Distributed Detection in Wireless Sensor Networks
Using Complex Field Network Coding

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Abstract—The signal transmission and the information fusion in the wireless sensor networks (WSN) are conventionally assumed to operate over orthogonal channels, which becomes bandwidth and throughput inefficient. To remedy this inefficiency and to improve the detection performance simultaneously, in this work, we first propose to use relaying with complex field network coding (CFNC), which operates over non-orthogonal channels. This provides not only the diversity in space but also diversity in time as well. In this method, each sensor is assigned a unique pre-determined signature and this provides robustness against the multi-access interference. Secondly, we derive the optimal likelihood ratio test (LRT) based fusion rule for the proposed system. Thirdly, we devise a way to choose the sensor signatures. Finally, we evaluate the detection performance of the proposed scheme and compare it with the performance of the conventional method. The simulation results show that we can obtain up to a 105% improvement using distributed detection with CFNC rather than employing the conventional method in literature. This suggests that the distributed detection with CFNC is a promising technique that can be used in next generation WSNs.

Keywords—Distributed detection, fading, relaying, network coding, wireless sensor networks

I. INTRODUCTION

A Wireless Sensor Network (WSN) involves spatially distributed sensor nodes, which work collaboratively to monitor an event in the region of interest (ROI) [1]. Basically, they extract some useful information about an event in order to send to a special node called “fusion center” (FC) which combines all sensor outputs and forms a global situational assessment.

Since sending raw data directly to the FC consumes more bandwidth and energy, the distributed detection (DD) becomes attractive for WSNs with limited resources [2] where the sensors first arrive at their own decisions and then, these decisions are transmitted to the FC to be fused optimally.

There exists a vast literature on the distributed detection. The optimal distributed signal detection under conditional independence assumption has been investigated both under Bayesian and Neyman-Pearson (N-P) criteria in [3]-[5]. Furthermore, the decision fusion under correlated observations assumption has been considered in [6]-[9]. These works assume that the communication channel between each sensor node and the FC is error-free. Since this is not a realistic assumption, Thomopoulos and Zhang [10] have come up with the idea of distributed detection with non-ideal communication channels. They have only considered the effect of the noise and assumed that the communication channel between each sensor and the fusion center is binary symmetric channel (BSC). However, fading is another source of degradation along with the receiver electronics noise in WSNs. Eventually, Chen et al. [11] have proposed the optimum and sub-optimum fusion rules for noisy and Rayleigh faded WSN by assuming that the channel state (CSI) information (i.e., fading coefficient) is exactly known at the fusion center. Notice that the DD works fine under fading and noise because the sensor nodes that give the same decision, provide spatial diversity since the FC is supplied with multiple copies of the transmitted signal over different channels that fade almost independently. This is very useful to combat the adverse effects of fading channels (e.g., multipath-fading, shadowing) in wireless sensor networks.

It is important to mention that in all previously mentioned studies, the signal transmission and the information fusion are assumed to occur over orthogonal channels to avoid multi-access interference (MAI) where only one sensor sends its decision at a certain time while others wait. This, however, decreases the throughput of the system, which is also turns out to be bandwidth inefficient particularly for large networks.

To improve the efficiency of WSNs, the decision fusion with non-orthogonal signaling have been analyzed extensively in [19]-[22] where the local sensors are directly send their decisions to the FC over a multiple access channel (MAC).

Additionally, the non-orthogonal signaling has been employed together with a network-coding (NC) method [18] to increase the throughput of the wireless networks more. In the literature, various NC schemes have been developed: XOR method, physical layer network coding (PNC) (a.k.a analog network coding), complex field network coding (CFNC)[13]-[18] etc. The CFNC provides the highest throughput (1/2 symbol per user per channel-use) compared to PNC and XOR
methods for communications over non-orthogonal channels since it uniquely allows decoding of user messages under multi-access interference [18]. In CFNC, each sensor is assigned a unique pre-determined complex number that we call signature. Each signature is used to weight the signal of a particular sensor before the signal transmission and this provides robustness against the multi-access interference.

Moreover, the performance of the DD in WSNs can be enhanced further with the use of relaying, which has been proposed to achieve both spatial and time diversity in multi-user communications [12]. In the relaying, the sensors first send their information over wireless medium, and because of the broadcast nature of the wireless channel both the relay node and the FC hear the information bearing signals of the sensors. Then, the relay node extracts the necessary information about the sensor decisions. Finally, in the subsequent time interval, the relay node forwards its signal to the FC by employing a relaying protocol [12].

Although the works in ([19]-[22]) show that non-orthogonal signaling has a potential to improve error performance distributed detection, none of them has considered the use of the relaying. Therefore, in this work, we consider CFNC (because of its nice features) with the non-orthogonal relaying for WSNs.

The contributions we have made can be summarized as follows. We first propose to incorporate CFNC to the WSN with a relay node operating over non-orthogonal wireless channels. Then, we derive the optimal LRT based fusion rule for the proposed system. After that, we find a solution for the signature selection problem. Finally, we have shown through the numerical experiments that the proposed method performs better than the classical approach that is available in the literature.

In the next Section, we give a background on the classical distributed detection (CDD) for a parallel WSN. In Section III, the system model for distributed detection with CFNC is described, and then we derive the LRT based optimum fusion rule. We also present our method on selecting the sensor signatures in the same Section. Performance evaluation of the proposed method through numerical simulations is given in Section IV. Finally, we conclude in Section V.

II. BACKGROUND ON CLASSICAL DISTRIBUTED DETECTION OVER ORTHOGONAL CHANNELS

In this section, the classical signaling formulation for the WSN with parallel topology is first reviewed, which operates over the orthogonal wireless channels. Then, the likelihood ratio test (LRT) based optimal fusion rule is explained for the sake of completeness. During this study, we focus on binary hypotheses: $H_1$ and $H_0$ (e.g., they may represent the existence and absence of a target respectively) at region of interest (ROI).

For this topology, we consider a network of $N$ sensors as shown in Fig. 1. Specifically, the $k^{th}$ sensor, $S_k$, first acquires its measurement $m_k$ from the ROI and arrives at its decision $u_k$, which is later modulated to produce $x_k$. Then, the modulated signal $x_k$ is distorted by the fading and noise and constitutes the signal $y_k$ at the FC. Finally, the FC combines all of the signals it has received according to a fusion rule, and casts a final decision $u$. We assume throughout the manuscript that all fading coefficients are modeled as complex Gaussian random variables with zero mean and unit variance and the receiver electronics noise is modeled as additive white Gaussian noise (AWGN) channel. Furthermore, we also assume that the channel state information (CSI) is available at the FC. Note that we just consider binary phase shift keying (BPSK) modulation for the modulated signal, but extension of the results to other modulation schemes is straightforward.

Therefore, the signal received by the fusion center due to the orthogonal signal transmission of the $k^{th}$ sensor can be mathematically expressed as

$$y_k = \sqrt{\gamma_k} h_k x_k + z_k$$  \hspace{1cm} (1)

where $\gamma_k$ is the path loss coefficient of the link between the $k^{th}$ sensor and FC; $h_k$ is complex Gaussian channel coefficient, $x_k$ is the BPSK modulated signal, which takes values of -1 and 1 respectively for $k^{th}$ sensor decision $u_k$ being 0 and 1 respectively, $z_k$ is zero-mean AWGN sample with variance of $\sigma^2 = N_0 / 2$ per dimension.

For this network topology and orthogonal signaling model, the optimal LRT based fusion rule [11] is given as

$$A(y) = \prod_{k=1}^{N} \frac{f(y_k | H_1, h_k)}{f(y_k | H_0, h_k)} = \prod_{k=1}^{N} \left( \frac{P_{D_k} e^{-\frac{(y_k - \sqrt{\gamma_k} h_k)^2}{2\sigma^2}}} {P_{F_k} (1 - P_{D_k}) e^{-\frac{(y_k + \sqrt{\gamma_k} h_k)^2}{2\sigma^2}}} \right)$$  \hspace{1cm} (2)

where $y = [y_1, \ldots, y_N]^T$ denotes the received signal vector, $P_{D_k}$ and $P_{F_k}$ denote the probability of detection and the probability of false alarm respectively of $S_k$, which can be written as

$$P_{D_k} = P(u_k = 1 | H_1)$$
$$P_{F_k} = P(u_k = 1 | H_0)$$  \hspace{1cm} (3)

The phase coherent detection formulation in reference [11] is equivalent to the complex representation in Eq.(2).
at the FC in time slot 2 due to the relaying; \( h_{s'FC} \), \( h_{sd} \), and \( h_{rd} \) denote the fading gains of \( S_i\)-\( R \), \( S_i\)-\( FC \) and \( R\)-\( FC \) links respectively, which are modeled as complex Gaussian random variables with zero mean and unit variance; the parameter \( \alpha \) determines the power allocated to the relay as a fraction of the total transmit power of all users; \( \gamma_{s} \), \( g_{s} \) and \( g_{r} \) denote respectively as the path-loss coefficients of \( S_i\)-\( FC \), \( S_i\)-\( R \), and \( R\)-\( FC \) links; \( z_{s} \) and \( z_{d} \) represent the noise samples at the relay node and FC respectively, which are modeled as additive white Gaussian noise (AWGN) with zero mean and variance of \( N_{0}/2 \) per dimension.

***DISTRIBUTED DETECTION WITH COMPLEX FIELD NETWORK CODING OVER NON-ORTHOGONAL CHANNELS***

In this section, we first propose distributed detection with complex field network coding (CFNC) over non-orthogonal channels, and derive LRT based fusion rule for the proposed method. Finally, we propose a method to select the complex sensor signatures optimally in the CFNC, which is obtained by minimizing the symbol-error rate (SER) at the FC.

A. System Model for Distributed Detection with CFNC

As mentioned earlier, the use of relay is beneficial to achieve cooperative diversity in order to combat the detrimental effects of fading channels. Hence, in this work, we propose to incorporate CFNC to the parallel WSN with a relay node \( (R) \) as depicted in Fig. 2 where all of information signals are transmitted over non-orthogonal wireless channels.

In this method, each sensor \( S_{k} \) is assigned a unique signature \( \theta_{x_{k}} \). Then the modulated decisions of sensors, \( x_{k} \)'s, are multiplied by the associated signature and the resultant signals of sensors are sent over non-orthogonal channels simultaneously in time slot 1, which causes interference both at the relay and at the FC. After that, based on the relaying policy (e.g., amplify and forward, estimate and forward etc.), the relay node sends its output to the FC in time slot 2. Therefore, the signals resultant from the non-orthogonal communications under the flat-fading can be written as

\[
y_{s} = \sum_{k=1}^{N} \sqrt{g_{s} h_{s,\theta_{x_{k}}}} x_{k} + z_{s}
\]

\[
y_{sd} = \sum_{k=1}^{N} \sqrt{g_{s} h_{sd,\theta_{x_{k}}}} x_{k} + z_{d}
\]

\[
y_{rd} = \sqrt{g_{r} h_{rd,\theta_{x_{k}}}} x_{k} + z_{d}
\]

where \( y_{s} \) and \( y_{sd} \) are the received signals at the relay node and the FC in time slot 1 respectively; \( y_{rd} \) is the signal acquired

![Fig. 2. A basic CFNC coded parallel sensor network with \( N \) sensors and one relay node.](image-url)
\[ \Lambda(y_d) = \frac{f(y_d \mid H_d)}{f(y_d \mid H_0)} \]

\[ = \sum_x f(y_d \mid H_1, x) \left( \sum_x f(y_d \mid H_1, \hat{x}) P(\hat{x} \mid x) \right) P(x \mid H_1) \]

\[ = \sum_x f(y_d \mid H_0, x) \left( \sum_x f(y_d \mid H_0, \hat{x}) P(\hat{x} \mid x) \right) P(x \mid H_0) \]

where \( y_d = [y_{d1}, \ldots, y_{dn}] \), \( x = [x_1, \ldots, x_n]^T \). Also, \( P(\hat{x} \mid x) \) is the probability that the relay decides \( \hat{x} \), although \( x \) is transmitted. To simplify the analysis, we assume that the decoding at the relay is perfect \( \hat{x} = x \) and thus the following LRT rule is employed at the FC.

\[ \Lambda(y_d) = \frac{f(y_d \mid H_1)}{f(y_d \mid H_0)} = \sum_x f(y_d \mid H_1, x) P(x \mid H_1) \]

Since the conditional probability density function of \( y_{d,i} \) is independent of hypothesis when \( x \) is given and sensor decisions are conditionally independent, the conditional distributions or probabilities in (10) can be written as

\[ f(y_{d,i} \mid x) = \frac{1}{2\pi \sigma^2} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \left( y_{d,i} - \sqrt{x_i} h_{d,i} \theta_{d,i} \right)^2 \right\} \]

\[ f(y_{d,i} \mid x) = \frac{1}{2\pi \sigma^2} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \left( y_{d,i} - \sqrt{x_i} h_{d,i} \theta_{d,i} \right)^2 \right\} \]

The FC produces its final decision \( u_0 \) as

\[ \Lambda(y_d) \xrightarrow{x_{u_0}} \tau \]

where \( \tau \) is the optimal threshold value used at the FC. In this paper, we consider minimum error probability detection at the FC. Hence, the optimal threshold value can be determined in terms of a priori probability of the event at the ROI as

\[ \tau = \frac{P(H_0)}{P(H_1)} \]

where \( P(H_0) \) and \( P(H_1) \) are a priori probabilities of \( H_0 \) and \( H_1 \) respectively. The average probability of error of the network can be determined as

\[ P_e = P(H_0) P_f + P(H_1)(1 - P_D) \]

where \( P_f \) and \( P_D \) are the false alarm and detection probability of the FC respectively, which can be expressed as

\[ P_f = P(\Lambda(y_d) > \tau \mid H_0) \]

\[ P_D = P(\Lambda(y_d) > \tau \mid H_1) \]

Note that the complexity of the fusion in CDD is linear in \( N \) whereas its complexity in CFNC-DD is exponential in \( N \). Although the CFNC-DD seems to be more complex, it has a better detection performance than CDD as we show in Section IV. This computation burden can be alleviated with the use of clustering [23]-[24].

B. Selection of Optimum Sensor Signatures and Relay Power

In this section, we propose a way to select the sensor-signatures optimally by minimizing the ML bound on the symbol error probability of the network. We denote the vector of signatures by \( \theta = [\theta_1, \ldots, \theta_N]^T \). Since each sensor decision is binary, there are \( 2^N \) possibilities for the sensor decision vector which can be put in an ordered list. Let \( \chi \) be the \( i \)th possible decision vector in the list for \( 1 \leq i \leq 2^N \). The CFNC coded symbol for the decision vector \( \chi \) due to the non-orthogonal signaling becomes

\[ c_i = 0^{T} \chi \]

Hence, the CFNC coded symbol takes \( 2^N \) distinct values.

Assuming CSI is known at the relay and ML relaying is used, the pair-wise symbol error probability of the relay node is

\[ P(c_i \rightarrow c_j \mid \textbf{h}_u) = Q(\frac{1}{2\sigma} \sum_{k=1}^{N} \sqrt{g_{i,k}} h_{u,k} \theta_{d,k} - d_{jk}) \]

where \( \textbf{h}_u = [h_{u,1}, \ldots, h_{u,N}]^T \), \( Q(x) \equiv (1/\sqrt{2\pi}) \int e^{-t^2/2} dt \) and \( d_{jk} \) is the difference between the \( i \)th and \( j \)th decision vectors i.e., \( d_{ij} = (\chi_i - \chi_j ) \). \( d_{jk} \) represents the \( k \)th component of \( d_{ij} \). Hence the symbol error probability of the relay can be bounded as

\[ P_e(\textbf{h}_u) \leq \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} P(c_i) Q(\frac{1}{2\sigma} \sum_{k=1}^{N} \sqrt{g_{i,k}} h_{u,k} \theta_{d,k} - d_{jk}) \]

where \( P(c_i) \) is the probability of the CFNC coded symbol for \( 1 \leq i \leq 2^N \), which depends on the false alarm and detection probabilities of the sensors as follows

\[ ^2 \text{This assumption can be justified under high SNR or by using an error correction code.} \]
\[ P(c_i) = P(c_i \mid H_0)P(H_0) + P(c_i \mid H_1)P(H_1) \]
\[ = P(x_i \mid H_0)P(H_0) + P(x_i \mid H_1)P(H_1) \]
\[ = \prod_{i=1}^{N} P_{\alpha_i} (1 - P_{\alpha_i})^{-\alpha_i}P(H_0) + \prod_{i=1}^{N} P_{\alpha_i} (1 - P_{\alpha_i})^{-\alpha_i}P(H_1) \]  
(22)

By using the Chernoff-bound [25] (i.e., \( Q(x) = \frac{1}{2}e^{-x^2} \)), the instantaneous CFNC symbol error rate (SER) or probability at the relay node is further upper-bounded as

\[ P_r'(h_{su}) \leq \frac{1}{2} \sum_{i,j} P(c_i) e^{\frac{\sum_{i,j}^N g_{ij} \theta_i d_{ij}^2}{8\sigma^2}} \]  
(23)

Consequently, a bound on the average SER can be obtained by averaging the upper-bound in (23) over fading gains of the sensors-to-relay links as

\[ \bar{P}_r' \leq \frac{1}{2} \sum_{i,j} P(c_i) \left( \sum_{i,j}^N g_{ij} \theta_i d_{ij}^2 / 1 + \frac{1}{8\sigma^2} \right) \]  
(24)

By employing ML sequence detection at the FC using the signals received in both time slots (i.e., \( y_{sd} \) and \( y_{ua} \)), the similar analysis results in a bound on the average SER at the FC as

\[ \bar{P}_r \leq \frac{1}{2} \sum_{i,j} P(c_i) \left( 0.5 \sum_{i,j}^N g_{ij} \theta_i d_{ij}^2 / 1 + \frac{1}{8\sigma^2} \right) \]
\[ + \frac{0.5}{\sum_{i,j}^N g_{ij} \theta_i d_{ij}^2 / 8\sigma^2} \]  
(25)

where \( \bar{P}_r'(\theta, \alpha) \) denotes the SER upper bound at the FC. It is important to notice that the bound in (25) on the average SER is convex in signatures \( \theta_i \). In order to determine the SER-optimized signatures, the minimum of the bound in (25) under the total transmit power constraint \( \sum_{i=1}^{N} \theta_i \) \( + \alpha(\sum_{i=1}^{N} \theta_i) \) should be attained. Additionally, since each sensor actively sends information, \( \theta_i \) should be strictly greater than zero. Also, since the information of each sensor can be recovered uniquely [18] from the CFNC coded symbol as long as all signatures are distinctively assigned to sensors. So, we can take this into account by putting the constraint: \( |\theta_i - \theta_l| > 0 \) for \( k \neq l \). Therefore, we can state the determination of SER-optimized signatures as the following constrained optimization problem.

\[ \minimize_{\theta, \alpha} B_r^0(\theta, \alpha) \]

such that

\[ P_r - \sum_{k=1}^{N} |\theta_k| - \alpha(\sum_{k=1}^{N} \theta_k^2) = 0 \]  
(26)

\[ |\theta_i - \theta_l| > 0 \]  
for \( k \neq l \)

\[ \alpha \geq 0 \]

One can easily see that the optimization in (26) is a convex program over the feasible region and thereby its unique global minimum exists, which should satisfy the Karush-Kuhn-Tucker (KKT) conditions. For a parallel network with \( N=2 \) sensors, we have proven by applying the KKT conditions that the optimal signatures should satisfy

\[ \arg(\theta_1) - \arg(\theta_2) = \cos^{-1}\left( \frac{(\rho - 1)(\sigma^2 + g_{12}(\theta_1^2 + \theta_2^2))}{2g_{12} \alpha \theta_1 \theta_2 (\rho + 1)} \right) \]  
(27)

where \( \rho = \frac{P(c_1) + P(c_2)}{P(c_1) + P(c_2)} \). The optimal values of \( \theta_1, \theta_2 \) and \( \alpha \), which satisfy KKT conditions are highly nonlinear and we determine them numerically by employing an active set algorithm in MATLAB.

IV. SIMULATION RESULTS

In this section, we investigate the performances of the CFNC distributed detection (CFNC-DD) over non-orthogonal signaling and classical distributed detection over orthogonal signaling by obtaining and comparing their probability of error plots and their receiver operating characteristics (ROC) curves. It is critical to note that the probability of error plots are obtained with the assumption of equal likely priors for various curves. It is assumed that sensors are identical in terms of signaling and classical distributed detection over orthogonal networks with \( N=2 \) sensors, we have proven by applying the KKT conditions that the optimal signatures should satisfy

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\[ |g_k - 0.5| < \gamma_k - 0.5 < g_k + 0.5 \]  
(28)
For this study, the signal-to-noise ratio of the network is defined in dB as

\[
\text{SNR (in dB)} = 10 \log_{10}\left(\frac{P_t}{\sigma^2}\right) \quad (29)
\]

Since the power budget is fixed, we realize different SNR values by changing the variance of electronics noise in (29).

In order to see the benefit of CFNC-DD over CDD, we first obtain probability of error as a function of SNR for a WSN with 2 sensors (i.e., \(N=2\)) and \(g_1=\gamma_2=0\) dB, \(g_2=g_4=10.45\) dB and \(g_3=3.10\) dB, which assumes that sensor nodes are equally separated from FC and the distance from the relay node to each sensor node is same. One can see from Fig. 3 that CFNC-DD outperforms CDD for all SNR values considered. Specifically, the CFNC-DD has provided an improvement up to 20.74% in average error probability over CDD. In addition to average error probability plots, we also obtained ROC curves as depicted in Fig. 4 for same network and different SNR values of -5, 0 and 5 dB. The proposed CFNC-DD method can obtain up to 10.34%, 11.76%, and 37.77% detection performance improvement over CDD for 5, 0 and -5 dB SNR respectively.

From the observations made above, we can say that by employing the proposed method, the probability of error gets better especially in the low-SNR regime as compared to using CDD, which can be explained as follows. In the CDD, each sensor signal is sent over an orthogonal channel and disturbed by one noise sample at the FC, which results in the availability of \(N\) noisy measurements at the FC. Contrary to that, CFNC-DD allows interference of sensor signals both at the relay and the FC but each of the interference signals in each time slot of CFNC-DD experiences a distortion due to one noise sample at the FC and thereby there are only two noisy measurements, which carry all sensor data, at the FC. Therefore, the noise has a worse impact on the performance of CDD. Moreover, CDD provides only spatial diversity using the sensors that give the same decision whereas CFNC-DD also achieves time diversity in addition to the spatial diversity since the relay node in CFNC-DD sends its decision to the FC at a different time slot. This is very beneficial to reduce the negative effect of the interference and noise on the detection performance of the network.

Secondly, we consider a scenario where sensors are located asymmetric with respect to the FC and relay node is closer to \(S_1\) and accordingly, parameters \(g_1, g_2\) and \(g_3\) are selected as 10.45 dB, 3.10 dB and 3.10 dB respectively. The value of \(\gamma_2\) is varied and decided to satisfy the triangle inequality in Eq. (28). Then, we obtain the probability of error results as a function of SNR for various values of \(\gamma_2\), which are shown in Fig. 5. Again, CFNC-DD outperforms CDD using the same \(\gamma_2\) value in all SNR regimes. For example, CFNC-DD has a error performance improvement of 8.43%, 20.92% and 22.98% respectively for \(\gamma_2\) value of 20 dB, 7.96 dB and -2.28 dB under the SNR of -5 dB. After that, the ROC curves are presented in Fig. 6 for different \(\gamma_2\) values and SNR of -5 dB. Consistently, CFNC-DD has a better detection performance than CDD for each \(\gamma_2\) value and a given false alarm probability. In particular, CFNC-DD results in detection performance improvements up to 32.14%, 42.10% and 105.50% can be obtained for 20 dB, 7.96 dB and -2.28 dB \(\gamma_4\) respectively. Therefore, our proposed method performs better even if sensor nodes are located far from the FC. As pointed out earlier, this is because the CFNC-DD results in both spatial diversity and the time diversity.

Next, we increase the number of sensors to \(N=4\) and select \(g_1=g_2=g_3=g_4=13.98\) dB in which the relay node is separated from the sensors equally. Also, the parameters \(\gamma_1, \gamma_2, \gamma_3, \gamma_4\) and \(g_1\) should be chosen to satisfy the triangle inequalities in Eq. (28) for which \(\gamma_1=0\) dB, \(\gamma_2=0.91\) dB, \(\gamma_3=0.91\) dB, \(g_4=0.91\) dB are used and the value of \(\gamma_4\) has let to change during the numerical experiments without conflicting the triangle inequalities. Then, we obtain the probability of error versus SNR curves for various values of \(\gamma_4\) as shown in Fig. 7. One can see from this figure that CFNC-DD decreases the error probability of CDD up to 20.56%, 25.92% and 21.69% for \(\gamma_4\) value of 20 dB, 6.02 dB and 0.91 dB respectively. Finally, we obtain the ROC curves in Fig. 8 by changing the a priori probability of the event in the region of interest for SNR of 0 dB and different values of \(\gamma_4\). CFNC-DD has a detection performance increase up to 15.21%, 19.88% and 11.32% detection performance improvement over CDD for 20 dB, 6.02 dB and 0.91 dB \(\gamma_4\) respectively.

Finally, we consider a WSN where the sensors are positioned equally apart on a horizontal line, which is separated 0.5 unit from the relay and has length of one unit. For this case, the detection probability of CFNC-DD is plotted in Fig. 9 as a function of number of sensors for the false alarm probability of 0.1 and SNR values of -5, 0 and 5 dB. As seen from this figure, the detection performance gets better by utilizing 8 sensors instead of employing 2 sensors with a factor of 95.68%, 64.67%, and 5.02% for SNR of -5, 0, and 5 dB, respectively. These results also suggest that CFNC-DD is more effective especially at low SNRs.

V. CONCLUSIONS

In this work, we first propose to incorporate the complex field network coding (CFNC) in order to improve the performance of the wireless sensor networks (WSN) with parallel topology under fading and noise. We then derive optimum LRT based fusion rule for the proposed system. Next, we propose to a way to sensor signatures in CFNC optimally, which is obtained by minimizing an upper bound on symbol error probability of the network. Finally, we have shown via simulations that proposed CFNC incorporated distributed detection (CFNC-DD) outperforms the classical distributed detection (CDD). Specifically, we obtain up-to 105.50% detection performance improvement by employing CFNC-DD rather than CDD. Therefore, CFNC-DD is a promising technique that increases the detection performance of wireless sensor network.
Fig. 3. Probability of error versus SNR curves of CFNC-DD and CDD for \( N=2 \), \( \gamma_1=\gamma_2=0 \text{ dB} \), \( g_1=g_2=10.45 \text{ dB}, g_r=3.10 \text{ dB} \).

Fig. 4. ROC curves of CFNC-DD and CDD under different SNRs for \( N=2 \), \( \gamma_1=\gamma_2=0 \text{ dB} \), \( g_1=g_2=10.45 \text{ dB}, g_r=3.10 \text{ dB} \).

Fig. 5. Probability of error versus SNR curves of CFNC-DD and CDD under various \( \gamma_2 \) values for \( N=2 \), \( \gamma_1=0 \text{ dB} \), \( g_1=g_2=10.45 \text{ dB}, g_r=3.10 \text{ dB} \).

Fig. 6. ROC curves of CFNC-DD and CDD under various \( \gamma_2 \) values for \( N=2 \), \( \gamma_1=0 \text{ dB} \), \( g_1=g_2=10.45 \text{ dB}, g_r=3.10 \text{ dB} \) and \( \text{SNR}=-5 \text{ dB} \).

Fig. 7. Probability of error versus SNR curves of CFNC-DD and CDD under various \( \gamma_4 \) values for \( N=4 \), \( \gamma_1=0 \text{ dB} \), \( \gamma_2=0.91 \text{ dB}, \gamma_3=0.91 \text{ dB}, \) and \( g_1=g_2=g_3=g_4=13.98 \text{ dB}, g_r=0.91 \text{ dB} \).

Fig. 8. ROC curves of CFNC-DD and CDD under various \( \gamma_4 \) values for \( N=4 \), \( \gamma_1=0 \text{ dB} \), \( \gamma_2=0.91 \text{ dB}, \gamma_3=0.91 \text{ dB}, \) and \( g_1=g_2=g_3=g_4=13.98 \text{ dB}, \) and \( \text{SNR}=0 \text{ dB} \).
Fig. 9. Probability of error versus Number of Sensors for CFNC-DD for 0.1 false alarm probability

REFERENCES