Sequential Detection of RGPO in Target Tracking by Decomposition and Fusion Approach

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Abstract—A modified decomposition-and-fusion approach for target tracking in the presence of range-gate-pull-off (RGPO) is proposed. The RGPO detection problem consists of two parts: onset detection and termination detection. The likelihood ratio test used in the decomposition-and-fusion approach is replaced with sequential change detection, such as the cumulative sum test and Shiryaev’s sequential probability ratio test. The proposed approach overcomes the deficiencies of the likelihood ratio test, such as uncontrollable detection probability and neglect of old information, and fits well with the RGPO detection problem. These detectors are evaluated. Simulation results show that the proposed solution substantially outperforms the original solution since miss detection rate is greatly reduced by sequential detection.

Keywords: target tracking, RGPO detection, sequential detection, ECM.

I. INTRODUCTION

Many modern military aircraft are equipped with Electronic Countermeasure (ECM) capabilities aiming to interfere with the tracking radar from obtaining true information. An effective category of range deception ECM techniques is the so-called range gate pull off (RGPO). It mimics the realistic target and retransmits a deception signal with a progressive time delay, thereby trying to pull the tracker’s range gate off of the true target return as time progresses. When the range gate is sufficiently removed from the actual target, the RGPO is turned off, thus forcing the radar to reacquire the target. By repeated reacquisition of the target, the track quality is degraded, causing the launch of missiles to be prevented or delayed, or target loss for a launched missile.

The problem of tracking in the presence of RGPO draws increasing attention since the second benchmark problem for radar target tracking [1], [2] was proposed, which provides a standardized radar system model for tracking maneuvering targets in the presence of ECM and false alarms in order to compare and evaluate different algorithms. Two types of ECMS were included: standoff noise jamming and RGPO. A method for neutralizing standoff noise jamming was proposed in [3]. In this paper, we focus on defeating RGPO countermeasures at the data processing (tracking) level. Several algorithms on this topic have been developed. The algorithm of [3] declares a RGPO if there exists two measurements with signal-to-noise-ratio (SNR) above a threshold, and then an “anti-RGPO discriminating function” is used to reduce the association probabilities of the more distant measurement to discount the RGPO measurements. In [4], a chi-square test was suggested. All measurements were compared pairwise, and then a RGPO was declared if the normalized angle difference squared of the measurement pair was smaller than a threshold. Reference [5] identified a RGPO from the occurrence of two or more closely spaced (in angle), high SNR observations that both fell within track gates for a given target family. Thereafter, the tracks on the true target and the RGPO were both maintained until there were three consecutive looks on which the RGPO criteria were not satisfied. Other methods are also available in [6], [7]. However, most of them are more or less heuristic, and lack a solid theoretical support.

Li et al. [8] proposed a general and systematic approach, called decomposition-and-fusion (DF) approach, which is effective against RGPO. It takes full advantage of the fact that the deception measurements for many radar systems (e.g., monopulse radars) have virtually the same angles as the target measurement. This approach has four fundamental components: (a) detection/determination of range deception measurements by hypothesis testing; (b) running one or more tracking filters using the detected range deception measurements only; (c) running a conventional tracking-in-clutter filter for the remaining measurements in the track gate; (d) fusing the tracking filters by a probabilistically weighted sum of their estimates. The implementation and analysis using the second benchmark problem were given in [9]. Simulation results showed that the tracking performance was indeed improved compared with the standard tracking algorithms.

The detection of RGPO was formulated as a binary simple hypothesis test and the likelihood ratio test was adopted in [8], [9]. However, three major problems have yet to be addressed. First, only current information is used in the likelihood ratio test. A small data size for hypothesis testing may cause performance degradation. Second, the detection probability cannot be improved given the type I error rate, which may cause severe miss detection of RGPO measurements. Third, the subsequent test after detection of RGPO is unclear. A

Research supported in part by ARO through grant W911NF-08-1-0409, ONR-DEPSCoR through grant N00014-09-1-1169, and Louisiana BoR through grant LEQSF(2009-12)-RD-A-25.

This work was completed during the first author’s visit to the University of New Orleans.
straightforward way is to repeat the test as before. Obviously, on account of the generation pattern of RGPO, it is unadvisable to ignore the previous decision and just continue the test independently at the next circle, especially when RGPO was detected. Actually, the RGPO measurement and non-RGPO measurement follow different statistical distributions, which implies a change occurring in the measurement distribution after the RGPO onset or termination. Moreover, the status will continue over a period of time after the change occurs. In other words, it is not the case that the change occurs frequently with alternate RGPO onset and termination. Considering these features, change detection should be well suited to this problem and should overcome the deficiencies mentioned above. Based on this idea, the problem in this paper is reformulated into two parts: RGPO onset detection and RGPO termination detection. They are both simple hypothesis testing problems since all parameters are attainable. So, abundant detection methods can be used here [10]. We choose sequential hypothesis testing methods for the following reasons [11]: (a) the measurements in the tracking systems are usually available sequentially; (b) a sequential test usually makes a decision faster than a non-sequential test on average under the same decision error rates; (c) a sequential test does not need to determine the sample size in advance; (d) the thresholds of SPRT-based sequential tests can be approximately determined without knowing the distribution of the test statistics. Cumulative sum (CUSUM) [12] test and Shiryayev’s sequential probability ratio test (SSPRT) [13] are two of the most popular approaches for sequential change detection. CUSUM is non-Bayesian while SSPRT is Bayesian. CUSUM minimizes the average detection delay given the decision error probabilities, and SSPRT minimizes a risk function at each time assuming that the change point has a geometric distribution a priori.

The paper is organized as follows. Section II gives statistical models of the measurement in order to utilize the DF approach. Hypotheses and distribution of the measurements are also derived. In section III, the problem is reformulated as a change detection problem. Sequential detection methods for RGPO onset and termination detection are presented in section IV. Section V provides simulation results and analysis of the detection and tracking performance, and a summary is given in the last section.

II. STATISTICAL MODELING OF MEASUREMENTS

A. Measurement model

We consider a phased array monopulse radar for single target tracking in this problem. Multiple measurements can be obtained at a given time point. Let the measurements at time \( k \) be defined as

\[
\mathbf{Z}_k = \left[ \mathbf{z}_k \right]
\]

where \( \mathbf{z}_k \) is the number of measurements, not larger than the total number of range cells. Each measurement is

\[
\mathbf{z}_k = \left[ \mathbf{r}_k, \mathbf{\theta}_k, \mathbf{\psi}_k \right]
\]

where \( \mathbf{r}_k, \mathbf{\theta}_k, \mathbf{\psi}_k \) denote range, bearing and elevation, respectively. The radar also provides the SNR for each measurement. Let \( \mathbf{c}_k, \mathbf{t}_k, \mathbf{f}_k \) denote clutter, true target measurement and false target (RGPO) at time \( k \), respectively.

Each individual measurement must be a return from a true target, clutter or false target. Here, the term “clutter” includes “clutter returns” (scattering off of objects in the environment such as land, water and buildings) and “false alarms” resulting from thermal noise in the radar receiver [8]. In order to perform a statistical hypothesis test in the DF approach, the following assumptions are made for the angle measurement of each type. It is reasonable to assume that bearing and elevation are independent for a given type of measurement.

Assumption 1. The bearing and elevation of a clutter measurement are independent of all other measurements, clutter or not. For a clutter return, bearing \( \mathbf{\theta}_k \) and elevation \( \mathbf{\psi}_k \) are uniformly distributed over \((-B_t/2 + \mathbf{\theta}_k k - 1, B_t/2 + \mathbf{\theta}_k k - 1)\) and \((-B_e/2 + \mathbf{\psi}_k k - 1, B_e/2 + \mathbf{\psi}_k k - 1)\), respectively, where \( B_t \) and \( B_e \) are the beamwidth in bearing and elevation, \( \mathbf{\theta}_k k - 1 \) and \( \mathbf{\psi}_k k - 1 \) are the predicted bearing and elevation. For a monopulse radar, the bearing and elevation of a false alarm have Gaussian distributions: \( \mathbf{\theta}_k \sim N(\mathbf{\theta}_k k - 1, \mathbf{\sigma}_\mathbf{\theta}_k^2) \), \( \mathbf{\psi}_k \sim N(\mathbf{\psi}_k k - 1, \mathbf{\sigma}_\mathbf{\psi}_k^2) \), where \( \mathbf{\sigma}_\mathbf{\theta}_k^2 \) and \( \mathbf{\sigma}_\mathbf{\psi}_k^2 \) are the variances of the predicted target bearing and elevation, often determined by radar signal processing, which depend on the SNR of the particular waveform used.

Assumption 2. The bearing \( \mathbf{\theta}_k \) and elevation \( \mathbf{\psi}_k \) of the true target measurement have Gaussian distributions: \( \mathbf{\theta}_k \sim N(\mathbf{\theta}_k k - 1, \mathbf{\sigma}_\mathbf{\theta}_k^2) \), \( \mathbf{\psi}_k \sim N(\mathbf{\psi}_k k - 1, \mathbf{\sigma}_\mathbf{\psi}_k^2) \), where \( \mathbf{\sigma}_\mathbf{\theta}_k^2 \) and \( \mathbf{\sigma}_\mathbf{\psi}_k^2 \) are the variances of the predicted target bearing and elevation, often determined by radar signal processing, which depend on the SNR of the particular waveform used.

Assumption 3. The bearing \( \mathbf{\theta}_k \) and elevation \( \mathbf{\psi}_k \) of a false-target (RGPO) measurement have Gaussian distributions: \( \mathbf{\theta}_k \sim N(\mathbf{\theta}_k k - 1, \mathbf{\sigma}_\mathbf{\theta}_k^2) \), \( \mathbf{\psi}_k \sim N(\mathbf{\psi}_k k - 1, \mathbf{\sigma}_\mathbf{\psi}_k^2) \), where \( \mathbf{\sigma}_\mathbf{\theta}_k^2 \) and \( \mathbf{\sigma}_\mathbf{\psi}_k^2 \) are obtained from the signal processor.

For a RGPO measurement, the bearing and elevation are nearly the same as the true target, but the range is progressively walking away from the true target return, denoted as \( r_k = r_0 + \Delta r \). The range walkoff \( \Delta r \) is determined by the time delay \( \tau \) of the RGPO program, that is, \( \Delta r = ct/2 \). In general, there are two basic models to generate the time delay [9]

\[
\tau = \begin{cases} 
\frac{v_0(t_d - t_0)}{a_0}, & \text{Linear} \\
\frac{v_0(t_d - t_0)^2}{2c}, & \text{Parabolic}
\end{cases}
\]

where \( t_0 \) is the start time of the walk-off program, \( c \) is the speed of light, \( v_0 \) is the velocity, and \( a_0 \) is the acceleration.

B. Hypotheses

In [8], it was assumed that the measurements always come in pairs when interfered with RGPO, one as a false target and the other as the true target or cover pulse (a RGPO technique with zero time delay, attempting to cover the actual target return), which is referred to as target and false-target (T-FT) measurement pair.

In a single-target environment, there are five possibilities, and thus five hypotheses, for any given pair of measurements with very close angles, referred to as the “common-angle measurements”: (a) a pure clutter pair (hypothesis \( H_c \)); (b) a clutter and target measurement pair (\( H_{ct} \)); (c) a false-target measurement pair (including the cover pulse) or a target and false-target measurement pair (\( H_{tf} \)), both referred as T-FT
measurement pair; (d) a multiple false-target measurement pair, excluding the cover pulse \( (H_{tf}) \); and (e) a clutter and false-target measurement pair \( (H_{ct}) \). Hypothesis \( H_{cc} \) actually consists of three sub-hypotheses \( H'_{cc} \) (a clutter return pair), \( H''_{cc} \) (a clutter return and a false alarm), and \( H'''_{cc} \) (a false alarm pair). Hypothesis \( H_{ct} \) actually consists of two sub-hypotheses \( H'_{ct} \) (a clutter return and a target), \( H''_{ct} \) (a false alarm and a target). Hypothesis \( H_{tf} \) actually consists of two sub-hypotheses \( H'_{tf} \) (a target and a false alarm), and \( H''_{tf} \) (a false-target pair, one being the cover pulse). We neglect \( H_{ct} \) due to its extremely small probability, which occurs when neither the target measurement nor the cover pulse is detected and a clutter measurement happens to have a “common” angle with the other false-target measurement.

### C. Probability distributions

The RGPO hypothesis test in the DF approach is performed based on the bearing and elevation difference for a given common-angle measurement pair,

\[
\Delta \theta = \theta_1 - \theta_2, \quad \Delta \psi = \psi_1 - \psi_2 \quad (4)
\]

Therefore, with the above assumptions of angle measurements, the hypotheses become

\[
H_{cc} : f(\Delta \theta, \Delta \psi | H_{cc}) = f_{cc}(\Delta \theta) f_{cc}(\Delta \psi) \quad (5)
\]
\[
H_{ct} : f(\Delta \theta, \Delta \psi | H_{ct}) = f_{ct}(\Delta \theta) f_{ct}(\Delta \psi) \quad (6)
\]
\[
H_{tf} : f(\Delta \theta, \Delta \psi | H_{tf}) = f_{tf}(\Delta \theta) f_{tf}(\Delta \psi) \quad (7)
\]
\[
H_{ff} : f(\Delta \theta, \Delta \psi | H_{ff}) = f_{ff}(\Delta \theta) f_{ff}(\Delta \psi) \quad (8)
\]

By using the total probability theorem and the independence of \( \theta_1 \) and \( \theta_2 \) in Eq. (4), we can derive the probability density function (pdf) of bearing difference under each hypothesis as:

\[
f_{cc}(\Delta \theta) = \text{Tri}(\Delta \theta; -B_b, B_b) P(H'_{cc}|H_{cc}) + U(\Delta \theta; -B_b + \hat{\theta}_{k|-1}, B_b + \hat{\theta}_{k|-1}) * N(\Delta \theta; \hat{\theta}_{k|-1}, \sigma^2_{\theta_{fa}}) P(H''_{cc}|H_{cc})
\]
\[
+ N(\Delta \theta; 0, 2\sigma^2_{\theta_{fa}}) P(H'''_{cc}|H_{cc})
\]
\[
= \text{Tri}(\Delta \theta; -B_b, B_b) P(H'_{cc}|H_{cc}) + \frac{1}{B_b} \left[ \Phi \left( \frac{\Delta \theta + B_b/2}{\sigma_{\theta_{fa}}^2} \right) - \Phi \left( \frac{\Delta \theta - B_b/2}{\sigma_{\theta_{fa}}^2} \right) \right] P(H''_{cc}|H_{cc})
\]
\[
+ N(\Delta \theta; 0, 2\sigma^2_{\theta_{fa}}) P(H'''_{cc}|H_{cc})
\]

\[
f_{ct}(\Delta \theta) = U(\Delta \theta; -B_b + \hat{\theta}_{k|-1}, B_b + \hat{\theta}_{k|-1}) * N(\Delta \theta; \hat{\theta}_{k|-1}, \sigma^2_{\theta_{fa}}) P(H'_{ct}|H_{ct}) + N(\Delta \theta; 0, \sigma^2_{\theta_{k}} + \sigma^2_{\theta_{fa}}) P(H''_{ct}|H_{ct})
\]
\[
= \frac{1}{B_b} \left[ \Phi \left( \frac{\Delta \theta + B_b/2}{\sigma_{\theta_{fa}}^2} \right) - \Phi \left( \frac{\Delta \theta - B_b/2}{\sigma_{\theta_{fa}}^2} \right) \right] P(H'_{ct}|H_{ct}) + N(\Delta \theta; 0, \sigma^2_{\theta_{k}} + \sigma^2_{\theta_{fa}}) P(H''_{ct}|H_{ct})
\]

\[
f_{tf}(\Delta \theta) = N(\Delta \theta; 0, \sigma^2_{\theta_{k}}) P(H'_{tf}|H_{tf}) + N(\Delta \theta; 0, 2\sigma^2_{\theta_{fa}}) P(H''_{tf}|H_{tf}) \approx N(\Delta \theta; 0, 2\sigma^2_{\theta_{fa}})
\]

\[
f_{ff}(\Delta \theta) = N(\Delta \theta; 0, 2\sigma^2_{\theta_{fa}})
\]

where the four pdfs are all mixtures, \( \text{Tri}(\Delta \theta; -B_b, B_b) \) stands for a symmetric triangular pdf of a width \( 2B_b \) and height \( 1/B_b \), 
\( U * N \) is the convolution of an uniform pdf and a Gaussian pdf, and \( \Phi \) is the standard Gaussian cumulative distribution function. \( P(H'_{cc}|H_{cc}) \), \( P(H''_{cc}|H_{cc}) \), \( P(H'''_{cc}|H_{cc}) \) in Eqs. (9) and (10) are obtainable depending on the false alarm rate and clutter return density [8]. In Eq. (11), the approximation is based on \( P(H''_{tf}|H_{tf}) \approx 1 \) since the cover pulse rather than the skin return is usually detected. Similar results hold for the elevation difference \( \Delta \psi \) of (4).

We merge \( H_{tf} \) into \( H_{ff} \), since \( f_{ff}(\Delta \theta) \approx f_{tf}(\Delta \theta) \). Finally, only the hypotheses \( H_{tf}, H_{cc}, \) and \( H_{ct} \) remain.

### III. Problem formulation

#### A. Neyman-Pearson binary hypothesis test

In the original DF approach, the RGPO detection problem is formulated as a binary hypothesis test—the null hypothesis \( H_0 \) (the pair is not a T-FT measurement pair) against the alternative hypothesis \( H_1 \) (the pair is a T-FT measurement pair):

\[
H_0 : \Delta \gamma \sim f_0(\Delta \gamma)
\]
\[
H_1 : \Delta \gamma \sim f_1(\Delta \gamma)
\]

where \( \Delta \gamma = [\Delta \theta, \Delta \psi]' \), and

\[
f_c(\Delta \gamma) = f(\Delta \gamma | H_{cc}) P(H_{cc}|H_0) + f(\Delta \gamma | H_{ct}) P(H_{ct}|H_0)
\]
\[
= f_{cc}(\Delta \theta) f_{cc}(\Delta \psi) P(H_{cc}|H_0)
\]
\[
+ f_{ct}(\Delta \theta) f_{ct}(\Delta \psi) P(H_{ct}|H_0)
\]
\[
f_{tf}(\Delta \gamma) = f_{tf}(\Delta \theta) f_{tf}(\Delta \psi)
\]
\[
L_k = \frac{f(\Delta \gamma_k | H_1)}{f(\Delta \gamma_k | H_0)} = \frac{f_{tf}(\Delta \gamma_k)}{f_{cc}(\Delta \gamma_k)} \geq \lambda
\]
- Decide on $H_0$ if $L_k < \lambda$.

In the above test, $\lambda$ is an appropriate threshold, determined by the maximum allowable type I error probability $\alpha$.

The likelihood ratio test can also be extended to the case with multiple false-target measurements, and the key idea is to replace the angle difference $\Delta \gamma$ with the maximum absolute value $|\Delta \gamma|_{\text{max}}$ of all the common-angle measurement pairs. See [8] for details.

B. Problem reformulation

In the above likelihood ratio test, only the current measurement is used. It ignores old information. Moreover, the detection probability cannot be improved if the type I error rate is specified. This may cause serious miss detections, even track loss. In addition, this single-scan approach does not specify explicitly what should be done after the RGPO is detected. A simple way is to perform the test independently at every time point. However, it is not good to ignore the previous decision and just continue the test independently, especially when RGPO was detected, since the RGPO program tends to last for a period of time once it starts. Actually, both RGPO onset and termination represent a change in the measurement distribution. We can resort to change detection. Thus, this problem can be reformulated into two parts: RGPO onset detection and RGPO termination detection. It has several merits over the previous formulation: a) more measurements are used to detect the change; b) the detection probability can be improved at the expense of time delay; c) the previous decision is accounted for by conducting alternative RGPO onset detection and termination detection.

1) RGPO onset detection: Here, the null hypothesis is that there are no RGPO measurements up to the current time $k$, and the alternative hypothesis is that the RGPO has started at time $n_0$ ($n_0 \leq k$). Mathematically, it is

$$H_0 : \Delta \gamma_i \sim f_c(\Delta \gamma) \quad \text{for} \quad i = 1, 2, \ldots, k$$
$$H_1 : \left\{ \begin{array}{ll}
\Delta \gamma_i \sim f_c(\Delta \gamma), & \text{for} \quad i = 1, 2, \ldots, n_0 - 1 \\
\Delta \gamma_i \sim f_{f}(\Delta \gamma), & \text{for} \quad i = n_0, \ldots, k
\end{array} \right.$$  \hspace{1cm} (18)

where $n_0$ is the time at which RGPO starts, and $f_c(\Delta \gamma)$ and $f_{f}(\Delta \gamma)$ were given by (15)-(16).

2) RGPO termination detection: Here, the null hypothesis is that the RGPO measurements continue to exist, and the alternative hypothesis is that they have stopped at time $n_t$ ($n_t \leq k$):

$$H_0 : \Delta \gamma_i \sim f_{f}(\Delta \gamma) \quad \text{for} \quad i = 1, 2, \ldots, k$$
$$H_1 : \left\{ \begin{array}{ll}
\Delta \gamma_i \sim f_c(\Delta \gamma), & \text{for} \quad i = 1, 2, \ldots, n_t - 1 \\
\Delta \gamma_i \sim f_{f}(\Delta \gamma), & \text{for} \quad i = n_t, \ldots, k
\end{array} \right.$$  \hspace{1cm} (19)

where $f_c(\Delta \gamma)$ and $f_{f}(\Delta \gamma)$ were given in Eqs. (15) and (16).

An important feature of sequential detection is that nothing needs to be done when there is no decision. But we need a decision to take action in target tracking. To handle this contradiction, we choose $H_0$ as the default decision when there is no decision from hypothesis testing. It is rational because $H_0$ usually has a much larger probability than $H_1$ when there is not enough data to make a decision. In general, there is no RGPO at the beginning, so the RGPO onset detection is performed first, and then the termination test is activated once RGPO is detected. Similarly, we will restart the onset procedure right after RGPO is declared terminated, and then repeat this process.

Note that $\Delta \gamma$ in this formulation can also be replaced with the maximum absolute value $|\Delta \gamma|_{\text{max}}$ for multiple false target measurements, and the pdfs of $|\Delta \gamma|_{\text{max}}$ under two hypotheses can be derived in the same way as the LR test [8].

IV. SEQUENTIAL DETECTION FOR RGPO ONSET AND TERMINATION

Sequential change detection methods for binary hypothesis testing problems have been successfully applied to maneuver detection in target tracking [15]-[18], fault detection in control systems [13], [19] and many other practical problems. Wald’s sequential probability ratio test (SPRT) [20] is the basis of such methods. But the problem formulation of SPRT does not fit change detection. Therefore, extended versions of SPRT for change detection have been proposed, two most well known of which are the cumulative sum (CUSUM) test and the Shiryayev’s sequential probability ratio test (SSPRT). They are both optimal for simple binary hypothesis testing under different criteria.

A. CUSUM-based detector

The typical behavior of the cumulative sum of the log-likelihood ratio shows a negative drift before change, and a positive drift after change, and thus the change time corresponds roughly to the time when cumulative sum reaches its minimum. The CUSUM test is in a non-Bayesian framework, which assumes the unknown change time $n$ is deterministic.

One of its derivations is based upon a repeated use of SPRT with the lower threshold log $A$ equal to 0 and the upper threshold equal to $B$, depending on decision error rates. The key difference is to restart the SPRT whenever $H_0$ is declared, which makes it fit to change detection problems.

The cumulative sum of the log-likelihood ratio is

$$L^k = \max\{L^{k-1} + \log f(\Delta \gamma_k|H_1, \Delta \gamma^{k-1}), 0\}, \quad L^0 = 0$$

- If $L^k \geq \lambda$, declare $H_1$ and then the stopping time $\hat{n} = \min\{k : L^k \geq \lambda\}$ is taken as the RGPO onset (or termination) time.
- Else, continue the test ($k \rightarrow k + 1$) if $L^k < \lambda$.

It has been proved that CUSUM asymptotically minimizes the worst-case expected detection delay for simple hypotheses subject to a lower bound on the mean time between type I errors.

B. SSPRT-based detector

The SSPRT minimizes an expected cost at each time. It is the quickest detection of a change in a sequence of conditionally independent measurements under the given decision cost. SSPRT is a Bayesian approach, which assumes the change time is random. It needs to know the prior probability $p_0$ of the change at $n = 0$ and the transition probability $\pi$ from $H_0$ to $H_1$. The posterior probability, defined as $p_k^n = P\{n \leq k | \Delta \gamma^k\}$, stands for RGPO onset (termination) at an unknown time $n$ no later than time $k$ given the available measurements, $\Delta \gamma^k = \{\Delta \gamma_1, \ldots, \Delta \gamma_k\}$. Then $p_k^n = 1 - p_k^{\parallel}$.
is the posterior probability that no change occurred up to time $k$.

The test statistic of the SSPRT is obtained recursively by

$$P_k = \frac{p_k}{p_0} = \frac{f(\Delta \gamma_k | H_1, \Delta \gamma_k^{k-1}) P_{k-1} + \pi}{f(\Delta \gamma_k | H_0, \Delta \gamma_k^{k-1})} \frac{1 - \pi}{\pi}$$

where $\Delta \gamma_k$ is an appropriate threshold.

and the decision rule is

- Declare $H_1$, if $P_k \geq P_T$
- Else, continue the test $k \rightarrow k + 1$

Note that it is difficult to obtain the prior information for the RGPO for $H_0$ is set at $0.9$ and transition probability from $H_0$ to $H_1$ for RGPO onset detection is set at $0.02$.

A. Simulation parameters

To evaluate the performance of sequential detection of RGPO, we design the following simulation scenario. There is only one T-FT pair, but multiple clutter returns and false alarms during target tracking. We set the radar revisit time to constant at $0.1s$. The entire track lasts $100s$. The RGPO is active from time $k = 201$ to $400$ and from $k = 501$ to $700$. The measurement pair used in the test is recognized as the one which has the minimum angle difference (maximum $\gamma_k$).

B. Simulation results

1) Average detection delay: The average RGPO onset detection delays $(\bar{n} - n)$ of CUSUM, SSPRT, and the LR test are shown in Table I under different type I error rates. It is clear that the average detection delay decreases as the type I error rate increases for the three detectors. The detection delays of CUSUM and SSPRT are basically the same, but SSPRT is slightly faster than CUSUM. The LR test has “better” performance in detection delay, but this is at the price of a much worse miss detection rate than the other two detectors at the times they have decisions. Indeed, it is quick to use only one measurement in the LR test to make a decision, but it may miss detect the RGPO at the next time although the RGPO is declared before since the LR test is performed independently at each time. This causes serious miss detections during the entire tracking. While the sequential tests need to wait until enough measurements are collected to make a decision, resulting in a longer detection delay to ensure a
low miss detection probability. This is verified by Table II, which reveals that the number of miss detections decreases dramatically with the time for sequential tests but the decrease is not significant for the LR test. Moreover, the LR test has much more miss detections than the sequential tests at $k = 204$, when the sequential tests have decisions. Table III shows the average RGPO termination detection delays. It is amazingly small compared with the onset detection delay. This is because the detection probability of RGPO termination is much higher than that of RGPO onset (see the ROC curves in Fig. 3 and Fig. 4 for details), thus it is easy to detect the termination.

Table I: Average delay for RGPO onset detection

<table>
<thead>
<tr>
<th>Detector</th>
<th>Type I error rate $P_{fa}$</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
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<tr>
<td>LR test</td>
<td>2.00</td>
<td>1.16</td>
<td>0.58</td>
<td>0.24</td>
<td>0.11</td>
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<td>CUSUM test</td>
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<td>2.47</td>
<td>2.32</td>
<td>2.14</td>
<td>2.06</td>
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<tr>
<td>SSPRT test</td>
<td>2.82</td>
<td>2.43</td>
<td>2.31</td>
<td>2.11</td>
<td>2.06</td>
<td></td>
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</tbody>
</table>

2) ROC curves: The RGPO was turned on at time $k = 201$. The ROC curves of RGPO onset detection at $k = 202$, 203, 204 are shown in Fig. 3. $P_d$ is computed as the percentage of the number of correct decision over the number of total trials. For the sequential tests, if no decision is made it is counted as $H_0$. Obviously, the LR detector outperforms the sequential tests at $k = 202$ since the onset detection delays of the sequential tests are greater than 2 time steps. At this time, most runs of the sequential tests can not make a decision. The plot only accounts for the correct decision part. But the weakness here is not serious because the track degradation is usually small at the beginning of RGPO. At $k = 203$, the sequential tests have comparable performance with the LR test. Later (e.g., at $k = 204$), the sequential detectors have greater detection probabilities than the LR test. It also verifies that SSPRT outperforms the CUSUM but the difference is insignificant, which can be observed from the enlarged ROC curve from $P_{fa} = 0.01$ to $P_{fa} = 0.1$ at $k = 204$ in Fig. 3.

Table II: Miss detections for RGPO onset at $k = 202, 203, 204$ with 1000 runs

<table>
<thead>
<tr>
<th>Detector</th>
<th>Type I error rate $P_{fa}$</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR test</td>
<td>(206, 403, 403)</td>
<td>73</td>
<td>74</td>
<td>59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUSUM test</td>
<td>(976, 884.48)</td>
<td>909</td>
<td>173</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSPRT test</td>
<td>(976, 874.51)</td>
<td>907</td>
<td>161</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table III: Average delay for RGPO termination detection

<table>
<thead>
<tr>
<th>Detector</th>
<th>Type I error rate $P_{fa}$</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR test</td>
<td>0.12</td>
<td>0.10</td>
<td>0.09</td>
<td>0.08</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>CUSUM test</td>
<td>0.18</td>
<td>0.17</td>
<td>0.16</td>
<td>0.14</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>SSPRT test</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
The ROC curves of RGPO termination detection at \( k = 401, 402, 403 \), and 404 are shown in Fig. 4. The RGPO was off at \( k = 401 \). It is clear that the SSPRT detector has the best performance for RGPO termination detection. It detects the change immediately after it happens. As time goes, the CUSUM detector has better and better performance until it surpasses the LR detector.

3) Tracking performance: We utilized two scenarios to compare the tracking accuracy of the three detectors: Scenario 1 with a linear RGPO walk-off program with \( v_0 = 200 \text{m/s} \) and Scenario 2 with a parabolic walk-off RGPO with a constant acceleration \( a_0 = 10 \text{m/s}^2 \). We used the extended Kalman filter (EKF) to track the target using the common-angle measurements. The Strongest Neighbor (SN) filter was used before RGPO was detected and the Nearest Neighbor (NN) filter was adopted to deal with the remaining measurements after RGPO onset detection. The thresholds for RGPO onset detection and termination detection were determined by type I error \( P_{fa} = 0.1 \) and \( P_{fa} = 0.01 \), respectively. Fig. 5 shows position and velocity RMSE of tracking in the presence of RGPO but without any RGPO detector. The radar would lose the target without a RGPO detector. Hence, it is necessary to utilize a RGPO detector to track the target.

For Scenario 1, the position and velocity RMSE of the tracks with three detectors are given in Fig. 6. The RMSE goes up gradually when RGPO starts and goes down quickly after RGPO ends for the LR detector. It is clear that the tracking accuracy of the LR detector is worse than the sequential tests because it has much more miss detections. This can be seen from Table II. Considering the linearly increasing nature of the range measurements of RGPO, the later a miss detection happens, the more accuracy loss will be. It can be made up partially by correct detection. That is why the LR detector
is more accurate than without any detector but less than the sequential tests. For CUSUM and SSPRT, their tracking performance is close to the case without any RGPO.

For Scenario 2, Fig. 7 shows the position and velocity RMSE of the detectors. Results and analysis similar to Scenario 1 hold for this case. The small fluctuations are due to detection delay and false alarms. In general, the tracking performance with CUSUM and SSPRT detectors is significantly better than that of the LR detector.

VI. SUMMARY

The decomposition-and-fusion approach for tracking in the presence of range deception ECM [8] is general and systematic, but its RGPO detection part as in [8] has drawbacks such as small data size, ignoring old decision and uncontrollable detection probability. In essence, the measurement distributions after RGPO onset and termination are greatly changed, and thus it is better to be formulated as a change detection problem. Based on this, in this paper the problem has been reformulated as RGPO onset detection and RGPO termination detection. For change detection CUSUM and SSPRT have been applied. Simulation results show that CUSUM and SSPRT have comparable performance, but SSPRT is slightly better than CUSUM in terms of average detection delay and ROC curve, and they both substantially outperform the likelihood ratio test.

REFERENCES