in \mathcal{F}(\mathcal{X})
$$

$$
Z_k = \{ z_k^i : i = 1 \ldots N(k) \} \in \mathcal{F}(\mathcal{Z})
$$

in which $M(k)$ and $N(k)$ represent the number of objects and measurements, $\mathcal{X}$ and $\mathcal{Z}$ are the state and observation spaces, and $\mathcal{F}(\cdot)$ is the ensemble of all subsets. In Bayesian recursion to compute the posterior density of the multi-object state propagated from a given prior density, FISST framework provides consistent notions of integration and density for RFSs [24].

The general form of the multi-object Bayes filter is computationally intractable [24] and a number of practical approximations have been suggested. The probability Hypothesis Density (PHD) filter [25] and its extended version, the cardinalized PHD filter [26] were introduced by Mahler. Both PHD and CPHD filters propagate the first moment of the multi-object density while CPHD provides a means for estimation of the cardinality density. In addition to the PHD and CPHD filters, Mahler proposed the multi-Bernoulli filter (MeMBer) [24]. Unlike the PHD-based filters, MeMBer filter is based on approximating the posterior multi-object density by multi-Bernoulli densities. Later, Mahler’s MeMBer filter was analytically shown to overestimate the cardinality of the multi-object state, and the cardinality balanced MeMBer (CB-MeMBer) filter was proposed [14]. Different sequential Monte Carlo (SMC) and Gaussian mixture (GM) implementations for RFS-based filters have also been proposed. Some examples are SMC and GM implementations of PHD and CPHD filters [27]–[31] complemented with investigation of their convergence [32], [33], and SMC and GM implementations of CB-MeMBer filter [14].

B. Adaptive CB-MeMBer Filter

A multi-Bernoulli RFS is the ensemble of $M$ independent Bernoulli RFSs. Each Bernoulli RFS is characterized by the probability of existence of a possible element $r^{(i)}$ and the probability density function (pdf) of the state of that element $p^{(i)}(\cdot)$ ($i = 1, \ldots, M$). Mahler showed that all statistical properties of a multi-Bernoulli RFS can be completely characterized by the set of $M$ parameter pairs [24]. He also derived a multi-Bernoulli Bayesian filter, and showed that if the prior distribution of the multi-object random set state is multi-Bernoulli, the predicted and updated posteriors are also approximately multi-Bernoulli. Vo et al. [14] presented Cardinality-Balanced MeMBer (CB-MeMBer) filter to tackle the problem of bias in cardinality estimates given by the original MeMBer filter. In these implementations, clutter intensity and probability of detection are assumed to be known a priori. Vo et al. [16] have recently tackled the problem of multi-Bernoulli filtering in cases where clutter intensity and sensor FoV are assumed to be unknown and non-homogeneous. The underlying idea is as follows:

- Propagate probability of detection for sensor FoV: Modify the original filter to implicitly estimate the unknown detection probability. It has been achieved by accommodating a state dependent variable as an augmented state. The space of detection probability is denoted by $\mathcal{X}^{(\Delta)} = [0, 1]$.
- Accommodate an unknown and non-homogeneous clutter intensity: Extend the original filter to incorporate a set of clutter generators in which they dynamically distribute themselves inside the surveillance area. This is carried out by Poisson assumption for clutter random sets and modelling the clutter generators behaviour similar to actual objects in a hybrid space denoted by $\tilde{\mathcal{X}} = \mathcal{X}^{(0)} \cup \mathcal{X}^{(1)}$ ($0$ for clutter generators state space and $1$ for actual objects state space). In this set up, the clutter generator has its own transition and observation model.

The new extended state space is Cartesian product of actual objects space, clutterers space and probability of detection space denoted by:

$$
\tilde{\mathcal{X}} = \mathcal{X}^{(\Delta)} \times \mathcal{X} \times \{0, 1\},
$$

and any arbitrary function on the new state space is shown by:

$$
\tilde{f} = f(a, x)
$$

where $a$ and $x$ denote clutter and object state values, respectively. The integral of a function $\tilde{f} : \tilde{\mathcal{X}} \to \mathbb{R}$ is given by

$$
\int_{\tilde{\mathcal{X}}} f(\tilde{x}) d\tilde{x} = \int_{\mathcal{X}^{(\Delta)} \times \mathcal{X}} f^{(0)}(x) da dx + \int_{\mathcal{X}^{(\Delta)} \times \mathcal{X}} f^{(1)}(x) da dx.
$$
where for $i,j$ the multi-Bernoulli density of the multi-object state is given by \( \{(v_{i,j}^{(i)}(x))\}_{i=1}^{M_{k-1}} \) and each $p_{k-1}^{(i)}(u)$ for $i = 1, \ldots, M_{k-1}$ and $u = 0, 1$ is approximated by a set of particles:

$$
\pi_{k|k-1} = \sum_{u=0,1} \left\{(r_{P,k}^{(i)}(x), \delta_{k-1}^{(i)}(x), x)\right\}_{i=1}^{M_{k-1}} \cup \{(r_{P,k}^{(i)}(x), \delta_{k-1}^{(i)}(x))\}_{i=1}^{M_{k-1}}
$$

where existence probabilities and distributions of the predicted Bernoulli components are given by:

$$
r_{P,k}^{(i)}(x) = \sum_{u=0,1} \left[ \sum_{j=1}^{L_{i,k}^{(u)}} \delta_{y_{i,j}^{(u)}}(x) \pi_{k|k-1} \right] P_{k-1}^{(i,j)}(x),
$$

$P_{k-1}^{(i)}(x)$ is birth model parameters

$$
\pi_{k|k-1} = \sum_{u=0,1} \left[ \sum_{j=1}^{L_{i,k}^{(u)}} \delta_{y_{i,j}^{(u)}}(x) \pi_{k|k-1} \right] P_{k-1}^{(i,j)}(x),
$$

where for $u = 0, 1$

$$
a_{r_{P,k}^{(i)}} = \sum_{j=1}^{L_{i,k}^{(u)}} \delta_{y_{i,j}^{(u)}}(x) \pi_{k|k-1} P_{k-1}^{(i,j)}(x)
$$

If at time $k$, the predicted multi-Bernoulli distribution is given by \( \{(v_{i,j}^{(i)}(x))\}_{i=1}^{M_{k-1}} \) and $p_{k|k-1}^{(i)}(a,x) = \sum_{j=1}^{L_{i,k}^{(u)}} \delta_{y_{i,j}^{(u)}}(x) \pi_{k|k-1} P_{k-1}^{(i,j)}(x)$, then updated multi-Bernoulli is represented by the union of legacy tracks and measurement-corrected tracks [16] as follows:

$$
\pi_{k} = \left\{(r_{L,k}^{(i)}(x), \delta_{k}^{(i)}(x))_{i=1}^{M_{k-1}} \cup \{(U_{k}(z), P_{U,k}(z))\}_{z\in Z_{k}} \right\}.
$$

with the following existence probabilities and singleton densities:

$$
r_{L,k}^{(i)} = \sum_{u=0,1} \delta_{k}^{(i)} P_{k}^{(i)}(u),
$$

$$
\pi_{L,k}^{(i)}(a,x) = \sum_{u=0,1} \left[ \sum_{j=1}^{L_{i,k}^{(u)}} \delta_{y_{i,j}^{(u)}}(x) P_{k-1}^{(i,j)}(x) \right] P_{k}^{(i)}(u),
$$

where for $u = 0, 1$

$$
\delta_{k}^{(i)} = \sum_{j=1}^{L_{i,k}^{(u)}} \delta_{y_{i,j}^{(u)}}(x) P_{k-1}^{(i,j)}(x) P_{k}^{(i)}(u),
$$

Similar to the original CB-MeMBer filter, its adaptive form includes a resampling step for each track to obviate degeneracy problems. The numerical explosion of the generated particles is prevented by thresholding, the existence probabilities and the low probability tracks are removed while similar ones are merged. Having the updated multi-Bernoulli parameters, EAP estimates of the cardinality and state of the multi-Bernoulli set can be calculated in similar fashion to CB-MeMBer, but using the parameters with $u = 1$. It has also been shown that the EAP
estimate of the clutter rate is given by [16]
\[ \hat{\lambda}_k = \sum_{i=1}^{M_k} \frac{L(i)(0)}{x(i)(0)} \sum_{j=1}^{W(i)(0)} u_{i,j}(0)\hat{u}_{i,j}(0), \]

(7)

III. SENSOR CONTROL

Sensor control problem is commonly defined as a stochastic control problem. In this problem a finite set of admissible control commands that would maximize the utility of the future measurements (quantified by an objective function) for a given estimation problem is sought [34]. Proper definition of an objective (reward or cost) function that is meaningful and computationally tractable, is at the core of any solution devised for this problem. Objective functions for sensor control are commonly defined in the partially observable Markov decision process (POMDP) framework with one step-ahead planning (myopic policy).

A. Sensor control framework

Following [1], we formulate the sensor control problem in the partially observable Markov decision process (POMDP) framework. In this, we use an adaptive state estimation method (presented in [16]) to concurrently solve the sensor control and multi-object estimation problems with unknown clutter intensity and sensor FoV.

POMDP is a generalized form of Markov decision process (MDP) [35] which is suitable for sensor planning manifested by a series of control commands (actions). In POMDP framework, there is no direct access to the states and decision are only arrived using uncertain observations. Mathematically, a POMDP at any time step k could be defined as a tuple:
\[ \Psi = \{X_k, S, f_{k|k-1}(x_k|x_{k-1}), Z, g_k(z|x), \vartheta(X_{k-1}, s, X_k)\}, \]

where \( X_k \) is a finite set of single-object states; \( S \) defines a finite set of sensor control commands; \( f_{k|k-1}(x_k|x_{k-1}) \) is a transition model for single-object state; \( Z \) comprises a finite set of observations; \( g_k(z|x) \) is a stochastic measurement model; and \( \vartheta(X_{k-1}, s, X_k) \) is an objective function and returns a reward or cost for transition from the multi-object state \( X_{k-1} \) to the state \( X_k \) by applying an action command \( s \in S \).

The purpose of sensor control is to find the control command \( \hat{s} \) that optimizes the objective function (cost or reward). In stochastic filtering, where the multi-object states \( X_{k-1} \) and \( X_k \) are characterized by their distributions, the control command \( \hat{s} \) is commonly chosen to minimize the statistical expectation of the cost function \( \vartheta(X_{k-1}, s, X_k) \) (or maximize in case of a reward function),
\[ \hat{s}_k = \arg\min_{s \in S} \{E_{X_{k-1}, X_k} [\vartheta(X_{k-1}, s, X_k)]\}. \]

(8)

B. Adaptive multi-Bernoulli sensor control

One of the first approaches to sensor scheduling for object tracking using sequential Monte Carlo is proposed in [36]. As it was mentioned earlier, for multi-object problems the sensor control solution proposed by Mahler [3] and Ristic et al. [6], assume that clutter intensity and sensor FoV are known parameters. In section II, we represented the lay out of an adaptive multi-Bernoulli filter [16], in which clutter intensity and sensor FoV are not required a priori. In this paper, we propose a sensor control solution that works within this adaptive multi-Bernoulli filtering scheme. The outline of proposed method is as follows. Suppose that at time \( k - 1 \), the posterior multi-object density is approximated by multi-Bernoulli RFS of the form of
\[ \pi_{k-1} = \left\{ \left( p_{k-1}^{(i)} \right)_{i=1}^{M_{k-1}} \right\}, \]

where \( M_{k-1} \) indicates the maximum possible number of targets, \( p_{k-1}^{(i)}(\ldots,u) = p_{k-1}^{(i)}(u,\ldots) \) for \( u = 0, 1 \) (\( u = 0 \) refers to clutter generators and \( u = 1 \) refers to actual objects) and it is approximated by \( L_{k-1} \) particles as shown in (2). Prediction is performed using the equations previously presented in section II. We denote the predicted multi-Bernoulli distribution by:
\[ \pi_{k|k-1} = \left\{ \left( (i)_{k|k-1}^{(i)} \right)_{i=1}^{M_{k|k-1}} \right\}, \]

where each \( p_{k|k-1}^{(i)} \) is approximated by \( L_{k|k-1} \) particles as described in (3) and (4).

The next step, before updating the distribution parameters and extracting object states, is to find the most desirable sensor command that would lead to the most useful measurements after the selected command is applied. As it was previously mentioned, selection of sensor command is carried out by optimizing an objective function that is at the core of the sensor control solution. We will present our proposed objective function in section III-C. However, we note that the value of the proposed function depends on future measurements which has stochastic variations even for a determined control command. Thus, we would need to produce synthetic measurements for each admissible command. Following [9], the MAP estimate of the measurement set was used to produce the required synthetic measurements via following steps:

- Cardinality and state of the multi-object RFS are estimated from the predicted multi-Bernoulli distribution. These will include actual target estimates and points estimates corresponding to generated clutters.
- Using the estimated number and states of the objects (from prediction), a set of clean (noise-free) measurements are created and denoted by \( Z \). This set would include clutter measurements associated with clutter generators that are not discriminated from real objects in the prediction step. Thus, an initial update step is taken, using \( Z \) as the measurement set, to arrive at updated multi-Bernoulli parameters which are labelled with \( u = 0, 1 \) and can be discriminated.
- The number of target objects and their states can now be estimated from the updated distribution.
- Using the estimated object states, for each admissible control command, \( s \), measurement sets are generated, and denoted by \( Z_s \). Each \( Z_s \) is the MAP estimate of the measurement set that could be acquired by the sensor after the control command \( s \) is applied.
Next, for each control command \( s \) using its corresponding measurement set \( Z_s \), the multi-Bernoulli multi-object density is updated. From each updated multi-object density, the objective function described in Section III-C can be calculated and the optimum control command can then be selected accordingly.

C. Objective function

In order to determine the best control command, we define a new objective function that directly quantifies the statistical expectation of an error calculated over all possible updated multi-object states. The error term chosen in this work is a linear combination of average uncertainty in cardinality estimates, \( \text{states} \) of objects as well as the average uncertainty in the estimated clutter rate.

In [9], the cost function included only the average uncertainty in cardinality estimates. In the proposed adaptive sensor control scheme, this term is now represented by the variance of cardinality of the updated multi-object state:

\[
\hat{\sigma}^2_{|X|}(s) = \sum_{i=1}^{M_k} \left[ r_{i,k}^{(i)} \left( 1 - s_{i,k}^{(i)} \right) \right].
\]

We note that the above term depends on the sensor control command, because the parameters of the updated multi-Bernoulli state (such as the \( r_{i,k}^{(i)} \) terms in the above equation) depend on the MAP estimate of the measurement set calculated for the chosen sensor control command. For the sake of simplicity in notation, we have removed the \( s \) dependence in our formulation of the error terms.

The second term is indicative of the variance of single object states. We note that we may be only interested in minimizing the average variance of some state components. For instance, in the case study presented in the next section, only the variance of location of targets is important, with the assumption that good accuracy in location guarantees sufficient accuracy in speed.

To quantify the uncertainty in single object state estimates, we first compute the following average error for each Bernoulli component in the updated distribution:

\[
\hat{\sigma}^2_{x,i}(s) = \sum_{j=1}^{L_k^{(i)}} \left[ u_k^{(i,j)}(x_k^{(i,j)})^2 - \left( u_k^{(i,j)}(1-x_k^{(i,j)}) \right)^2 \right]
\]

where squaring the single-object state vector means summing the squares of the components of interest. Then, the total state estimation error is given by the weighted average of the errors of single Bernoulli components, where the weights are the updated probabilities of existence:

\[
\hat{\sigma}^2_x(s) = \sum_{i=1}^{L_k^{(0)}} \left[ r_{i,k}^{(0)} \frac{1}{\hat{\sigma}^2_{x,i}(s)} \right] \sum_{i=1}^{L_k^{(0)}} r_{i,k}^{(0)}
\]

The variance of the clutter rate estimate is approximated by updated particles for generated clutter samples, as below:

\[
\hat{\sigma}^2_{\lambda}(s) = \sum_{i=1}^{M_k} r_{i,k}^{(i)} \left( \sum_{j=1}^{L_k^{(i)}} \left( 1 - r_{\lambda,k}^{(i,j)} \right) \right).
\]

where \( r_{\lambda,k}^{(i,j)} = u_k^{(i,j)}(0) \).

Finally, the proposed objective function is defined as the following cost function:

\[
\vartheta(s) = \eta_{\lambda} \sigma^2_{\lambda}(s) + \eta_x \sigma^2_x(s) + \eta_{\lambda} \sigma^2_{\lambda}(s)
\]

where \( \eta_{\lambda} > 0, \eta_x > 0 \) and \( \eta_{\lambda} > 0 \) are the user-defined importance weights given to minimization of each of the variances. To have a normalized weighted sum of the variances, the weights are chosen to satisfy \( \eta_{\lambda} + \eta_x + \eta_{\lambda} = 1 \). Since precision of the estimated number of objects and their locations are in priority, it is reasonable to put more emphasis on the weight of the variance of cardinality of the updated multi-object and their states (\( \eta_{\lambda} \) and \( \eta_x \)). The control command is then chosen by minimizing the above cost function:

\[
\hat{s} = \arg\min_s \vartheta(s).
\]

IV. SIMULATION RESULTS

A non-linear multi-object scenario reported in [7] is employed to evaluate the performance of the proposed adaptive CB-MeMBer sensor control. In this scenario, a controllable moving sensor returns range and bearing measurements of the form \( z_k = [\theta_k, \rho_k, \lambda_k]^T \), where \( \tau \) denotes matrix transpose. A total of 5 objects appear on the scene and manoeuvre on the half disc of radius 1000m. Each single object state is composed of the unknown detection probability \( a_k \), location and velocity components in \( x \) and \( y \) directions, and detection type label \( u_k \in \{0,1\} \), all denoted by \( [a_k, x_k, y_k, \dot{x}_k, \dot{y}_k]^T \).

Although the following case study only involves one sensor, generalization of the proposed adaptive CB-MeMBer sensor control for multiple sensors is straightforward. In the following model parametrization, \( \mathcal{N}(::m,\Sigma^2) \) and \( \mathcal{B}(::\alpha_1,\alpha_2) \) denote a Gaussian density with mean and covariance \( m, \Sigma \) and a Beta density with parameters \( \alpha_1, \alpha_2 \), respectively.

The sensor location is denoted by \( s = [x_s, y_s]^T \). The sensor enters the surveillance area at position \( (10 \text{ m}, 10 \text{ m}) \). The sensor detection probability is unknown to the filter, but the measurement data is generated with the following probability that depends on the location of both the sensor and object locations:

\[
p_D(s, o) = \begin{cases} 
1, & \text{if } ||o - s|| \leq R_0 \\
\max\{0, \xi(||o - s|| - R_0)\}, & \text{otherwise}
\end{cases}
\]

where

\[
\begin{align*}
- & R_0 = 320 \text{ m}, \\
- & \xi = 25 \times 10^{-5} \text{ m}^{-1}, \\
- & \text{measurement model:} \\
\end{align*}
\]

\[
\text{atan2} \left( ||o - s|| + \sigma_R \right)
\]

where:

\[
\begin{align*}
- & \sigma_R \sim \mathcal{N}(0, \sigma^2) , \\
- & \sigma = \sigma_0 + \eta ||o - s||^2 , \\
- & \sigma_0 = 1 \text{ m}, \\
- & \eta = 5 \times 10^{-5} \text{ m}^{-1} ,
\end{align*}
\]

Here, the cost function includes only the variance of location of targets with expected values of 10 m, 10 m and variances of 0.5 m and 0.5 m for the object movements. The control command is chosen as a linear velocity vector.

The generalization of the proposed adaptive CB-MeMBer sensor control for multiple sensors is straightforward. In the following model parametrization, \( \mathcal{N}(::m,\Sigma^2) \) and \( \mathcal{B}(::\alpha_1,\alpha_2) \) denote a Gaussian density with mean and covariance \( m, \Sigma \) and a Beta density with parameters \( \alpha_1, \alpha_2 \), respectively.
and \( \sigma_\theta = \frac{\pi}{180} \text{rad} \), and \( \text{atan2} \) denotes the 4-quadrant arc tangent function.

**Model for actual objects:** Actual objects move according to the constant velocity model. These objects are positioned relatively close to each other in the surveillance area. Their initial locations and velocities are: \([800, 600, 1, 0]^T\), \([650, 500, 0.3, 0.6]^T\), \([620, 700, 0.25, -0.45]^T\), \([750, 800, 0, 0.6]^T\), and \([700, 700, 0.2, 0.6]^T\), where the units of \( x \) and \( y \) are meters and \( \dot{x} \) and \( \dot{y} \) are m/s. Transition density of actual objects is modelled by the following Gaussian density:

\[
f_{k|x_{k-1}}^{(X)(1)}(x_k|x_{k-1}) = \mathcal{N}(x_k; F x_{k-1}, Q),
\]

where

\[
F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} T^2 & 0 & T \frac{T^2}{2} & 0 \\ 0 & T^2 & 0 & T \frac{T^2}{2} \\ T^2 & 0 & T & 0 \\ 0 & T^2 & 0 & T \end{bmatrix}
\]

with \( q = 0.05 \) and \( T = 1 \) s. The transition density for the augmented part of the state is:

\[
f_{k|x_{k-1}}^{(A)(1)} = \beta(a_k; \alpha_1^{(1)}(k-1), \alpha_2^{(1)}(k-1)),
\]

where

\[
\begin{align*}
\alpha_1^{(1)}(k-1) &= \left( \mu_2^{(1)}(a, \sigma_2^{(1)}(k-1)) \sigma_1^{(1)}(a, \sigma_2^{(1)}(k-1)) \right)^2 \\
\alpha_2^{(1)}(k-1) &= \left( \mu_2^{(1)}(a, \sigma_2^{(1)}(k-1)) \sigma_1^{(1)}(a, \sigma_2^{(1)}(k-1)) \right)^2 (1 - \mu_2^{(1)}(a, \sigma_2^{(1)}(k-1))),
\end{align*}
\]

where,

- \( \mu_2^{(1)}(a, \sigma_2^{(1)}(k-1)) = a_{k-1} \),
- \( \sigma_1^{(1)}(a, \sigma_2^{(1)}(k-1)) = 0.07 \).

The survival probability for actual objects is fixed through the scenario \( P_a = 0.99 \).

**Model for clutter generators:** Clutter generators moment is modelled by a simple random walk. State vector of clutter generators is established by these elements: their positions \( x \), \( y \) and an augmented variable as a unknown probability of detection all denoted by \( x_c = [a_c, x_c, y_c]^T \). Clutters transition density is given by:

\[
f_{k|x_{c,k-1}}^{(C)}(x_c | x_{c,k-1}) = \mathcal{N}(x_c; x_{c,k-1}, Q_c),
\]

where \( Q_c = \text{diag}(\sigma_c^2, \sigma_c^2) \) and \( \sigma_c^2 = 1000 \text{ m}^2 \) and \( \sigma_c^2 = 500 \text{ m}^2 \). The transition density of augmented part is same as the augmented part of actual objects, except \( \sigma_0^{(0)}(a, \sigma_0^{(0)}(k-1)) = 0.07 \). The survival probability for actual objects is fixed through the scenario \( P_{c_a} = 0.90 \). The measurement model for clutter generators is similar to the actual model except \( \sigma_{\theta} = 400^\circ \) and \( \sigma_{\theta} = 20 \left( \frac{\pi}{180} \right) \text{ rad} \). The clutter generators birth process is as follows:

\[
r_{T_0, k}^{(0)} = \left( r_{T_0, k}^{(0)(i)}, P_{T_0, k}^{(0)(i)} \right)_{i=1}^{20},
\]

where

- \( r_{T_0, k}^{(0)} = 0.1 \).

**Filter parameters:** The filter applies a maximum of \( L_{max} = 1000 \) and minimum of \( L_{min} = 100 \) particles per Bernoulli component or track. The actual number of particles in each component or track is proportional to its existence probability. Track pruning is performed with a threshold of \( T_{min} = 10 \times 10^{-4} \). The finite set of admissible control commands are also same as reported in [7].

In this case study, not only the measurements noise power is relatively high (as noted in [7]), but also the clutter rate is high and both the clutter rate and probability of detection are unknown to the filter. These factors make this scenario a very challenging case of sensor control in multi-object estimation scheme in which many state-of-the-art techniques would fail.

**Results:** We ran the proposed adaptive CB-MeMBer sensor control method and computed the number and locations of the targets for the times \( k = 1, \ldots, 35 \). Intuitively, we expect the sensor to move toward the objects and remain in the point central to them (where the most accurate measurements can be acquired due to small distances between the sensor and targets). Figure 1 shows sensor movements from initial position to time \( k = 11 \), after which, the sensor remains amongst the objects. Figure 2 shows the average of cardinality and clutter rate estimates over 200 Monte Carlo runs of the proposed adaptive CB-MeMBer sensor control. We observe that the clutter rate estimates shown in Fig. 2(a) gradually approach the ground-truth value of 10 and fluctuate around it with a relatively small standard deviation.

To demonstrate the improvement achieved by our proposed adaptive sensor control, we compared its estimation accuracy with the original CB-MeMBer sensor control method of [9]. With the original CB-MeMBer sensor control method, we used presumed fixed values of 15 and 0.9 for the clutter rate and probability of detection. We note that the ground truth of clutter rate is 10 and the probability of detection not fixed and varies with the sensor-object distance. In Fig. 2(b), the average cardinality estimates given by both methods are plotted, along with the ground-truth cardinality (which is 5). We observe that, the average cardinality estimates given by the original CB-MeMBer sensor control are close to 4, i.e. the rounded cardinality estimate given by that method is 4 at most of the times. More precisely, inaccurate assumptions for clutter rate and detection properties led to one object being missed in most of the times. On the contrary, our adaptive sensor control results in cardinality estimates that are dominantly rounded to 5, since the average over 200 MC runs is very close to 5. Thus, estimation accuracy is significantly improved by using the adaptive sensor control method.

Another significant advantage of the proposed multi-Bernoulli sensor control is that the computational complexity is less than the competing methods. The reason is that in contrast to sensor control methods that are based on information theoretic objective functions, the adaptive CB-MeMBer
sensor control method does not need to sample the space of measurement sets and to repeat the update computations for each sample. This means significant savings in computation as the creation and processing of numerous Monte Carlo samples of measurement sets is no longer needed in our method. Furthermore, computing the cardinality and single-object state in a multi-Bernoulli scheme is straightforward compared to PHD and CPHD methods.

V. CONCLUSION

In this paper, a novel sensor control solution was proposed. The sensor control is particularly formulated to perform within the recently proposed adaptive multi-Bernoulli filtering scheme [16]. Our sensor control method is an extension of the recently proposed multi-Bernoulli sensor control [9], that does not need any prior information on clutter and sensor field-of-view parameters. In this work, we propose a sensor control strategy the minimizes the statistical expectation of the uncertainties (quantified by variance) of the cardinality, and object state estimates as well as the estimated rate of clutter. Simulation results show that as a result of minimizing this cost function, the sensor control drives the sensor toward locations where more accurate measurements can be acquired. Our simulations, showed that this leads to more accurate cardinality and state estimates and better detection of clutters and more accurate clutter rate estimates. An important aspect of our method is that our proposed implementation does not need to perform Monte Carlo sampling in the space of measurement sets. This leads to significant computational savings. Our method is particular useful in space object tracking applications where there is limited information on the distribution of traceable objects in the space and clutters, and our method can effectively handle such uncertainties.

Fig. 2: Monte Carlo averages of clutter rate and cardinalities

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