Estimating Sensor Performance and Target Population Size with Multiple Sensors

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Abstract—A Bayesian inference approach to exploit multisensor target detections for the estimation of sensor performance is presented. Additionally, it is shown that a closed form approximate Bayesian estimator for the target population can be derived. This Bayesian method is applied to real data in the case of satellite detection of Automatic Identification System messages resulting in a global map of sensor performance. The resulting sensor detection performance estimates are also suitable for the tuning of tracking and fusion algorithms which make use of sensor models.

I. INTRODUCTION

In Multi-Target Tracking (MTT), one of the challenges is correctly tuning parameters of the tracking algorithm used. One of the important parameters used in tracking algorithms is the probability of detection, which is defined as the probability that given an observation, the sensor gives a positive indication of the presence of a target. When the sensor characteristics are not completely known, it is sometimes possible to calibrate the sensor to tune the probability of detection parameter. For example, calibration of the probability of detection in the case of a controlled surveillance environment such as a short range coastal radar. However, for many sensors such as High Frequency (HF) radar, or satellite sensing, it is not possible to achieve a controlled calibration. In these cases, one must often tune the probability of detection manually.

Target detections by a sensor are usually accompanied by some localization of the target (with some uncertainty of measurement), however, this paper does not focus on the precision of tracking but instead on the sensor performance in terms of detecting targets. For example, in Multi-Hypothesis Tracking (MHT), the probability of detection is used to score the various multi-frame assignment possibilities via a score function [1], [2]. Another example is in Joint Probabilistic Data Association (JPDA), one uses the probability of detection to compute the association probabilities [2].

The impact of the probability of detection in tracking is such that if the parameter is configured to be higher than the true probability, then the tracker will be more likely to lose or break a track on a target earlier when there are some missed detections. If the probability of detection parameter is configured lower than the true probability, then the tracker is more likely to spawn false tracks.

It has been shown [3] that using an improved estimate of sensor probability of detection results in superior tracking with the Probabilistic Data Association (PDA) tracker and a single sensor. In this paper, we show that in a multi-sensor environment, it is possible to exploit the independence of sensors to implement a Bayesian estimator for sensor probabilities of detection without the need to know the target population. However, in the multi-sensor case, it will also be required that detected targets from each sensor be effectively associated to one another. Therefore, the formulations here must assume that one already has some kind of effective multisensor data association capability. This work extends on initial progress made in [4].

Another parameter for MTT is the target population. Some tracking algorithms, such as the Probability Hypothesis Density (PHD) Filter [5] do not require this parameter. In fact, it is possible to estimate the target population dynamically with the PHD Filter by integrating the PHD function density. However, due to the complexity of the filter, it is difficult to apply to situations where the target density is excessively large.

It has been shown that given sensor performance and a set of detected targets, it is also possible to calculate the size of the target population [6]. For effective Situational Awareness (SA), one is often forced to resort to using the number of tracks in a tracking system as the estimate of the number of targets in a surveillance region. However, this does not necessarily provide any indication of the effectiveness of surveillance (namely, if any targets are not detected and should be), nor any indication of any surveillance “gaps”.

This paper will present an initial evaluation and demonstration of the estimation technique in the form of an analysis of real data from the Automatic Identification System (AIS). Ships and vessels exceeding a certain gross tonnage are equipped with AIS transponders for position reporting, as established by the Safety of Life at Sea (SOLAS) Convention [7]. Ships repeatedly broadcast their name, position and other details for automatic display on nearby ships. While this allows ships to be aware and keep track of other ships in their immediate vicinity, coastal states will also be able to receive, plot and log the data by means of receiving stations along the coast, in the air, or in space. In such a context, most targets are cooperative.

This paper is organized as follows. Section II introduces the mathematical notation followed by the derivations of the sensor models for the cases of static and pseudo-static.
Section III presents an analysis of real AIS data using the derived formulations. The paper is then summarized in the conclusion Section IV.

II. MATHEMATICAL FORMULATION

The following notation will be used: probabilities are indicated with \( \Pr(\cdot) \); \( f_X(x) \) denotes the probability density function (pdf) of the random variable \( X \), if it is continuous, or the probability mass function (pmf), if it is discrete; the \( T \) sensors are referred with the index \( i = 1, \ldots, T \); a detection \( D \) consists in the time of detection \( t \), identifier \( y \), position \( x \) (longitude and latitude), course \( \theta \) and speed \( v \) of the target; the set of actual targets in the surveillance area is indicated with \( \mathcal{N} \) and \( N \triangleq |\mathcal{N}| \); time is discretized in bins of length \( t_s \) and \( t_k \) denotes the final time of the bin \( k \); \( S_i(k) \) is the set of detections from sensor \( i \) during timestep \( k \) and \( S_i \triangleq |S_i(k)| \); finally the probability of detection of a target by sensor \( i \) is denoted with \( P_i \).

Sensors are modeled as having two key characteristics: Persistence and Mobility, which leads to four possible classifications of sensor types.

- **Persistence:**
  - persistent: a sensor is persistent if it is always observing an area; the sensor may have a scan-rate, or a revisit rate, but the time between observations is short. An example of a persistent sensor is a SAR satellite;
  - non-persistent: a sensor is non-persistent if there is a relatively long time between observations. An example would be a “snap-shot” of an area from a satellite;

- **Mobility:**
  - mobile: a mobile sensor is characterized by a changing field of view and surveillance area; possible mobile sensors include sensors on platforms, or on satellites;
  - fixed: a fixed sensor always observes the same surveillance area;

The derivation for the Bayesian estimator will start with the simplest case first. That is, multiple persistent and fixed sensors. Then the assumption of time and space synchronization will be slightly relaxed to include an approximate solution for the case of mobile sensor and nearly synchronous observations.

**A. Overlapping Sensors**

The simplest case to consider is the case of two synchronized sensors \( T = 2 \) which have just observed the same surveillance area. The resulting situation can be summarized as follows. In this case we omit the time index \( k \) because of the synchronization of the sensors.

For the set of detectable targets \( \mathcal{N} \), each sensor \( i \) will observe a set of targets \( S_i \).

Given \( N \) and the probabilities \( P_1 \) and \( P_2 \), \( S_1 \) and \( S_2 \) are binomially distributed:

\[
f_{S_i}(s \mid N, P_i) = \binom{N}{S_i} P_i^{S_i} (1 - P_i)^{N - S_i}, \quad i = 1, 2. \tag{1}
\]

Estimating \( P_i \) using Eq. (1) requires an a-priori knowledge of \( N \), that means

\[
f_{S_i}(s \mid P_i) = E_N \left[ f_{S_i}(s \mid N, P_i) \right], \tag{2}
\]

where \( E_N \left[ \cdot \right] \) is the expected value w.r.t. \( N \).

1) **Sensor Independence Assumption:** It is reasonable to assume that the detection by a given sensor is statistically independent of the detection by any other sensor. Note that this may not be the case in sensor cross-cuing, in which case this assumption will not hold and therefore invalidate the results.

In the case of independent sensors, the conditional probability of detection is

\[
P_i = \Pr(\text{targ. det. by } 1) = \Pr(\text{targ. det. by } 1 \mid \text{targ. det. by } 2).
\]

It is possible to use \( S_2 \) as “conditional” number of target to write the pmf of \( S_{1 \cap 2} \) as binomially distributed:

\[
f_{S_{1 \cap 2}}(s \mid P_1) = \binom{S_2}{s} P_1^s (1 - P_1)^{S_2 - s}, \tag{3}
\]

where \( S_{1 \cap 2} \triangleq |S_1 \cap S_2| \) and is referred to as the number of successes of sensor \( 1 \) with respect to the detection of sensor \( 2 \).

2) **Derivation of \( \hat{P}_i \):** In our case, we wish to estimate \( P_1 \) from the set of observations. This can be achieved using Bayesian inference

\[
f_{P_1}(p \mid S_{1 \cap 2}) = \frac{f_{S_{1 \cap 2}}(s_{1 \cap 2} \mid p) f_{P_1}(p)}{\int_0^1 f_{S_{1 \cap 2}}(s_{1 \cap 2} \mid p) f_{P_1}(p) \, dp}. \tag{4}
\]

The prior can be determined in many ways. Given no previous information before the first observation, it is reasonable then to assume an uninformed prior for \( P_1 \) such that initially, any value is equally possible. That is, we choose a uniform prior.

The marginal for \( S_{1 \cap 2} \) can be derived as follows

\[
f_{S_{1 \cap 2}}(s) = \int_0^1 f_{S_{1 \cap 2}}(s \mid \ell) f_{P_1}(\ell) \, d\ell. \tag{5}
\]

Thus substituting Eq. (3) and Eq. (5) in Eq. (4) and using the uniformity of the prior of \( P_1 \), we get

\[
f_{P_1}(p \mid S_{1 \cap 2}) = \frac{\binom{S_2}{s_{1 \cap 2}} p^{s_{1 \cap 2}} (1 - p)^{S_2 - s_{1 \cap 2}}}{\int_0^1 \binom{S_2}{s_{1 \cap 2}} p^{s_{1 \cap 2}} (1 - p)^{S_2 - s_{1 \cap 2}} \, dp}. \tag{6}
\]

Canceling out the constants in the numerator and the denominator, the integral is the definition of the Beta Function, and the final solution is simply a Beta Distribution. This result can be written as

\[
f_{P_1}(p \mid S_{1 \cap 2}) = \frac{1}{B(\alpha, \beta)} p^{\alpha - 1} (1 - p)^{\beta - 1}, \tag{7}
\]

where \( B(\alpha, \beta) = \int_0^1 t^{\alpha - 1} (1 - t)^{\beta - 1} \, dt \) is the beta function and \( \alpha \) and \( \beta \) are parameters of the distribution.

1For example, a Synthetic Aperture Radar (SAR) satellite

2Nearly synchronous observations here means that the time between observations is short such that the target has not moved a great distance.
Defining the number of failures of sensor 1 with respect to the detection of sensor 2 as

\[ F_{1\cap 2} \triangleq |S_2 - S_1|, \]

then in our case the values of the parameters of the Beta distribution are

\[ \alpha = S_{1\cap 2} + 1 \]

and

\[ \beta = F_{1\cap 2} + 1. \]

We estimate \( P_1 \) as the mode of Eq. (6), that is known in closed form:

\[ \hat{P}_1 = \arg \max_p f_{P_1}(p \mid S_{1\cap 2}) = \frac{S_{1\cap 2}}{S_{1\cap 2} + F_{1\cap 2}}. \]

The method can be easily extended to the case of \( T > 2 \) sensors. For each sensor \( i \) in overlap, we sum the successes with respect to the detection of all other sensors:

\[ S_{i,\text{tot}} = \sum_{j=1, j\neq i}^T |S_i \cap S_j| \]

and in the same way the number of failures is computed as

\[ F_{i,\text{tot}} = \sum_{j=1, j\neq i}^T |S_j - S_i|. \]

Given our distribution for \( P_i \), the maximum likelihood estimator is:

\[ \hat{P}_1 = \arg \max_p f_{P_1}(p \mid S_{i,\text{tot}}, F_{i,\text{tot}}) = \frac{S_{i,\text{tot}}}{S_{i,\text{tot}} + F_{i,\text{tot}}}. \]

3) Estimating \( N \): Given observations by sensors, it is also possible to estimate the population of targets \( N \) if the sensors’ \( P_i \), \( i = 1, \ldots, T \), are known.

In the case of \( T = 2 \) sensors, the pmf of the number of detections \( S_1, S_2 \) and \( S_{1\cap 2} \) is a multinomial:

\[ f_{S_1, S_2, S_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid N, P_1, P_2) = \frac{N!}{s_{1\cap 2}! s_1! s_2! s_{1\cap 2}!} P_{1\cap 2}^{s_{1\cap 2}} P_1^{s_1} P_2^{s_2}, \quad (9) \]

where \( s_{1\cap 2} \triangleq s_1 \cap s_2, s_2 \triangleq s_2 - s_1, s_{1\cap 2} \triangleq N - s_1 - s_2 + s_{1\cap 2}, P_{1\cap 2} \triangleq P_1(1 - P_2), P_2 \triangleq P_2(1 - P_1). \)

Eq. (9) can be rewritten as follows:

\[ f_{S_1, S_2, S_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid N, P_1, P_2) = \frac{N!}{s_{1\cap 2}! s_1! s_2! s_{1\cap 2}!} P_{1\cap 2}^{s_{1\cap 2}} P_2^{s_2}(1 - P_1)^{N-s_1}(1 - P_2)^{N-s_2}. \]

Now, an approximation is required. Given the sets of observations \( S_i, i = 1, \ldots, T \), the estimated population of ships can be derived, using the estimated performance \( \hat{P}_i \), \( i = 1, \ldots, T \). Thus in the following we omit the last two conditionings from the pmf, e.g. \( f_{\hat{S}_1, \hat{S}_2, \hat{S}_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid N) \triangleq f_{\hat{S}_1, \hat{S}_2, \hat{S}_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid N, \hat{P}_1, \hat{P}_2). \)

The posterior pmf of \( N \) is

\[ f_N(n \mid S_1, S_2, S_{1\cap 2}) = \frac{f_{\hat{S}_1, \hat{S}_2, \hat{S}_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid n) f_N(n)}{\sum_{\ell=0}^\infty \hat{f}_{\hat{S}_1, \hat{S}_2, \hat{S}_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid \ell) f_N(\ell)}. \]

Next, if one assumes that there are no false detections in our observations, then the summation in the denominator goes not from 0 to infinity, but from \( S_{1\cap 2} \triangleq S_1 + S_2 - S_{1\cap 2} \) to infinity. Furthermore, assuming that the prior of \( N \) is uniformly distributed, Eq. (10) can be rewritten as

\[ f_N(n \mid S_1, S_2, S_{1\cap 2}) = \frac{f_{\hat{S}_1, \hat{S}_2, \hat{S}_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid n)}{\sum_{\ell=S_{1\cap 2}}^\infty \hat{f}_{\hat{S}_1, \hat{S}_2, \hat{S}_{1\cap 2}}(s_1, s_2, s_{1\cap 2} \mid \ell) f_N(\ell)}. \]

It is shown in the appendix that the denominator in Eq. (11) is equal to

\[ \frac{S_{1\cap 2}!}{(S_{1\cap 2}!)^2 (1 - \hat{P}_{1\cap 2})(1 - \hat{P}_1)\hat{P}_2 S_{1\cap 2} \hat{P}_1 S_{1\cap 2}} \]

and Eq. (11) becomes

\[ f_N(n \mid S_1, S_2, S_{1\cap 2}) = \frac{\hat{P}_1^{n-S_{1\cap 2}} (1 - \hat{P}_{1\cap 2})(1 - \hat{P}_1) S_{1\cap 2}^{n-S_{1\cap 2}}}{(S_{1\cap 2}!)^{n-S_{1\cap 2}} (1 - \hat{P}_{1\cap 2})(1 - \hat{P}_1) S_{1\cap 2}}, \]

where \( \hat{P}_{1\cap 2} \triangleq (1 - \hat{P}_1)(1 - \hat{P}_2) \). Eq. (12) is the negative binomial distribution:

\[ f_N(n \mid S_1, S_2, S_{1\cap 2}) = \text{NB}(S_{1\cap 2} + 1, n - S_{1\cap 2}, \hat{P}_{1\cap 2}). \]

Thus the ML estimate for \( N \) can be derived in closed form starting from the mode of the negative binomial:

\[ \hat{N} = \left\lceil \frac{S_{1\cap 2}}{1 - \hat{P}_{1\cap 2}} \right\rceil. \]

In the general case of \( T > 2 \) sensors, the posterior pmf of \( N \) is

\[ f_N(n \mid S_{\cup i}) = \frac{\hat{P}^{n-S_{\cup i}}(1 - \hat{P}_{\cup i}) S_{\cup i}^{n+1}}{S_{\cup i}!} \]

where \( S_{\cup i} \triangleq \bigcup_{j=1}^T S_j \) and \( \hat{P}_{\cup i} \triangleq \bigcap_{j=1}^T (1 - \hat{P}_j) \). The ML estimate of \( N \) is

\[ \hat{N} = \left\lceil \frac{S_{\cup i}}{1 - \hat{P}_{\cup i}} \right\rceil. \]

4) Achievements: There are many advantages to the formulation presented:

- It was shown that it is possible to estimate \( P_i, i = 1, \ldots, T \), without having a “truthing” sensor. By using a Bayesian formulation, it is also the case that the greater the number of sensors, the better the estimate for \( P_i \).
- Given some running knowledge of sensor performances, it is also possible to estimate \( N \) given a set of detections by a set of sensors.
- Another feature of this method is that to estimate \( P_i \), one uses only the intersecting set of detection with other sensors. In the case in which sensor \( i \) is “unclassified” and
sensor $j$ is “classified”, it is possible to use the restrictive sensor to evaluate the performance of sensor $i$ without revealing any information about the performance of the classified sensor.

5) **Limitations:** There are limitations to this formulation that are important to note.

- It does not account for uncertainty in the detections (the set of detections are true or false).
- In order to identify the set of intersecting detections, effective multi-sensor fusion and tracking is required. This, of course, is not always achievable and therefore any inefficiencies or errors in the fusion and tracking can bias the results if not properly addressed.
- The estimation for $N$ is based off of absolute knowledge of the sensor performances. A method for estimating $N$ which makes use of the uncertainty in the sensor performance would provide a more accurate measure of $N$. It is, however, unlikely that a closed-form solution for this more accurate measure would be possible. Future work will cover such an improved method.
- This method treats all targets as being homogeneous and does not consider key variables which likely affect the $P_i$ for a given target. This could include target features (which can be empirically included in the estimation by binning the detections), and target-sensor geometry (including geographic regions). The next section will extend the formulation to include geographical distribution and near-overlapping of sensors.

**B. Extending to Pseudo-Static Case**

It is worth noting that in a real set-up with mobile sensor it is unlikely to have synchronized fields of view and so the previous method is applicable only in limited time periods, during which the fields of view overlap.

In this section we present an alternative method to overcome this limitation.

The surveillance area is binned into a grid of cells with width and height of equal angular size. That is, the cells are not equal-area, but the longitude and latitude are the same width and height of equal angular size. That is, the cells are of the same size.

An extension of the notation is needed: each cell is identified by longitude ($u$) and latitude ($v$) indexes and the region within is denoted by $C_{uv}$; $S_{i,uv}(k) \triangleq \{D \in S_i(k) : x \in C_{uv}\}$ and $S_{i,uv}(k) \triangleq |S_{i,uv}(k)|$; the set of actual target in a given cell $u,v$ during timestep $k$ is denoted by $N_{uv}(k)$ and $N_{uv}(k) \triangleq |N_{uv}(k)|$; $P_{i,uv}$ is the probability of detection in the cell $u,v$.

For each sensor the probability of detection of observed cells is estimated at each step.

While a fixed sensor always covers the same cells, the cells covered by a mobile sensors change, so it needs to update continuously its field of view to know which cells are covered at a given time. To keep track of the covered cells the Last Visit Matrices (LVM) are used. There is a LVM for each sensor and it contains an element for each cell, which contains the last time the cell was observed by the sensor.

At timestep $k$ the following operations are performed:

1) **Update of LVMs:** If the sensor is not persistent and has no detections in the current timestep, its LVM is not updated.

If the sensor is mobile, we search for the covered cells in the interval $[t_k - t_s, t_k]$ and for each of those we set the corresponding element of the LVM to the time of the last visit.

If the sensor is fixed, we consider only those cells in which there are detections during the current timestep and set the corresponding elements in the LVM to $t_s$. This approach for fixed sensors is not ideal, however, it is a first approximation for observation range.

2) **Generation of $S_{i,uv}(k)$:** For sensor $i = i_1, \ldots, T$, for each covered cell, namely the corresponding elements in the LVM have values greater than $t_k - t_s$, the set $S_{i,uv}(k)$ is constructed.

If a sensor is not persistent, then for all cells that even if covered have no detections in the current timestep, the set is not generated.

3) **Update of Successes and Failures Counts:** The update of the success and failure counts of the generic sensor $i$ is performed as follows for each updated cell:

   i. For each sensor $j \neq i$;
   ii. Construct the set $S_{j,uv}(k)$, if it doesn’t exist (see Section II-B6);
   iii. If $i$ didn’t detect any ship in the current cell and $i$ is persistent, add $S_{j,uv}(k)$ to the failures count $f_{i,uv}$;
   iv. For each identity in $S_{j,uv}(k)$, if it was detected by $i$, increment the successes count $s_{i,uv}$, else increment the failures count $f_{i,uv}$.

4) **Update of $P_{i,uv}$:** The probability of detection in each updated cell is estimated as

$$\arg\max_p f_{p_{i,uv}}(p \mid s_{i,uv}, f_{i,uv}),$$

where

$$f_{p_{i,uv}}(p \mid s_{i,uv}, f_{i,uv}) = \frac{1}{B(s_{i,uv} + 1, f_{i,uv} + 1)} p^{s_{i,uv}} (1 - p)^{f_{i,uv}}.$$  

5) **Calculation of $N_{uv}$ over time:** For the pseudo-static case, the distribution of targets over space will vary over time. For this reason, it is more effective to show the density of targets over a longer period of time. By accumulating the estimate of $N_{uv}$ over the analysis period, one obtains a cumulative estimate of the target population density ($N_{uv}$).

At a specific time $k$, the estimate of the target population is achieved in the same way as for the static case. The only difference is that only the terms are accumulated up to $k$ such that the cumulative density at time $k$ is

$$\overline{N}_{uv}(k) = \left[ \frac{s_{i,uv}(k)}{1 - P_{i,uv}} \right],$$  

where $s_{i,uv}(k) = \sum_{\ell=1}^{k} \sum_{T_{i,uv}(\ell)}$ and $P_{i,uv}(k) = \prod_{\ell=1}^{k} (1 - P_{i,uv}(\ell))$. In this way, the total count of estimated targets from $k$ looks is accumulated for each cell $uv$. At the final time $k^*$,
Fig. 1. Logarithm of the number of detections by a S-AIS satellite over a period of one month in 2011. Note the decreased detection density near, for example, China and Europe. This is due to AIS message collisions interfering with detections from space [8].

Fig. 2. Estimated probability of detection of an AIS transmission from an AIS satellite (i.e. raw data). The blue background area indicates regions of no data or no sensor overlaps. It is clearly observed that the probability quantifiably decreases near coastal regions as expected.
where we have defined

\[ \frac{N_{uv}}{k^*}, \]  

where we have defined

\[ N_{uv} \triangleq N_{uv}(k^*). \]  

6) Track Prediction: To count successes and failures for a given sensor \( i \) in a given updated cell \( u, v \), we need the sets \( S_{j,uv}(k), j = 1, \ldots, T \). These sets are not always available, because not necessarily all sensors observe the same cell during the same timestep\(^3\). Track prediction is introduced to overcome this issue and to estimate the hypothetical detections of a given sensor in the cell and the time step under analysis, starting from the real detections in the neighboring cells, that are within a maximum traveled distance \( d \) from the one under analysis. We solve the problem using the course and speed data, that are part of the detections as described below:

i. For each other sensor \( j \neq i \), we calculate the difference in time \( (t_d) \) between the time of the last visit of \( i, t_v \), and the time of the last visit of \( j^4 \):

ii. The maximum traveled distance is defined as the product between \( t_d \) and the maximum allowed speed of a target \( v_{\text{max}} \):

iii. We consider the last detections by \( j \) in the neighboring cells up to the distance \( d \) and for each of those target we predict a new position based on its last known \( x, \theta \) and \( v \) and the difference between \( t_v \) and \( t \):

iv. The set of those ships that fall into the cell under analysis constitute the estimation of \( S_{i,uv}(k) \) used for the successes and failures counts.

7) Limitations: It is worth noting that for a given sensor, this method can’t estimate performance in a cell if at least one of the following conditions occurs:

- the cell is never covered by the sensor;
- if the cell is covered, there are never detections, and the sensor is not persistent;
- there are detections from only one sensor in the cell.

Furthermore the track prediction approach used here is quite simple and there is much room for improvement. For instance, a kinematic filtering algorithm such as a Kalman Filter can be used to more accurately predict ship positions with uncertainties. Context based filtering [9], [10], which takes advantage of historical data on ship movements is a possible further improvement.

If the predicted target states include the uncertainty in prediction over the period of time between consecutive sensor observations, it would be possible to better adjust the Bayesian updates by using a distribution vice a simple success or failure approach. This idea is to be explored in future work.

The methodology was applied to the case of satellite sensing of AIS transponder messages. Four different commercial satellites and a network of coastal sensors were processed over a period of one month in 2011. In the case of AIS, the fusion and tracking problem is greatly simplified since the detections include nearly unambiguous\(^5\) identification information.

The raw detections from one AIS satellite are shown in Figure 1. It should be noted that a greater number of detections by the sensor does not necessarily indicate better detection performance. In fact, the raw data in Figure 1 is a convolution of both the sensor detection performance and the target population. In addition to the detections from AIS transmitting ship, it can also be observed near the polar regions that a few detections are outside the expected surveillance area. This is evidence that there are some corrupted or false detections in the data from this sensor. Fortunately, since the method relies on multi-sensor verification of detections, as given in Eq. 8, the impact of these false detections in the performance estimate is minor.

To de-convolve the sensor performance from the data, the detections were evaluated against other sensors observing an overlapping surveillance region during the same time period. Figure 2 shows the calculated probability of detection of an AIS transmitted message by a single satellite sensor. Note that in areas of high message traffic near the coast lines, the sensor performance decreases. This is a known effect [8], and is observably quantified here in the sensor detection performance model.

Figure 3 shows the cumulative target population density over the one month period of analysis. This figure represents the proportional target density in an area derived from the estimated target distribution over time, using Eq. 14. Not shown here is the uncertainty in the estimate. The artifacts where single cell high density estimates are present are due to low quantities of data, and in fact, those cells have a very high uncertainty associated with the estimate. However, near the equatorial regions (~ 50°N and 40°S) the density estimate uncertainties are low, as this is the region which had the highest AIS coverage.

IV. Conclusion

In this paper, a Bayesian estimation approach was presented which exploits observations by multiple independent sensors. Having multiple sensors makes realizable the possibility to estimate the sensor performance without having knowledge of the sea truth. We demonstrate the application of the method by estimating the performance of a satellite AIS receiver geographically.

Furthermore, we show that given the estimated sensor performance, a closed form approximate estimator for the total target population can be derived. Even with poor performing sensors, it is then possible to estimate the population of targets.

\(^3\)This is the motivation of this second method.

\(^4\)These values of time are available in the relative LVMs.

\(^5\)The phrase “nearly unambiguous” is used here since it is possible to have transponder misconfiguration or message corruption in the detection of AIS even though the AIS messages provide identification information.
Some initial progress towards a more generalized Bayesian estimator was also presented. Future work will involve further developing this estimator to remove some of the limitations in the approximate method. Another avenue of investigation will be the coupling of the multi-sensor data association process with the probability of detection estimation process. It remains a future goal to improve the tracking performance using sensors with non-uniform or time varying probabilities of detection through positive feedback.

**APPENDIX**

**DERIVING THE TARGET POPULATION POSTERIOR DISTRIBUTION**

The denominator in Eq. (11), i.e. the marginal probability, is

$$\sum_{\ell=S_{1/2}}^{\infty} \frac{\ell!}{S_{1-2}!S_{2-1}!S_{1/2}!S_{\sim 12}!} \times \hat{P}_1^{S_1} \hat{P}_2^{S_2} (1 - \hat{P}_1)^{\ell - S_1} (1 - \hat{P}_2)^{\ell - S_2}.$$

By setting

$$C \triangleq \frac{\hat{P}_1^{S_1} \hat{P}_2^{S_2} (1 - \hat{P}_1)^{-S_1} (1 - \hat{P}_2)^{-S_2}}{S_{1-2}!S_{2-1}!S_{1/2}!},$$

and $k \triangleq \ell - S_{1/2}$, it can be written as

$$C \sum_{k=0}^{\infty} \frac{(k + S_{1/2})!}{k!} \hat{P}_1^{k+S_{1/2}} \hat{P}_{\sim 12}^k.$$ The sum can be further rearranged using the binomial coefficient as

$$CS_{1\sim 12}! \hat{P}_{\sim 12}^{S_{1/2}} \sum_{k=0}^{\infty} \binom{k + S_{1/2}}{k} \hat{P}_{\sim 12}^k.$$

By noting the Taylor Series expansion centered at zero of the function

$$\frac{1}{(1 - x)^{a+1}} = \sum_{k=0}^{\infty} \binom{k + a}{k} x^k,$$

the sum can be evaluated:

$$CS_{1\sim 12}! \hat{P}_{\sim 12}^{S_{1/2}} \frac{1}{(1 - \hat{P}_{\sim 12})^{S_{1/2}+1}}.$$

Resubstituting $C$ results with the closed form marginal probability

$$\frac{S_{1\sim 12}! \hat{P}_1^{S_1} \hat{P}_2^{S_2} (1 - \hat{P}_1)^{S_1 - 1} (1 - \hat{P}_2)^{S_2 - 1}}{(1 - \hat{P}_{\sim 12})^{S_{1/2}+1}}.$$

The posterior probability is calculated according to Eq. (11):

$$\frac{n!}{S_{1-2}!S_{2-1}!S_{1/2}!} \hat{P}_1^{S_1} \hat{P}_2^{S_2} (1 - \hat{P}_1)^{S_1 - 1} (1 - \hat{P}_2)^{S_2 - 1}.$$

This complicated looking formula simplifies to

$$\binom{n}{S_{1/2}} \hat{P}_1^{n - S_{1/2}} (1 - \hat{P}_{\sim 12})^{S_{1/2}+1},$$

which is the negative binomial distribution.
REFERENCES


