Robust Sensor Registration Based on Bounded Variables Least Squares

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Abstract—The ill conditioning problem of sensor registration is considered. We analyze the ill conditioning in the dense-registration scenario and the dense-sensor scenario, respectively, and present a robust registration method based on the bounded variables least squares (BVLS). The proposed approach can reduce the influence of ill conditioning by means of inserting prior constraints on the desired solution. Compared with the traditional least squares (LS) method and the recently proposed azimuth-LS method, the BVLS approach can accommodate the ill conditioning in dense-target and (or) dense-sensor scenarios. In the meantime, it is consistent with the LS estimator when the registration problem is good-conditioned. Simulation results demonstrate the superior performance of the proposed method.

Keywords—Sensor registration, bounded variables least squares (BVLS), ill conditioning, dense-target scenario, dense-sensor scenario.

I. INTRODUCTION

In multi-sensor surveillance systems, data fusion can supply more comprehensive and accurate information about the battlefield situation than any single sensor alone. However, the presence of sensor biases could result in misassociations and redundant tracks. Sensor registration is a prerequisite for successful multi-sensor data fusion [1].

Various algorithms for sensor registration in the literature can be mainly classified into two categories. The first one is the relative registration with the assumption that the local sensor is bias free and all biases belong to the remote sensor [2]–[4]. The Cramér-Rao lower bound for relative sensor registration process is raised in [5]. The drawback is that, biases of the local sensor could not be corrected by means of this approach. The second category performs absolute registration to fix this problem. Several algorithms have been developed including the real time quality control (RTQC) routine, the least squares (LS) method [6], the generalized least squares (GLS) approach [1], the exact maximum likelihood (EML) approach [7], the new least squares registration (NLSR) algorithm [8], the kalman filter (KF) approach for time-varying biases [9], etc. In [10], the hybrid Cramér-Rao lower bound (HCRLB) for absolute registration is presented.

However, the problem of numerical ill conditioning, meaning the sensitivity of the estimation results to small changes in the input, is neglected by these aforementioned algorithms. Gao et al. [11] point out that in the dense-target or dense-sensor scenario, the sensor registration problem tends to be ill-conditioned, and it has a notably large condition number. Actually, dense targets in formation and (or) dense sensors are common in practice, so the ill-conditioned registration problem is an important issue in real-life applications. To the best of our knowledge, this problem has not been well studied in the current literature yet. The minimum eigenvalue analysis on the observation matrix was used to examine whether satisfactory estimation accuracy of the radar biases could be obtained [12]. Similarly, [13] made use of eigenvalue to check the observability of the sensor biases. Recently, a practical solution, the azimuth generalized least squares (azimuth-GLS) approach, was developed by Du [14] to deal with the ill conditioning in dense-target scenarios. Nevertheless, the performance of the azimuth-GLS approach deteriorates seriously under the dense-sensor scenario as shown in the simulation results of this paper.

In this work, we firstly analyze the ill conditioning in the dense-target scenario and the dense-sensor scenario, respectively. Then on the basis of the bounded variables least squares (BVLS), a robust registration approach is developed to reduce the influence of ill conditioning by means of inserting prior constraints on the desired solution. Since the sensor biases are quite small compared to sensor measurements in the application of anti-air surveillance, bounds constraints exist naturally.

The paper is organized as follows. Section II briefly describes the sensor registration problem and also describes the LS approach and the azimuth-LS approach. Section III firstly analyzes the ill conditioning theoretically in dense-target scenarios and dense-target scenarios, respectively, and then reveals the relation between ill conditioning and scenario parameters based on an illustrative scenario model. Section IV discusses the proposed BVLS registration method. Numerical simulation is given in Section V to demonstrate the validity of our method. Section VI concludes the paper.

II. SENSOR REGISTRATION PROBLEM DESCRIPTION

Consider two sensors A and B, which measure the range and azimuth of common targets. Assume the sensor locations are precisely known. Due to the communication bandwidth limitations, each sensor carries out sensor-level tracking independently without registration, and transmits the unregistered local tracks to the fusion center where track-level registration are in operation.

It is assumed that both sensors have biases modeled as additive constants on their range and azimuth measurements.
For sensor $s \in \{A, B\}$, one has the bias vector as

$$ b_s = [b_r^s, b_\theta^s]^T, \quad (1) $$

where $b_r^s$ and $b_\theta^s$ represent the range and azimuth bias of sensor $s$, respectively. The registration geometry is shown in Fig. 1. Without loss of generality, we assume that sensor A is located at the origin, and sensor B is located at $(u,v)$. It is worth mentioning that the north direction is the positive orientation for azimuth, clockwise is the positive direction. $(x_k, y_k)$ is the true position of target $k$ in the system plain (cartesian coordinate), while $(\hat{x}_sk, \hat{y}_sk)$ denotes the true range and azimuth in the local polar coordinate system of sensor $s$. $(\hat{x}_sk, \hat{y}_sk)$ is the position estimate of target $k$ from sensor $s$, while $(\hat{\theta}_sk, \hat{\theta}_sk)$ denotes the local range and azimuth estimate at sensor $s$.

Directly influenced by sensor biases, only the position estimates will be used in registration. Using the first-order approximation, the position estimate of target $k$ available at sensor $s$ at a given time (without the time argument, for simplicity) will be [15]

$$ \hat{x}_sk \approx x_k + \bar{x}_sk + B_{sk}b_s, \quad (2) $$

where $x_k$ is the true position; $\bar{x}_sk \sim N(0, P_{sk})$ is the position estimation error in the local tracker $s$, $P_{sk}$ is the state estimation error covariance matrix; $B_{sk}b_s$ is the additional error induced by sensor biases. With the estimated range $\hat{r}_sk$ and azimuth $\hat{\theta}_sk$ of target $k$ at sensor $s$, one has

$$ B_{sk} = \begin{bmatrix} \sin(\hat{\theta}_sk) & \hat{r}_sk \cos(\hat{\theta}_sk) \\ \cos(\hat{\theta}_sk) & -\hat{r}_sk \sin(\hat{\theta}_sk) \end{bmatrix}. \quad (3) $$

For target $k$, we can get two equations by substituting $A$ and $B$ for $s$ in Eq. (2) respectively, then combine them (ignoring the effect of random errors) to get

$$ \Delta \hat{x}_k \approx B_k b, \quad (4) $$

where $\Delta \hat{x}_k = \hat{x}_{Ak} - \hat{x}_{Bk}$. $B_k = [B_{Ak}, -B_{Bk}]$, and $b = [b_r^A, b_r^B]^T$. Using the track sets of $N$ targets, we have

$$ \Delta \hat{x} \approx Bb, \quad (5) $$

where $\Delta \hat{x} = [\Delta \hat{x}_1^T, \Delta \hat{x}_2^T, \ldots, \Delta \hat{x}_N^T]^T$, and $B = [B_1^T, B_2^T, \ldots, B_N^T]^T$. The LS registration approach is equivalent to solving

$$ \min_b \sum_{i=1}^{2N} r_i^2 = \| \Delta \hat{x} - Bb \|^2_2, \quad (6) $$

where $r_i$ is the residual of the $i$th case. Consequently, the LS solution of Eq. (5) is given by

$$ b_{LS} = (B^T B)^{-1} B^T \Delta \hat{x}. \quad (7) $$

The GLS approach is more general than LS, in the sense that it weights the measurements according to their noise variances [1]. However, in sensor registration, especially for real-life applications, the GLS method does not make any significant improvement over the LS approach [16]. Therefore, the LS registration method and the azimuth-LS [14] method are used in this paper for the purposes of comparison.

Expanding Eq. (4) into the scalar form, we get

$$ x_{Ak} - x_{Bk} \approx \hat{x}_{Ak} - \hat{x}_{Bk} \approx \hat{r}_{Ak} \cos \hat{\theta}_{Ak} - \hat{r}_{Bk} \cos \hat{\theta}_{Bk} \approx \hat{r}_{Ak} \sin \hat{\theta}_{Ak} - \hat{r}_{Bk} \sin \hat{\theta}_{Bk}. \quad (8) $$

In Eq. (8) and Eq. (9), the following conditions are generally satisfied in practical applications:

$$ \sin \hat{\theta}_{Ak} \ll \hat{r}_{Ak} \cos \hat{\theta}_{Ak}, \quad (10) $$

$$ \sin \hat{\theta}_{Bk} \ll \hat{r}_{Bk} \cos \hat{\theta}_{Bk}. \quad (11) $$

Therefore, the impact of range bias $b_r^s$ on $\Delta \hat{x}_k$ is much less than that of azimuth bias $b_\theta^s$, $s \in \{A, B\}$. Ignoring the terms containing $b_r^s$ in Eq. (8) and Eq. (9), we get the simplified equations:

$$ \hat{x}_{Ak} \approx \hat{x}_{Bk} \approx \hat{r}_{Ak} \cos \hat{\theta}_{Ak} - \hat{r}_{Bk} \cos \hat{\theta}_{Bk} \approx \hat{r}_{Ak} \sin \hat{\theta}_{Ak} - \hat{r}_{Bk} \sin \hat{\theta}_{Bk}. \quad (12) $$

Let $b^\theta = [b_\theta^A, b_\theta^B]^T$, and

$$ B^\theta_k = \begin{bmatrix} \hat{r}_{Ak} \cos \hat{\theta}_{Ak} - \hat{r}_{Bk} \cos \hat{\theta}_{Bk} \\ \hat{r}_{Ak} \sin \hat{\theta}_{Ak} - \hat{r}_{Bk} \sin \hat{\theta}_{Bk} \end{bmatrix}, \quad (14) $$

then we have

$$ \Delta \hat{x}_k \approx B^\theta_k b^\theta, \quad (15) $$

which is the basis of the azimuth-LS approach [14]. Using the track sets of $N$ targets, the azimuth-LS solution is [14]

$$ b^\theta_{azimuth-LS} = (B^\theta T B^\theta)^{-1} B^\theta T \Delta \hat{x}, \quad (16) $$

where $B^\theta = [B^\theta_1^T, B^\theta_2^T, \ldots, B^\theta_N^T]^T$.

The azimuth-LS method only estimates the azimuth biases. It is more stable than the LS method, due to the reduction of the number of regression coefficients. However, there are two problems with the azimuth-LS approach. First, it sacrifices the accuracy of range bias estimation by setting the range bias estimate to be zero all along. When the problem is well-conditioned, it is not able to give satisfactory estimation result and even worse than the LS approach. Second, it is specially designed for the dense-target scenario, so it cannot adapt itself to a situation with dense sensors.
III. ILL CONDITIONING ANALYSIS

In this section, we first explain the ill conditioning in the dense-target scenario and the dense-sensor scenario, respectively, and then elaborate some more in-depth understanding about the relation between ill conditioning and scenario parameters on the basis of an illustrative scenario model.

A. Ill conditioning in the dense-target scenario

Consider the extreme dense-target situation with $N$ targets residing at the same place, i.e.,

$$\hat{\theta}_{i1} = \hat{\theta}_{i2} = \cdots = \hat{\theta}_{iN},$$  \hspace{1cm} (17)

$$\hat{r}_{i1} = \hat{r}_{i2} = \cdots = \hat{r}_{iN},$$  \hspace{1cm} (18)

where $i \in \{A, B\}$. Under this extreme scenario, the even rows and the odd rows of $B$ are totally identical in their inner parts, respectively, resulting in

$$\text{rank}(B) = 2.$$  \hspace{1cm} (19)

Then there are an infinite number of solutions to the LS problem (6), so some special techniques must be applied to choose a reasonable one.

With closely spaced targets, even if the $B$ is full column rank, there are strong collinearities between its odd rows and even rows, respectively. Therefore, $B^T B$ has a close-to-zero eigenvalue and a large condition number.

B. Ill conditioning in the dense-sensor scenario

Let $d_S$ represents the distance between sensor $A$ and $B$, then we have

$$d_S = \sqrt{u^2 + v^2},$$  \hspace{1cm} (20)

with the assumption that sensor $A$ is located at $(0, 0)$, and sensor $B$ is located at $(u, v)$. In multi-sensor surveillance applications, when sensor $A$ and $B$ are closely spaced, observing a remote target $k$, we have

$$r_{Ak} \approx r_{Bk},$$  \hspace{1cm} (21)

$$d_S \ll r_{Ak},$$

$$d_S \ll r_{Bk}.$$  \hspace{1cm} (23)

Generally,

$$|\theta_{Ak} - \theta_{Bk}| \approx \frac{d_S}{r_{Ak}} \approx \frac{d_S}{r_{Bk}} \approx 0.$$  \hspace{1cm} (24)

That is,

$$\theta_{Ak} \approx \theta_{Bk}.$$  \hspace{1cm} (25)

Since

$$\hat{\theta}_{Ak} = \theta_{Ak} + \theta^*_A,$$

$$\hat{\theta}_{Bk} = \theta_{Bk} + \theta^*_B,$$  \hspace{1cm} (26)

$$\theta^*_A - \theta^*_B \approx \hat{\theta}_{Ak} - \hat{\theta}_{Bk}.$$  \hspace{1cm} (28)

Eq. (28) means that the relative azimuth bias $\theta^*_A - \theta^*_B$ is approximately equal to the difference between the azimuth estimates on a remote target $k$ of the two closely spaced sensors. Therefore, parameters $\theta^*_A$ and $\theta^*_B$ are nearly collinear under a dense-sensor scenario. So it is difficult to have accurate registration results under this situation.

C. An illustrative scenario model

To uncover the relationship between scenario parameters and performance of traditional registration (see Eq. (7)), an illustrative scenario model is given in Fig. 2, where $d_T$ is the distance between two targets, $d_S$ is the distance between sensor $A$ and $B$, $d_{ST}$ is the distance between midpoints of the two line segments connecting the sensors and targets, respectively. Sensor biases are set as: $b^*_A = 1 km, b^*_B = -1 km, b^*_A = 1^\circ, b^*_B = -1^\circ$. Random errors of all local reports are independently identically distributed (i.i.d) normal with mean zero and covariance diag$(625 m^2, 625 m^2)$, where diag$(d_1, d_2)$ is a diagonal matrix with diagonal elements $d_1$ and $d_2$. All results are based on 100 Monte Carlo runs.

Fig. 3 demonstrates the performance of sensor bias estimation as a function of $d_S$, with $d_T = 80 km$ and $d_{ST} = 80 km$. Since the scenario is apt to be dense-sensor with quite a small $d_S$ and dense-target with quite a large $d_S$, both small and large $d_S$s are able to bring about large estimation errors and large cond$(B^T B)$, which represents the condition number of $B^T B$. In other words, the dense-sensor scenario and dense-target scenario are relative concepts. Similar conclusion can be obtained from Fig. 4, which demonstrates the performance of sensor bias estimation as a function of $d_T$, with $d_S = 100 km$ and $d_{ST} = 80 km$. Fig. 5 shows the performance of sensor bias estimation as a function of $d_{ST}$ with $d_S = 80 km$ and $d_T = 80 km$. The estimation performance deteriorates seriously with the increase in $d_{ST}$, because the scenario with quite a large $d_{ST}$ tends to be both dense-sensor and dense-target. According to Fig. 3(a) and Fig. 4(a), the registration performance can be regarded to be ill-conditioned, when cond$(B^T B)$ is larger than $10^7$. That is, $10^7$ can be used as a threshold of the condition number to roughly judge whether there is ill conditioning in a scenario, which agrees with [11]. The theoretical analysis and proof about this threshold are left for future.

Intuitively, more information can make better estimation results. It is natural to question whether ill conditioning can be avoided by increasing measurements. Let $N$ targets evenly distribute across the line segment between $(-\frac{d_T}{2}, d_{ST})$ and $(\frac{d_T}{2}, d_{ST})$. Fig. 6 shows the registration performance as a function of target number $N$ with $d_S = 20 km$, $d_T = 20 km$ and $d_{ST} = 100 km$. With the increase in $N$, no significant improvement is obtained. Therefore, target number $N$ is not a pivotal parameter about ill conditioning.
Fig. 3. Performance of sensor bias estimation as a function of \(d_S\)

Fig. 4. Performance of sensor bias estimation as a function of \(d_T\)

Fig. 5. Performance of sensor bias estimation as a function of \(d_{ST}\)

Fig. 6. Performance of sensor bias estimation as a function of target number \(N\)
IV. ROBUST REGISTRATION BASED ON BVLS

Various biased estimators have been developed to combat ill-conditioning of the estimation problem in literature, including ridge regression [17], Tikhonov regularization [18], James-Stein estimator [19], the truncated singular value decomposition (TSVD) [20], the Liu-type estimator [21], etc. However, these approaches are not free of turning parameters, which always lack clear physical meaning. In this paper, we employ the BVLS constrained optimization algorithm to address the ill-conditioned problem through inserting prior constraints on the solution space [22]. The constraints are the permissible range of sensor biases. In registration, we can put forward the solution space [22]. The constraints are the permissible range of sensor biases. In registration, we can put forward the solution space [22].

The BVLS approach with soft constraints is used to accommodate different sensor-target configurations. We call the BVLS approach with soft constraints by soft-BVLS.

A. BVLS registration with hard constraints

Let $\mathbf{u}^{max}$ represent the prior upper bound of the sensor bias parameters. A simple method of determining the bound for the BVLS method is to directly use the prior knowledge as hard constraints, which keep unchangeable even if the sensor-target configuration changes. That is,

$$\mathbf{u} = \mathbf{u}^{max}.$$

We call the approach with hard constraints by hard-BVLS.

The more accurate prior knowledge about the bounds we obtain, the better estimation result we will get. However, it is difficult to assess the quality of the prior knowledge about the bounds in practice. To guarantee the true value of each variable lies in the corresponding interval, loose bound constraints are usually preferred. When the problem is good-conditioned, the BVLS approach with loose bound constraints is approximately equivalent to the LS method. But it could not work equally well for an ill-conditioned problem.

B. BVLS registration with soft constraints

To overcome the drawback of the hard-BVLS method, soft constraints are used to accommodate different sensor-target configurations. We call the BVLS approach with soft constraints by soft-BVLS.

In the framework of soft-BVLS, there are two basic requirements for setting the bound constraints:

- loose bound constraints are preferred for a good-conditioned problem to ensure the estimator is consistent with the LS approach;
- tighter bound constraints should be used for an ill-conditioned problem to reduce the influence of ill-conditioning.

In fact, many functions or their amended versions can satisfy the two conditions above. In this paper, we use the sigmoid function in the artificial neural network community [26] for reference, and set the component of $\mathbf{u}$ in the soft-BVLS algorithm as

$$u_i = \frac{u_i^{max}}{1 + \exp(\alpha \times \beta \times \log_{10}(\text{cond}(\mathbf{B}^T \mathbf{B}) - \beta))}$$

where $u_i$ and $u_i^{max}$ are the $i$th component of $\mathbf{u}$ and $\mathbf{u}^{max}$, respectively; $\alpha$ and $\beta$ are two adjustable parameters.

As shown in Fig. 11, the parameter $\alpha$ decides the drop-down speed of the bound constraint curve. If $\alpha$ is too large, the sensor bias estimates are apt to be confined to zero under an ill-conditioned problem; If $\alpha$ is set too small, the ill-conditioning could not be circumvented well. The parameter $\beta$ should correspond to the intersection point between the ill-conditioned and good-conditioned problems. In accordance with [11] and the discussion in section III-C, we can set $\beta = \log_{10}(7) = 7$. In future, research on the ill-conditioning evaluation will be investigated to design an accurate constraint function. On the basis of Eq. (34), the bound constraints can be adjusted according to the actual sensor-target situation.

Fig. 7. The bound constraint curve of Eq. (34), with $u_i^{max} = 5$, $\alpha = 2$, and $\beta = 7$. 

where $v$ is the BVLS problem as

$$\min \| \Delta \mathbf{x} - \mathbf{B} \mathbf{b} \|^2_2$$

s.t.

$$\mathbf{b} \preceq \mathbf{u}$$

$$\mathbf{b} \succeq \mathbf{v}.$$
biases are set as $b_A^b = -0.6\text{km}$, $b_A^a = 0.8^\circ$, $b_B^b = 0.7\text{km}$ and $b_B^a = -0.8^\circ$. The prior upper bound about sensor biases are $u_{\max} = [5\text{km, }5^\circ, 5\text{km, }5^\circ]$. The hard-BVLS and soft-BVLS approaches can work well irrespective of the existence of sensor biases. The hard-BVLS is superior to other methods, since it even better than the hard-BVLS.

C. Varying the size of target region and the distance between sensors simultaneously

Let $d_S = l_T$. Fig. 10 illustrates the performance of different registration approaches in the light of RMSE as a function of $l_T$ (or $d_S$). When both $l_T$ and $d_S$ are small, the scenario tends to be both dense-target and dense-sensor.

Under both dense-target and dense-sensor situation, both azimuth-LS and LS work badly, and loose hard bound constraints could not improve the estimation performance any more. The soft-BVLS is superior to other methods, since it adjusts the bound constraints according to the actual situation. When $d_S$ and $l_T$ are enough large, the BVLS approaches are consistent with the LS method.

D. Impact of the tightness of bound constraint on BVLS

Let $d_S = l_T$. Bound constraint for the hard-BVLS approach is set as $u_{\max} = u_{\max} [1\text{km, }1^\circ, 1\text{km, }1^\circ]^T$, where $u_{\max}$ is varied to obtain different bound constraints in the simulation. Eq. (30) is satisfied as default. Fig. 10 illustrates the performance of hard-BVLS with four different bound constraints in the light of RMSE as a function of $l_T$ (or $d_S$).

Smaller RMSE is caused when the bound constraint gets tighter (compare the cases with $u_{\max} = 5, 3, 1$ in Fig. 11) but appropriate. The word "appropriate" means that the upper bound should not be too tight to be smaller than the true value of the parameters. Obviously, the bound constraint with $u_{\max} = 0.5$ is not appropriate. It causes fairly large RMSE about the azimuth bias estimation. Only when the scenario is severely ill-conditioned, the bound constraint with $u_{\max} = 0.5$ makes the smallest RMSE on the range bias estimation, but
In this paper, we analyze the ill conditioning theoretically in the dense-target scenario and the dense-sensor scenario, respectively, and propose a robust registration approach based on BVLS. To reduce the influence of ill conditioning, the proposed approach inserts prior constraints on the desired solution. Compared with the traditional LS method and the recently proposed azimuth-LS method, the BVLS approach can overcome ill conditioning in complex scenarios, and in the meantime it could be consistent with the LS estimator when the problem is good-conditioned. Since the proposed registration approach is performed in the system plane at the fusion center, it can be used in more generalized, moving collection platforms or sensors with the condition that the sensors can be precisely located dynamically.

Apart from the ill-conditioning in some registration scenarios which causes high estimation variance, misassociations in track-to-track association can also lead to degradation of the registration process, because registration needs correctly associated data. Registration with presence of both misassociations and ill-conditioning is much more challenging, which is our major concern for the future work.

VI. CONCLUSION

In this paper, we analyze the ill conditioning theoretically in the dense-target scenario and the dense-sensor scenario, respectively, and propose a robust registration approach based on BVLS. To reduce the influence of ill conditioning, the proposed approach inserts prior constraints on the desired solution. Compared with the traditional LS method and the recently proposed azimuth-LS method, the BVLS approach can overcome ill conditioning in complex scenarios, and in the meantime it could be consistent with the LS estimator when the problem is good-conditioned. Since the proposed registration approach is performed in the system plane at the fusion center, it can be used in more generalized, moving collection platforms or sensors with the condition that the sensors can be precisely located dynamically.

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