A Sequential Tracking Filter without Requirement of Measurement Decorrelation

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Abstract—A sequential measurement processing method in nonlinear state estimation is of benefit to both estimation accuracy and computational efficiency. When the measurement errors are correlated, the decorrelation is required to operate based on the covariance of the measurements in previous methods in order for correct sequential processing. In this paper, a new sequential processing method without decorrelation before filtering is presented. The measurements are processed directly by corresponding approaches, while the correlation is handled in the measurement update under the minimum mean squared error (MMSE) estimation framework. Simulations demonstrate the effectiveness of the new method.

Keywords—state estimation; nonlinear filtering; sequential processing; correlation; RMSE

I. INTRODUCTION

Target motion is best modeled in Cartesian coordinates, while the measurements are usually reported in polar or spherical coordinates. In this case, tracking in Cartesian coordinates using the original measurements is a nonlinear state estimation problem. Various nonlinear filtering approaches such as the extended Kalman filter (EKF) [4][14] and the unscented Kalman filter (UKF) [6] are used to handle this problem. In a Doppler radar tracking system, the measurement consists of range, range rate (Doppler) and one or two angles. The range and angles can be converted into a linear form in Cartesian states to avoid using nonlinear filtering approaches. This actually results in the so called converted measurement Kalman filter (CMKF) [7], which is considered to be superior to the nonlinear approaches (EKF and UKF) in most scenarios. But when the Doppler measurements are incorporated, nonlinear filtering techniques are suggested to be used since there is not yet a conversion from the Doppler measurements straightly linear in Cartesian states. The problem, whether CMKF or nonlinear method should be used, arises when the position measurements and Doppler are required to be used to estimate target states. Sequential measurement processing scheme, which is proven to be favorable to both estimation accuracy and computational efficiency[5][11][12][13], is used to process position measurements and Doppler measurements by the CMKF and nonlinear filtering techniques [3][4][6][14][15], respectively. In some radar systems, the range and range rate measurement errors are statistically correlated. To perform correct sequential measurement processing, decorrelation based on Cholesky factorization is used in [3][4][6][14][15], to produce a pseudo measurement with errors independent to the position measurements. The main advantage of the sequential processing method is that the approximation of nonlinearity operates based on the estimated state from the position measurements on the current stage, not the predicted state from the last stage in simultaneous measurement processing method. However, the decorrelation is carried out based on the measurements and their covariance during all of the processing stages. This may cause lose of estimation accuracy and require much computational cost.

In this paper, a new sequential processing technique, which does not require explicit decorrelation between parts of the measurement, is proposed. The position measurements and the converted Doppler measurements (i.e., the product of the range and range rate measurements) are used directly in the sequential processing scheme. The correlation is evaluated based on the covariance after CMKF instead of the origin measurement covariance, and is handled implicitly in the minimum mean squared error (MMSE) estimation framework. Monte Carlo simulations illustrate the effectiveness and benefits of this new method.

The rest of this paper is organized as follows. Section II describes the formulation of the tracking system. The measurement conversions and the converted measurement errors are presented in Section III. In Section IV, the new tracking filter is presented, including the formulations of the CMKF, the covariance between the state errors from CMKF and the converted Doppler measurements, and the final state update which handles the correlation implicitly. Simulations are performed in Section V followed by conclusions in Section VI.

II. PROBLEM FORMULATION

In Cartesian coordinates, the target motion can be modeled as

\[ X_k = \Phi X_{k-1} + \Gamma v_{k-1} \]  \hspace{1cm} (1)

where \( X_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T \), \( X_k \in \mathbb{R}^n \) is the state vector consisting of position components and corresponding velocity components along \( x \) and \( y \) directions, respectively, at time step \( k \). If maneuvering target is considered, the state vector can be augmented by other components such as acceleration.
Here, $\Phi \in \mathbb{R}^{n \times n}$ is the state transition matrix, $v_k$ is zero-mean Gaussian random process noise with known covariance $Q_{k-1}$, and $\Gamma$ is the noise gain matrix [2].

A 2D Doppler radar is assumed to report measurements of targets in polar coordinates, including range, range rate and azimuth. The measurement equation can be expressed as

$$Z_k = [x_k^a, \theta_k^a, \dot{r}_k^a]^\top$$

$$= h(X_k) + w_k = [r_k, \theta_k, \dot{r}_k]^\top + w_k$$

where

$$r_k = \sqrt{x_k^2 + y_k^2}$$

$$\theta_k = \tan^{-1}(y_k / x_k)$$

$$\dot{r}_k = [x_k \dot{x}_k + y_k \dot{y}_k] / \sqrt{x_k^2 + y_k^2}$$

$$w_k = [\tilde{r}_k, \tilde{\theta}_k, \tilde{\eta}_k]^\top$$

$\tilde{r}_k^a$, $\tilde{\theta}_k^a$ and $\tilde{\eta}_k^a$ are measurements of the true target range, azimuth and range-rate, respectively. In the above, $\tilde{r}_k$, $\tilde{\theta}_k$ and $\tilde{\eta}_k$ are the corresponding measurement noises, which are all assumed to be zero-mean white Gaussian noise with known variances $\sigma^2_{r_k}$, $\sigma^2_{\theta_k}$ and $\sigma^2_{\dot{r}_k}$, respectively. It is assumed that the measurement noises are mutually independent with the exception that $\tilde{r}_k$ and $\tilde{\eta}_k$ are statistically correlated [1] with correlation coefficient $\rho$, i.e.

$$\text{cov}[	ilde{r}_k, \tilde{\eta}_k] = \rho \sigma_{\tilde{r}_k} \sigma_{\tilde{\eta}_k}$$

The covariance matrix of the original measurements can be written as

$$R_k = \begin{bmatrix}
\sigma^2_{r_k} & \rho \sigma_{\tilde{r}_k} \sigma_{\tilde{\eta}_k} & 0 \\
\rho \sigma_{\tilde{r}_k} \sigma_{\tilde{\eta}_k} & \sigma^2_{\theta_k} & 0 \\
0 & 0 & \sigma^2_{\dot{r}_k}
\end{bmatrix}$$

III. MEASUREMENT CONVERSIONS

A. Measurement conversion equations

As well known, the position measurements including range and azimuth in polar coordinates can be transformed into linear forms in Cartesian coordinates by

$$Z_{i}^{\text{xx}} = [x_i^c, y_i^c]^\top$$

In the above,

$$x_i^c = r_i^c \cos \theta_i^c = x_i + \bar{x}_i$$

$$y_i^c = r_i^c \sin \theta_i^c = y_i + \bar{y}_i$$

where $\bar{x}_i$ and $\bar{y}_i$ are the errors of converted position measurements along $x$ and $y$ directions in the Cartesian coordinates respectively. The converted Doppler measurement is given as

$$Z_k^{\text{ax}} = [x_k^a, \theta_k^a, \dot{r}_k^a]^\top = \eta_k + \tilde{\eta}_k$$

where $\eta_k$ is the converted Doppler (i.e., the product of range and range rate), with a quadratic function of Cartesian states as

$$\eta_k = d(X_k) = x_k \dot{x}_k + y_k \dot{y}_k$$

and $\tilde{\eta}_k$ is the error of the converted Doppler measurement $\eta_k^a$. Incorporating the position and Doppler measurements, the whole converted measurements from the original sensor reports can be expressed as

$$Z_k^c = [x_k^c, y_k^c, \eta_k^a]^\top = [x_k, y_k, \eta_k^a] + [\tilde{x}_k, \tilde{y}_k, \tilde{\eta}_k]^\top$$

B. Converted Measurement Errors

In order to perform effective filtering, appropriate error computation should be taken to obtain accurate measurement errors matching the true statistics.

Denote the bias and covariance of the whole converted measurement vector (14), respectively, as

$$\mu_k = [\mu_k^x, \mu_k^y, \mu_k^\theta]^\top = [\mu_k^x, \mu_k^y, \mu_k^\theta]^\top$$

and

$$R_k^c = \begin{bmatrix}
R_k^x & R_k^{xy} & R_k^y \\
R_k^{yx} & R_k^{yy} & R_k^{yx} \\
R_k^{xy} & R_k^{yx} & R_k^y
\end{bmatrix}$$

The expressions of the elements in (15) and (16) are given as

$$\mu_k^x = r_k^c \cos \theta_k^c \left( e^{\sigma_{\dot{r}_k}^2 / 2} - e^{-\sigma_{\dot{r}_k}^2 / 2} \right)$$

$$\mu_k^y = r_k^c \sin \theta_k^c \left( e^{\sigma_{\dot{r}_k}^2 / 2} - e^{-\sigma_{\dot{r}_k}^2 / 2} \right)$$

$$R_k^x = (r_k^c)^2 e^{2\sigma_{\dot{r}_k}^2} \left\{ \cos^2 \theta_k^c \left( \cosh (2\sigma_{\theta_k}^2) - \cosh (\sigma_{\theta_k}^2) \right) \right\}$$

$$+ (r_k^c)^2 e^{-2\sigma_{\dot{r}_k}^2} \left\{ \sin^2 \theta_k^c \left[ \sinh (2\sigma_{\theta_k}^2) - \sinh (\sigma_{\theta_k}^2) \right] \right\}$$

$$+ \sigma_{\dot{r}_k}^2 e^{-2\sigma_{\dot{r}_k}^2} \left\{ \cos^2 \theta_k^c \left( 2 \cosh (2\sigma_{\theta_k}^2) - \cosh (\sigma_{\theta_k}^2) \right) \right\}$$

$$+ \sigma_{\dot{r}_k}^2 e^{2\sigma_{\dot{r}_k}^2} \left\{ \sin^2 \theta_k^c \left[ 2 \sinh (2\sigma_{\theta_k}^2) - \sinh (\sigma_{\theta_k}^2) \right] \right\}$$

$$R_k^y = (r_k^c)^2 e^{2\sigma_{\dot{r}_k}^2} \left\{ \sin^2 \theta_k^c \left( \cosh (2\sigma_{\theta_k}^2) - \cosh (\sigma_{\theta_k}^2) \right) \right\}$$

$$+ (r_k^c)^2 e^{-2\sigma_{\dot{r}_k}^2} \left\{ \cos^2 \theta_k^c \left[ \sinh (2\sigma_{\theta_k}^2) - \sinh (\sigma_{\theta_k}^2) \right] \right\}$$

$$+ \sigma_{\dot{r}_k}^2 e^{-2\sigma_{\dot{r}_k}^2} \left\{ \sin^2 \theta_k^c \left( 2 \cosh (2\sigma_{\theta_k}^2) - \cosh (\sigma_{\theta_k}^2) \right) \right\}$$

$$+ \sigma_{\dot{r}_k}^2 e^{2\sigma_{\dot{r}_k}^2} \left\{ \cos^2 \theta_k^c \left[ 2 \sinh (2\sigma_{\theta_k}^2) - \sinh (\sigma_{\theta_k}^2) \right] \right\}$$

$$R_k^{xy} = \sin \theta_k^c \cos \theta_k^c e^{4\sigma_{\dot{r}_k}^2} \sigma_{\dot{r}_k}^2$$

$$+ \sin \theta_k^c \cos \theta_k^c e^{4\sigma_{\dot{r}_k}^2} \left\{ (r_k^c)^2 + \sigma_{\dot{r}_k}^2 \right\} \left( 1 - e^{-\sigma_{\dot{r}_k}^2 / 2} \right)$$
\[
\begin{align*}
\mu_t^{i} &= \rho \sigma_r, \sigma_r \\
R_0^i &= R_0^{ii} = (\sigma_r^2 + \rho^2 \sigma_r^2) \cos \theta_t e^{-\sigma_r^2/2} \\
R_0^p &= R_0^{ip} = (\sigma_r^2 + \rho^2 \sigma_r^2) \sin \theta_t e^{-\sigma_r^2/2} \\
R_0^p &= R_0^{pp} = (\rho^2)^2 \sigma_r^2 + \sigma_r^2 (\rho^2) + 3(1 + \rho^2) \sigma_r^2 \sigma_r^2 + 2(\rho^2) \sigma_r^2 \rho \sigma_r + \rho^2 \sigma_r^2 
\end{align*}
\]

IV. SEQUENTIAL TRACKING FILTER

A. Position measurement filtering

The converted position measurements \(Z_{k}^{\text{cp}}\) are processed by the de-biased CMKF [7] to produce Cartesian state estimates. The measurement equation is expressed as

\[
Z_{k}^{\text{cp}} = HX_k + w_k^{p}
\]

where \(H\) is the measurement matrix and for a constant velocity

The measurement noise \(w_k^{p} = [\tilde{x}_k, \tilde{y}_k]^T\) is zero-mean Gaussian noise with unknown covariance \(R_k^{p}\) in (18).

The time update and measurement update of the target state are implemented by follows:

\[
\begin{align*}
\hat{X}_{k+1} &= \Phi \hat{X}_{k} + K_{k} [Z_{k}^{\text{cp}} - \mu^{p} - H \Phi \hat{X}_{k}] \\
P_{k+1} &= (I - K_{k} H)P_{k+1} \quad \text{(22)} \\
K_{k} &= P_{k+1}H^{T} [HP_{k+1}H^{T} + R_{k}^{p}]^{-1} \quad \text{(23)} \\
\hat{X}_{k+1} &= \hat{X}_{k+1} + K_{k} [Z_{k}^{\text{cp}} - \mu^{p} - H \Phi \hat{X}_{k}] \quad \text{(25)} \\
P_{k+1} &= (I - K_{k} H)P_{k+1} \quad \text{(26)}
\end{align*}
\]

B. Covariance after position measurement filtering

To implement the sequential after the position measurement filtering, the covariance between the state errors from the CMKF, described in the subsection above, and the converted Doppler measurements should be evaluated.

From the formulations of CMKF, (1), (19)-(26), one can rewrite the state estimate of CMKF at time \(k\) as

\[
\hat{X}_{k} = \Phi \hat{X}_{k-1} + K_{k} [Z_{k}^{\text{cp}} - \mu_{k}^{p} - H \Phi \hat{X}_{k-1}] \quad \text{(27)}
\]

Then the corresponding estimation error is

\[
\begin{align*}
\tilde{X}_{k} &= X_k - \hat{X}_{k} \\
&= (\Phi X_{k-1} + \Gamma \eta_k) - \Phi \hat{X}_{k-1} - K_{k} \{ \} \\
&= (I - K_{k} H) \Phi \tilde{X}_{k-1} + (I - K_{k} H) \Gamma \eta_k - K_{k} w_{k}^{cp}
\end{align*}
\]

Multiply the above by the error of the converted Doppler measurements, one can get the covariance as

\[
P_{k+1} \triangleq \text{cov} (\tilde{X}_{k+1}, \tilde{Z}_{k}) = E [\tilde{X}_{k+1} (-\eta_k)] = \\
E[(I - K_{k} H) \Phi \tilde{X}_{k-1} (-\eta_k) + (I - K_{k} H) \Gamma \eta_k - K_{k} w_{k}^{cp} (-\eta_k)] = K_{k} R_{k+1}^{p}
\]

In the above, the assumption that the estimation error and process noise at the last time is independent to the measurement errors at the current time is used and thus the corresponding terms vanish.

C. Final estimation

Considering the Cartesian states \(\hat{X}_{k+1}\) at time \(k\) from the CMKF as the prior \(\hat{X}_{k}\), the state update by the converted Doppler measurements is derived under the minimum mean squared error (MMSE) estimation framework as

\[
\hat{X} = \hat{X}_{k} + P_{k+1}^{cp} (Z_{k}^{\text{cp}} - \tilde{Z}_{k}^{p})
\]

Expanding \(Z_{k}^{p}\) around \(\hat{X}_{k}\), in a Taylor series with terms up to the second order as

\[
\hat{X}_{k+1} = d(X_k) + \eta_k = \{d(\hat{X}_{k}) + \tilde{D}(X_k - \hat{X}_{k})\} + \frac{1}{2} (X_k - \hat{X}_{k})^{\text{T}} \tilde{D}(X_k - \hat{X}_{k}) + \text{HOT} \}
\]

where \(d\) is the nonlinear function of the converted Doppler measurements with respect to the Cartesian states. The Jacobian and Hessian of \(d\) are defined, respectively, as

\[
\tilde{D} = \frac{\partial d}{\partial X_{k-1-1}} \quad \text{(32)}
\]

and

\[
\tilde{D} = \frac{\partial^2 d}{\partial X_{k-1}} \quad \text{(33)}
\]

The higher-order terms \(\text{HOT}\) are zero, since converted Doppler is quadratic in Cartesian states. The prior mean of measurement is obtained by taking expectation of (31) as

\[
Z_{k}^{p} = d(\hat{X}_{k}) + \frac{1}{2} \text{tr}(\tilde{D}P_{k+1}^{p}) \quad \text{(34)}
\]

The covariance between the states to be estimated and the measurement can be obtained as

\[
P_{x} \to E[(X - \hat{X})(Z_{k}^{p} - \tilde{Z}_{k}^{p})] = P_{k+1}^{x} \tilde{D}^{V} - P_{k+1}^{p} \quad \text{(35)}
\]

and the covariance of the measurement is

\[
P_{z} \to E[(Z_{k}^{p} - \tilde{Z}_{k}^{p})(Z_{k}^{p} - \tilde{Z}_{k}^{p})^{\text{T}}] = \tilde{D}P_{k+1}^{p} \tilde{D}^{V} + R_{k+1}^{p} + \frac{1}{2} \text{tr}(\tilde{D}P_{k+1}^{p} \tilde{D}P_{k+1}^{p} - \tilde{D}P_{k+1}^{p} \tilde{D}P_{k+1}^{p}) \tilde{D}^{V} \quad \text{(36)}
\]

Then, the final state estimates can be provided by (30) and the covariance associated with this combined estimate is

\[
P_{k+1} = P_{k+1}^{x} - P_{x} (P_{z}^{p})^{\text{T}} (P_{z}^{p}) \quad \text{(37)}
\]

D. Filter initialization

The basic idea of the two-point differencing initialization method [2] is to estimate the initial position and velocity components of the state using the first two sets of position measurements and find the initial covariance by the
measurement covariance.

The initial state is given as
\[
\hat{x}_{k|k} = \left( Z_{k}^{i,p} - \mu_{k}^{i,p} \right) / T\]
and the initial state is
\[
P_{k|k} = \begin{bmatrix}
R_{k}^{i} & R_{k}^{i} / T \\
(R_{k}^{i})^T / T & 2R_{k}^{i} / T^2
\end{bmatrix}
\]

V. SIMULATIONS

In order to evaluate the performance of the new sequential nonlinear tracking filter, which is referred to as SEKF-NEW, target with nearly CV and constant acceleration (CA) motion are considered. The transition matrix for CV model is
\[
\Phi = \begin{bmatrix}
1 & 0 & T & 0 \\
0 & 1 & 0 & T \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
and that for CA model is
\[
\Phi = \begin{bmatrix}
1 & 0 & T & T^2 / 2 & 0 \\
0 & 1 & 0 & T & T^2 / 2 \\
0 & 0 & 1 & 0 & T \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Sampling interval is \( T = 1s \). All of the process noises are generated as zero-mean uncorrelated Gaussian white noise with standard deviation 0.001m/s^2.

To avoid a large number of figures, only the root mean squared error (RMSE) of position and velocity are used to examine performance of the proposed filter and are defined as
\[
e_P = \sqrt{ \frac{1}{M} \sum_{i=1}^{M} \left[ (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right] }
\]
and
\[
e_V = \sqrt{ \frac{1}{M} \sum_{i=1}^{M} \left[ (\dot{x}_i - \hat{\dot{x}}_i)^2 + (\dot{y}_i - \hat{\dot{y}}_i)^2 \right] }
\]
respectively. The tracking performance of the proposed SEKF-NEW is compared with the sequential nonlinear filtering approach based on decorrelation with Cholesky factorization (SEKF-CF) presented in [4].

For the CV model, the target moves with a speed of 20m/s and a heading of 60deg, the initial of the target is [30km, 30km], and the acceleration is [0.02m/s^2, 0.02m/s^2]. The sensor is located at the origin of Cartesian coordinates to report position and Doppler measurements of targets. The measurement errors are set as \( \sigma_r = 0.3km \), \( \sigma_\theta = 0.3deg \) and \( \sigma_v = 0.1m/s \). The correlation coefficient \( \rho \) between the range and range rate errors are considered as the influencing factors to be investigated. The tracking performance for \( \rho = 0 \) and \( \rho = 0.9 \) are displayed in Fig. 1 and Fig. 2, respectively. It is shown that the proposed SEKF-NEW method can provide velocity estimation performance close to the SEKF-CF and position estimation performance better than the SEKF-CF, especially for the case with strongly correlated measurement errors.

For the CA model, the target moves with a speed of 10m/s and a heading of 60deg, the initial of the target is [5km, 5km], and the acceleration is [0.02m/s^2, 0.02m/s^2]. The sensor is located at the origin of Cartesian coordinates to report position and Doppler measurements of targets. The measurement errors are set as \( \sigma_r = 0.3km \), \( \sigma_\theta = 0.3deg \) and \( \sigma_v = 0.1m/s \). The correlation coefficient \( \rho \) between the range and range rate errors are considered as the influencing factors to be investigated. The tracking performance for \( \rho = 0 \) and \( \rho = 0.9 \) are displayed in Fig. 3 and Fig. 4, respectively. Conclusion similar to the CV case can be drawn for the CA model. The proposed SEKF-NEW method outperforms SEKF-CF in position RMSE, especially for the case of large correlation coefficient.
Fig. 2 Tracking performance in case of $\rho = 0.9$ for CV model

Fig. 3 Tracking performance in case of $\rho = 0$ for CA model

Fig. 4 Tracking performance in case of $\rho = 0.9$ for CA model
VI. CONCLUSIONS

A new sequential processing method, without requirement of measurement decorrelation before filtering, is presented. The case of tracking in Cartesian coordinates with position and Doppler measurements observed in polar coordinates is investigated. First, the converted position measurements are processed by the CMKF. The covariance between the state errors of CMKF and converted Doppler measurements is evaluated based on the filtering gain and measurement covariance. Then, a final estimation method is derived, based on Taylor expansion and minimum mean squared error estimation, where the nonlinearity and correlation are handled simultaneously and implicitly. Simulations demonstrate the benefits from using this new sequential processing method.

REFERENCES