Minimized Euclidean Error Data Association for Multi-Target and Multisensor Uncertain Dynamic Systems

Xiaojing Shen, Yunmin Zhu Yingting Luo
Dept. of Mathematics, Sichuan University,
Chengdu, Sichuan, 610064, P. R. China.
Email: shenxj@scu.edu.cn, ymzhu@scu.edu.cn

Jiazhou He
Jiangsu Automation Research Institute,
PO-box: 86210,
Lianyungang, Jiangsu, 222006, China

Abstract — In this paper, data association and tracking problems with multi-target and multisensor uncertain dynamic systems are considered. The methods developed in [1] for state estimation of single target systems will be extended to data association and tracking for multi-target systems in terms of minimizing Euclidean absolute error. Assume that a nominal system model, bounds of parameters uncertainty/biases and noises are known. This type of uncertain models have also many applications. For example, uncertain biases of measurements and time stamps may be described by a bounded set. Obviously, this uncertainty framework is significantly different from that of the combination of IMM and JPDA estimators. The latter assumes that a true target model is one of several possible precise maneuvering models given the transition probabilities among these models and probability density functions of all model noises. However, the former only knows that the true model is an element of a bounded uncertain model set so that there are infinite model candidates.

Besides, the optimization criterion for the latter is conventional MSE of the state estimation, but the former is to minimize Euclidian error. Clearly, removing model uncertainty or biases requires enough well-associated measurement data in advance. However, to obtain a good data association, one has to well estimate and remove the model uncertainty or biases. Since the two problems are mutually dependent and influenced, such data association and estimation problems cannot be solved well by the existing data association methods. In this paper, two minimized Euclidean-error data association (MEEDA) algorithms for single sensor and multi-sensor systems are proposed respectively. Quite a few numerical examples are given to reveal the major factors influencing the performance of MEEDA algorithms.

I. INTRODUCTION

In radar, sonar, and other target tracking applications, multi-target data association is a basic topic. Many results have been obtained in different situations (see [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]). For instance, the probabilistic data association (PDA) [4] was thought to be a very good method for tracking multiple targets in dense clutter without using powerful processor and huge memory for multiple hypothesis tracking. It is based on computing the posterior probability of each candidate measurement found inside a validation tracking gate, assuming that only one real target is present and all other measurements are Poisson-distributed clutters. To solve the problem of two or more targets in the same validation gate, an extension was derived called joint probabilistic data association (JPDA) [6]. Because of considerations such as reliability, survivability, and communication bandwidth, a distributed version of the JPDA algorithm was proposed in [8]. Multiple hypothesis tracking (MHT) is also generally accepted as the preferred method for solving the data association problem [5], [9], [10]. More improved results and applications can be seen in [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21].

On the other hand, many practical application problems can be described by uncertain system models. There are two classes of model uncertainty at least. The first one assumes that a true moving target model is one of several possible precise maneuvering models given the transition probabilities among these models and probability density functions of all model noises. Besides, its optimization criterion is to minimize MSE of the state estimation. This class of problems is often called the interacting multiple model (IMM) problem. In multi-target situations, IMM-JPDA (the combination of IMM and JPDA) and IMM-MHT (the combination of IMM and MHT) methods have also been developed and extensively used in maneuvering target tracking (see, e.g., [9], [10], [17], [22]). When the aforementioned probability prior knowledge can be well known in maneuvering multi-target tracking, sufficiently using these knowledge is very reasonable as done by IMM-JPDA and IMM-MHT to improve the performance of tracking algorithms. The second one is that one can only know the nominal model, the bounds of uncertain parameters and noises. The true model is just one of an infinite number of possibilities. Since there is no any prior statistical knowledge in this uncertain systems, a reasonable optimization criterion is minimizing Euclidian error of the state estimation. Therefore, the uncertain systems considered in this paper are completely different from the uncertain systems in IMM-JPDA and IMM-MHT.

There are also many applications for the second class of uncertain problems. For example, measurements of moving or aging equipments are time-varying uncertain. This may be described by uncertain measurement models. When measurement time stamps are biased, it can be described by uncertain state models (see the numerical example in [1]). Clearly, removing
model uncertainty or biases requires enough well-associated data to estimate them in advance. However, to obtain a good data association, one has to well estimate and remove the model uncertainty or biases first. Since the two problems are mutually dependent and influenced, such data association and estimation problems cannot be solved well by the existing data association methods. Thus, previously they are often divided into two individual problems. First, assume that the model uncertainty or biases are estimated and removed well, and then make data association as done in most of the reference papers aforementioned. Second, assume that the data is well associated, and then estimate the model uncertainty/biases (see [23], [24], [25]). Obviously, when the data association problem and the second class of uncertainty problems are suffered together, these approaches are not applicable simultaneously. In this paper, we focus on solving such a class of data association and estimation problems.

The bounded noises and uncertain models with linear fractional representation (LFR) form [26] are assumed. In the bounded perturbation framework, set-valued state estimation—an estimation set that is guaranteed to contain the state vector to be estimated has been extensively studied (see, for instance, [27], [28], [29], [30], [31]). In many practical applications, the assumption of the bounded noises is reasonable indeed. For example, for an aircraft, it is acceptable that its position, velocity and acceleration have bounded interfered changes. Besides, when the models are uncertain, probability density functions (pdf) of noises are difficult to be obtained in general. Thus, measurements of too far away a distance are usually ignored and the measurement noises are assumed to be bounded. Furthermore, the LFR of uncertainty is a general uncertainty description including, for instance, additive uncertainty and general rational matrix functions of a vector of uncertain parameters (see [26], [32]). Under the above two assumptions and in the criterion of minimizing Euclidean error, [1] proposed the state bounding box/ellipsoid estimation by sufficient use of complementary advantages of multisensor and multi-algorithm, which is only suitable for single target uncertain systems without data association problems that requires to deal with missing measurements (due to the uncertainty of detection) and false measurements from both discrete interfering sources (other targets) and random clutters.

In this paper, we consider the data association problems of multi-target and multisensor uncertain dynamic systems. For the probabilistic data association approach (e.g. JPDA and MHT), a high-probability (e.g. 95%) validate gate is derived based on an typical assumption of Gaussian models for measurement error statistics and target dynamics. However, for the bounded data association approach here, a 100% probability validate gate is derived based on bounded model noises and uncertain parameters. The key is how to minimize the size of validate gate. Obviously, the smaller the size of the validate gate is, the more false measurements from both discrete interfering sources (other targets) and random clutters can be eliminated. Firstly, the predictive measurement bounding boxes\(^1\) (i.e., validation gates of measurements) are derived by solving a semidefinite program (SDP) problem which can be efficiently computed in polynomial-time by interior point methods and related softwares. To guarantee the bounding box certainly to contain the true target, all possible measurements including other target measurements and clutters inside a validation gate are taken as the candidate measurements which are used to estimate the target by the state bounding box estimation in [1] respectively. Since none of them can be ignored, the union of all of the optimized state bounding boxes based on the candidate measurements respectively is taken as the state bounding box which then is propagated to next time. Moreover, the single sensor MEEDA algorithm is presented. To minimize the size of validate gate by using the complementary advantage of multisensor, the distributed MEEDA algorithm is presented. Finally, a number of numerical examples are given to reveal the major factors influencing the performance of the MEEDA algorithms. They show that bounds of noises and uncertain parameters, the number of the targets, spurious/false measurements and missing measurements are important factors influencing the algorithm performance. It is also shown that minimizing the sizes of validate gates and state bounding boxes by sufficient use of complementary advantages of multisensor and multi-algorithm fusion is really beneficial to improve the performance of data association algorithms.

The rest of the paper is organized as follows. In Section II, preliminaries are presented. In Sections III, the predictive measurement bounding box estimation is derived. Moreover, the single sensor MEEDA algorithm and distributed fusion MEEDA algorithm are proposed respectively. In Section IV, numerical examples are given. In Section V, conclusions are made.

II. Preliminaries

A. Formulation of multi-target and multisensor uncertain dynamic systems

Let us consider an integrated dynamic system with \(T\) targets and \(L\) sensors, the \(t\)-th target can be formulated as follows:

\[
\begin{align*}
    x_{k+1} &= M_{x_k}(\Delta x_k) \left[ \begin{array}{c} x_k \\ w_k \end{array} \right], \\
    y_{k,i} &= M_{y_{k,i}}(\Delta y_{k,i}) \left[ \begin{array}{c} x_k \\ v_{k,i} \end{array} \right], \quad i = 1, \ldots, L, 
\end{align*}
\]

(see footnote \(^2\)) where \(x_k \in \mathcal{R}^n\) is the state of system at time \(k\); \(w_k \in \mathcal{R}^{n_w}\) is a bounded uncertain process noise taking value in a unit sphere, i.e., \(\parallel w_k \parallel_\mathcal{R}^n \leq 1\); \(y_{k,i} \in \mathcal{R}^{n_i}\).

\(^1\)The approach of the propagating bounding ellipsoids can also be used. However, the following proposed data association algorithms will frequently use the union and intersection of sets which are easier based on the bounding box than based on the bounding ellipsoid, since the former can obtain an analytic solution and the latter requires solve an optimization problem. In addition, [1] has shown that two approaches have no dominated advantages. Therefore, we focus on the bounding boxes.

\(^2\)To avoid heavy notations, we suppress the superscript \(t\) from the target state \(x_k^t\) and measurements \(y_{k,i}^t\) for \(t = 1, \ldots, T\), if necessary we will clarify it.
is the measurement of state \( x_k; v_{k,i} \in \mathbb{R}^{n_{u,i}} \) is the bounded uncertain measurement noise of the \( i \)-th sensor taking value in a unit sphere, i.e., \( ||v_{k,i}|| \leq 1 \). The uncertainty on the system parameter matrices is assumed to be represented in LFR form, for any given \( \Delta_{x_k} \in \mathbb{R}^{n_p \times n_q}, \Delta_{y_{k,i}} \in \mathbb{R}^{n_p \times n_q} \),

\[
M_{x_k}(\Delta_{x_k}) = [F_k \ A_k] + L_{x_k} \Delta_{x_k} (I - D_{x_k} \Delta_{x_k})^{-1} [R_{Fx} \ R_{Ax}],
\]

\[
M_{y_{k,i}}(\Delta_{y_{k,i}}) = [H_{k,i} \ B_{k,i}] + L_{y_{k,i}} \Delta_{y_{k,i}} (I - D_{y_{k,i}} \Delta_{y_{k,i}})^{-1} [R_{Hy} \ R_{By}], \quad i = 1, \ldots, L,
\]

where \( I \) is an identity matrix with compatible dimensions; \( F_k \in \mathbb{R}^{n_{u,n}}, A_k \in \mathbb{R}^{n_{u,n}}, H_{k,i} \in \mathbb{R}^{n_{u,n}}, B_{k,i} \in \mathbb{R}^{n_{u,n}}, L_{x_k} \in \mathbb{R}^{n_{p,n}}, L_{y_{k,i}} \in \mathbb{R}^{n_{p,n}}, D_{x_k} \in \mathbb{R}^{n_{p,n}}, D_{y_{k,i}} \in \mathbb{R}^{n_{p,n}}, R_{Fx} \in \mathbb{R}^{n_{u,p}}, R_{Ax} \in \mathbb{R}^{n_{u,p}}, R_{Hy} \in \mathbb{R}^{n_{y,q}}, R_{By} \in \mathbb{R}^{n_{y,q}} \) are known time-varying matrices. The uncertainty time-varying matrices \( \Delta_{x_k}, \Delta_{y_{k,i}} \) are in general structured and bounded \( \Delta_{x_k} \in \tilde{\Omega}_{x_k} \triangleq \{ \Delta \in \Omega_{x_k} : ||\Delta|| \leq 1 \}, \Delta_{y_{k,i}} \in \tilde{\Omega}_{y_{k,i}} \triangleq \{ \Delta \in \Omega_{y_{k,i}} : ||\Delta|| \leq 1 \}, \) where \( \Omega_{x_k} \subset \mathbb{R}^{n_p \times n_q}, \Omega_{y_{k,i}} \subset \mathbb{R}^{n_p \times n_q} \) are structure subspaces. Let these LFRs be well-posed over \( \Omega_{x_k} \) and \( \Omega_{y_{k,i}} \), respectively, meaning that \( \det(I - D_{x_k} \Delta_{x_k}) \neq 0, \forall \Delta_{x_k} \in \tilde{\Omega}_{x_k}, \det(I - D_{y_{k,i}} \Delta_{y_{k,i}}) \neq 0, \forall \Delta_{y_{k,i}} \in \tilde{\Omega}_{y_{k,i}} \), a sufficient condition for well-posedness can be seen in \( \mu \) analysis problems [26]. The above LFR of the uncertainty is widely used in control theory and has great generality including the additive uncertainty of the form \( M_{x_k}(\Delta_{x_k}) = [F_k + \Delta F \ A_k + \Delta A] \) if let \( L_{x_k} = I, D_{x_k} = 0, \Delta_{x_k} = \text{diag}(\Delta F, \Delta A), [R_{Fx} \ R_{Ax}] = [I \ I] \) (see [26], [32]). Associated to the structure subspaces, we will use the scaling subspaces

\[
\mathcal{P}(\Omega_{x_k}) = \{(S, T, G) : \forall \Delta \in \Omega_{x_k}, S = \Delta T, G = -\Delta T G^T \},
\]

\[
\mathcal{P}(\Omega_{y_{k,i}}) = \{(S, T, G) : \forall \Delta \in \Omega_{y_{k,i}}, S = \Delta T, G = -\Delta T G^T \}.
\]

**B. State bounding box estimation of single sensors**

Suppose that the initial state \( x_0 \) belongs to a given bounding box for the \( i \)-th sensor:

\[
B_{x_{0,i}} = \{ x \in \mathbb{R}^n : |x(j) - \hat{x}_{0,i}(j)| \leq \frac{b_{x_{0,i}}(j)}{2}, \quad j = 1, \ldots, n \},
\]

where \( \hat{x}_{0,i} \) is the center of \( B_{x_{0,i}}, \) \( b_{x_{0,i}} \) is the vector of the side lengths of \( B_{x_{0,i}} \). The edges of the box parallel to the axes of the coordinate system. At time \( k \), given that \( x_k \) belongs to a current bounding box:

\[
B_{x_{k,i}} = \{ x \in \mathbb{R}^n : |x(j) - \hat{x}_{k,i}(j)| \leq \frac{b_{x_{k,i}}(j)}{2}, \quad j = 1, \ldots, n \},
\]

where \( \hat{x}_{k,i} \) is the center of \( B_{x_{k,i}}, \) \( b_{x_{k,i}} \) is the vector of the side lengths of \( B_{x_{k,i}} \). At next time \( k + 1 \), the \( i \)-th sensor can receive the measurements \( y_{k+1,i} \) and needs to determine a bounding box \( B_{x_{k+1,i}} \), i.e, to find \( \hat{x}_{k+1,i}, b_{x_{k+1,i}} \) such that the state \( x_{k+1} \) belongs to

\[
B_{x_{k+1,i}} = \{ x \in \mathbb{R}^n : |x(j) - \hat{x}_{k+1,i}(j)| \leq \frac{b_{x_{k+1,i}}(j)}{2}, \quad j = 1, \ldots, n \},
\]

whenever I) \( x_k \) is in \( B_{x_k} \), II) the process and measurement noises \( w_k, v_{k+1,i} \) are bounded in a unit sphere, i.e., \( ||w_k|| \leq 1, ||v_{k+1,i}|| \leq 1 \) and III) (1) and (2) hold for some \( \Delta_{x_k} \in \tilde{\Omega}_{x_k}, \Delta_{y_{k,i}} \in \tilde{\Omega}_{y_{k,i}} \). Moreover, we provide a state estimation box by minimizing the “size” of the box. The corresponding result have been proposed by Theorem 3.2 in [1].

**C. State bounding box estimation based on distributed fusion**

The \( i \)-th local sensor can use the measurements \( Y_{k+1,i} \triangleq \{ y_{1,i}, \ldots, y_{k+1,i} \} \) to obtain the bounding box \( B_{x_{k+1,i}} \) at time \( k + 1 \) by the single sensor recursive method. Then, \( B_{x_{k+1,i}} \) are sent to the fusion center for \( i = 1, 2, \ldots, L \) without communication delay.

Suppose that the initial state \( x_0 \) belongs to a given bounding box:

\[
B_{x_0} = \{ x \in \mathbb{R}^n : |x(j) - \hat{x}_0(j)| \leq \frac{b_{x_0}(j)}{2}, \quad j = 1, \ldots, n \},
\]

where \( \hat{x}_0 \) is the center of \( B_{x_0}, b_{x_0} \) is the vector of the side lengths of \( B_{x_0} \). At time \( k \), given that \( x_k \) belongs to a current bounding box:

\[
B_{x_k} = \{ x \in \mathbb{R}^n : |x(j) - \hat{x}_k(j)| \leq \frac{b_{x_k}(j)}{2}, \quad j = 1, \ldots, n \},
\]

where \( \hat{x}_k \) is the center of \( B_{x_k}, b_{x_k} \) is the vector of the side lengths of \( B_{x_k} \). At next time \( k + 1 \), the fusion center can receive the the bounding boxes of the sensors \( B_{x_{k+1,i}}, i = 1, 2, \ldots, L \) and is to determine a bounding box \( B_{x_{k+1}} \), i.e, to find \( \hat{x}_{k+1}, b_{x_{k+1}} \) such that the state \( x_{k+1} \) belongs to

\[
B_{x_{k+1}} = \{ x \in \mathbb{R}^n : |x(j) - \hat{x}_{k+1}(j)| \leq \frac{b_{x_{k+1}}(j)}{2}, \quad j = 1, \ldots, n \},
\]

whenever I) \( x_k \) is in \( B_{x_k} \), II) \( x_{k+1} \) is in \( B_{x_{k+1,i}}, i = 1, \ldots, L \), III) the process noises \( w_k \) are bounded in a unit sphere, i.e., \( ||w_k|| \leq 1 \) and IV) (1) holds for some \( \Delta_{x_k} \in \tilde{\Omega}_{x_k} \). Moreover, we provide a state estimation box by minimizing the “size” of the box. The corresponding result have been proposed by Theorem 3.4 in [1].

**D. Formulation of data association**

At time \( k \), once we obtain the single sensor/distributed fusion bounding box \( B_{x_k} \) of the state \( x_k \) (of the \( t \)-th target), which is used to determine predictive measurement bounding boxes/"validation gates" \( B_{y_{k+1,i}}, i = 1, \ldots, L \) respectively, i.e,
whenever $x_k$ is in $B_{x_k}$, II) the process and measurement noises $w_k, v_{k+1,i}$ are bounded in a unit sphere, i.e. $\|w_k\| \leq 1, \|v_{k+1,i}\| \leq 1$ and III) (1) and (2) hold for some $\Delta_{x_k} \in \Omega_{x_k}, \Delta_{y_{k+1,i}} \in \Omega_{y_{k+1,i}}$. Moreover, we provide a measurement predicative bounding box by minimizing the “size” of the box.

Note that, for the probabilistic data association approach (e.g. JPDA and MHT), a high-probability (e.g. 95%) validate gate is derived based on an typical assumption of Gaussian models for measurement error statistics and target dynamics. However, for the bounded data association approach here, a 100% probability validate gate is derived based on bounded model noises and uncertain parameters. The key is how to minimize the size of validate gate. Obviously, the smaller the size of the validate gate is, the more false measurements from both discrete interfering sources (other targets) and random clutters can be eliminated. In addition, one has to deal with missing measurements due to the uncertainty of detection.

In summary, the data association and tracking problems in the bounded setting are decomposed to consider the following three problems.

- How to minimize the sizes of validate gates and the sizes of state bounding boxes?
- How to make use of all candidate measurements (including false measurements from both discrete interfering sources and random clutters) in a minimized validate gate to derive a minimized state bounding box that is guaranteed to contain the estimated target?
- How to deal with missing measurements to derive a minimized state bounding box that is guaranteed to contain the estimated target?

The single sensor and distributed fusion approaches are discussed respectively.

### III. Data Association and Tracking Algorithms

We first present the predicative measurement bounding box as follows.

**Theorem 1:** For the $t$-th target, at the $i$-th sensor and time $k+1$, based on the single sensor/distributed fusion bounding box $B_{x_k}$, the $i$-th predicative measurement bounding box $B_{y_{k+1,i}} = \{y \in \mathbb{R}^{n_j} : |y(j) - \hat{y}_{k+1,i}(j)| \leq \frac{b_{y_{k+1,i}}(j)}{2}, j = 1, \ldots, n_i\}$ can be obtained by solving the optimization problem in the variables $b_{y_{k+1,i}}, \hat{y}_{k+1,i}, S, T, G, S_i, T_i, G_i$ nonnegative scalars $\tau_{r_j}^w \geq 0, \tau_{w}^w \geq 0, \tau_{j}^w \geq 0, r = 1, \ldots, n, j = 1, \ldots, n_i$

$$\min g(b_{y_{k+1,i}})$$

subject to

$$(S, T, G) \in \mathcal{P}(\Omega_{x_k}), S \geq 0, T \geq 0,$$

$$(S_i, T_i, G_i) \in \mathcal{P}(\Omega_{y_{k+1,i}}), S_i \geq 0, T_i \geq 0,$$

$$\tau_{r_j}^w \geq 0, \tau_{w}^w \geq 0, \tau_{j}^w \geq 0,$$

for $j = 1, \ldots, n_i$.

where $g(b_{y_{k+1,i}}) \triangleq \omega_1(b_{y_{k+1,i}}(1))^2 + \cdots + \omega_{n_i}(b_{y_{k+1,i}}(n_i))^2$ is a measure of the size of $B_{y_{k+1,i}}$ (see [1]); $\omega_j, j = 1, \ldots, n_i$ are nonnegative weights; $\Phi(\hat{y}_{k+1,i})(j, \cdot)$ means the $j$-th row of matrix $\Phi(\hat{y}_{k+1,i})$; $b_{y_{k+1,i}}(j)$ means the $j$-th entry of vector $b_{y_{k+1,i}}$.

$$\Phi(\hat{y}_{k+1,i}) \triangleq \begin{bmatrix} H_{k+1,i} F_k \hat{x}_k - \hat{y}_{k+1,i} \\ H_{k+1,i} F_k \text{diag}(\frac{b_{y_k}}{2}) H_{k+1,i} A_k B_{k+1,i} \\ H_{k+1,i} L_{x_k} H_{y_{k+1,i}} \end{bmatrix},$$

$$\Xi_j \triangleq \text{diag}(1 - \sum_{r=1}^n \tau_{r_j}^w - \tau_{r}^w, \text{diag}(\tau_{1j}^w, \ldots, \tau_{nj}^w)), \quad \Pi \triangleq \Upsilon(S, T, G) + \Upsilon(S_i, T_i, G_i),$$

$$\Upsilon(S, T, G) \triangleq \hat{\Upsilon}^T \begin{bmatrix} T & G \end{bmatrix} \hat{\Upsilon},$$

$$\hat{\Upsilon} \triangleq \begin{bmatrix} R_F \hat{x}_k & R_F \text{diag}(\frac{b_{y_k}}{2}) & R_{A_k} \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_i(S_i, T_i, G_i) \triangleq (\hat{\Upsilon}_i)^T \begin{bmatrix} T_i & G_i \\ (G_i)^T & -S_i \end{bmatrix} \hat{\Upsilon}_i.$$
\[ \tilde{\mathbf{T}}_i \triangleq \begin{bmatrix} R_{H_{k+1,i}} F_k \hat{x}_k & R_{H_{k+1,i}} F_k \text{diag}(b_{k,i}) & R_{H_{k+1,i}} A_k \\ 0 & 0 & 0 \\ :R_{B_{k,i+1}}: & R_{H_{k+1,i}} L_{x_k}: & D_{y_{k,i+1}}: \\ 0: & 0: & \mathbb{I}_{I \in \mathbb{R}^n_+} \end{bmatrix} \]

The proof is deleted in the conference form due to the limited space.

Notice that the optimization problem is also an SDP problem.

Based on the Theorem 3.2 in [1] (the case of L=1) and Theorem 1, MEEDA and tracking algorithms for the single sensor system can be given as follows.

**Algorithm 1:** [Data association and tracking for single sensors]

**Step 1** (Initialization): Set \( k = 0 \). For the \( t \)-th target, choose weights in the objectives of Theorem 3.2 in [1] and Theorem 1 in terms of the importance of the entries of \( b_{x_{k+1,i}} \) and \( b_{y_{k+1,i}} \), respectively, form LFRs of the system (1)–(2), and find bases of the scaling subspaces \( \mathcal{P}(\Omega_{x_k}) \), \( \mathcal{P}(\Omega_{y_k}) \), \( i = 1, 2, \ldots, L \) respectively; Start with an initial bounding box \( B_{x_k} ; T \) targets are treated similarly.

**Step 2** (Predicative measurement bounding box/validation gate): Set \( k = k + 1 \). For the \( t \)-th target, and the \( i \)-th sensor, minimize the predicative measurement bounding boxes \( B_{y_k} \) by Theorem 1 based on \( B_{x_{k-1}} ; i = 1, 2, \ldots, L \). \( T \) targets are treated similarly.

**Step 3** (State bounding box): For the \( t \)-th target, and the \( i \)-th sensor, there are \( m_{k,i} \) candidate measurements from both discrete interfering sources (other targets) and random clutters in the validation gate \( B_{y_k} \), which are denoted \( z_{k,s}, s_i = 1, \ldots, m_{k,i}, i = 1, 2, \ldots, L \) respectively. Thus, \( m_{k,i} \) state bounding boxes can be obtained by Theorem 3.2 in [1] respectively, which are denoted by \( B_{x_{k,i}} \), \( i = 1, 2, \ldots, L \). If \( m_{k,i} = 0 \), i.e., the case of missing measurements due to the detection uncertainty, then the state predictive bounding box denoted by \( B_{x_{k,i}}^0 \) is taken as the state bounding box. \( T \) targets are treated similarly.

**Step 4** (Union box/predictive box): For the \( t \)-th target, the \( i \)-th sensor, the \( m_{k,i} \) state bounding boxes may be in one of the \( m_{k,i} \) state bounding boxes, the union box denoted by \( B_{x_k} \) \( \triangleq \text{Box}(\bigcup_{s=1}^{m_{k,i}} B_{x_{k,s}}) \) is taken as the state bounding box; If \( m_{k,i} = 0 \), then \( B_{x_k} \) \( \triangleq B_{x_k}^0 \) is taken as the state bounding box. Moreover, \( B_{x_k} \) is propagated to next time step; \( T \) targets are treated similarly. Go to Step 2.

To minimize the sizes of the valid gate and the state bounding box by using the complementary advantage of multisensor, MEEDA and tracking algorithms for the distributed fusion structure can be presented based on the Theorem 3.2 and 3.4 in [1] and Theorem 1 as follows.

\[ \text{Box}(\bigcup_{s=1}^{m_{k,i}} B_{x_{k,s}}) \] means the minimum box containing \( \bigcup_{s=1}^{m_{k,i}} B_{x_{k,s}} \).

**Algorithm 2:** [Data association and tracking based on distributed fusion]

**Step 1** (Initialization): Set \( k = 0 \). For the \( t \)-th target, choose weights in the objectives of Theorem 3.2 and 3.4 in [1] and Theorem 1 in terms of the importance of the entries of \( b_{x_{k+1,i}} \), \( b_{y_{k+1,i}} \), respectively, form LFRs of the system (1)–(2), and find bases of the scaling subspaces \( \mathcal{P}(\Omega_{x_k}) \), \( \mathcal{P}(\Omega_{y_k}) \), \( i = 1, 2, \ldots, L \) respectively; Start with an initial bounding box \( B_{x_k} ; T \) targets are treated similarly.

**Step 2** (Predicative measurement bounding box/validation gate): Set \( k = k + 1 \). For the \( t \)-th target and the \( i \)-th sensor, minimize the predicative measurement bounding boxes \( B_{y_k} \) by Theorem 1 based on \( B_{x_{k-1}} ; i = 1, 2, \ldots, L \). \( T \) targets are treated similarly.

**Step 3** (State bounding box): For the \( t \)-th target and the \( i \)-th sensor, there are \( m_{k,i} \) candidate measurements from both discrete interfering sources (other targets) and random clutters in the validation gate \( B_{y_k} \), which are denoted \( z_{k,s}, s_i = 1, \ldots, m_{k,i}, i = 1, 2, \ldots, L \) respectively. Thus, \( m_{k,i} \) state bounding boxes can be obtained by Theorem 3.2 in [1] respectively, which are denoted by \( B_{x_{k,i}} \), \( i = 1, \ldots, m_{k,i} \). If \( m_{k,i} = 0 \), i.e., the case of missing measurements due to the detection uncertainty, then the state predictive bounding box denoted by \( B_{x_{k,i}}^0 \) is taken as the state bounding box. \( T \) targets are treated similarly.

**Step 4** (Union box/predictive box): For the \( t \)-th target, and the \( i \)-th sensor, the \( m_{k,i} \) state bounding boxes may be in one of the \( m_{k,i} \) state bounding boxes, the union box denoted by \( B_{x_k} \) \( \triangleq \text{Box}(\bigcup_{s=1}^{m_{k,i}} B_{x_{k,s}}) \) is taken as the state bounding box; If \( m_{k,i} = 0 \), then \( B_{x_k} \) \( \triangleq B_{x_k}^0 \) is taken as the state bounding box. Moreover, \( B_{x_k} \) is sent to the fusion center, \( i = 1, 2, \ldots, L \); \( T \) targets are treated similarly.

**Remark 1:**
1) Note that the steps 3-4 have involved the cases of missing measurements and spurious/false measurements, which are also illustrated in the section of numerical examples.

2) Furthermore, the size of validate gate can be minimized by multi-algorithm fusion. Multi-algorithm fusion means that, in each optimization step, one can choose multiple groups of weights in optimization objectives (e.g., each group emphasizes one side of the bounding box) and solve multiple optimization problems in parallel, then the intersection box of multiple optimized bounding boxes is propagated to next step.

3) In addition, for distributed fusion MEEDA algorithm, if the sensor data are asynchronous, then the estimates of sensors can be predicted to the current fusion time respectively. Thus, the proposed distributed fusion MEEDA and tracking algorithm is also suitable for
asynchronous estimation fusion.

IV. SIMULATIONS

Let us consider a typical example of two targets in a plane and two sensors with uncertain systems. The target motion is modeled in Cartesian coordinates, where target state vector consists of position and velocity in each of the two coordinates, i.e., \( x_k = [x \ y \ \dot{x} \ \dot{y}]^T \). Targets 1 and 2 travel with (normalized) speed \( 450\sqrt{2} \text{ m/s} \) and initial state \([0 \text{ m} \ 450 \text{ m/s} \ 18000 \text{ m} \ -450 \text{ m/s}] \) and \([0 \text{ m} \ 450 \text{ m/s} \ 0 \text{ m} \ 450 \text{ m/s}] \) respectively. Their trajectories cross at about middle through the 40 s period, which results in severe interference. The similar examples with accurate models are also discussed in [7], [8]. Here, we also consider that the time stamp and measurement matrices are uncertain, where the uncertainty of the measurement depends on the distance between the target position and the sensor position.

The uncertain dynamic system is modeled as

\[
x_{k+1} = \begin{bmatrix} 1 & T + b_s T \delta_T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T + b_s T \delta_T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + A_k w_k,
\]

\[
y_{k,i} = \begin{bmatrix} 1 + b_m \delta_{i,i} & 0 & 0 & 0 \\ 0 & 1 & b_m \delta_{2i} & 0 \end{bmatrix} x_k + B_{k,i} v_{k,i}, \quad i = 1, 2,
\]

where \( T = 1s \) is the sample time interval, model uncertainty parameters \( \| \delta_T \| \leq 1, \| \delta_{i,i} \| \leq 1, \| \delta_{2i} \| \leq 1 \), \( i = 1, 2, w_k \) and \( v_{k,i} \) are the process noise and the measurement noises taking value in a unit spheres, i.e., \( \| w_k \| \leq 1, \| v_{k,i} \| \leq 1 \) respectively; \( b_s \) and \( b_m \) are the bounds of the state and the measurement uncertainty parameters respectively. If we denote

\[
Q_k = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} q^2, \]

\[
R_{k,i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} r^2,
\]

where \( q, r \) are parameters which determine the bounds of noises, then the matrices \( A_k, B_{k,i} \) are satisfied with \( Q_k = A_k A_k^T, R_{k,i} = B_{k,i} B_{k,i}^T \). The LFR uncertainty representation (3)-(4) specializes to

\[
L_{x_k} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad R_{F_k} = \begin{bmatrix} 0 & b_s T & 0 & 0 \\ 0 & 0 & 0 & b_s T \end{bmatrix},
\]

\[
D_{x_k} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta_{x_k} = \begin{bmatrix} \delta_T \\ 0 \end{bmatrix},
\]

\[
R_{A_k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

\[
L_{y_{k,i}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R_{H_{k,i}} = \begin{bmatrix} b_m & 0 & 0 & 0 \\ 0 & b_m & 0 & 0 \end{bmatrix},
\]

\[
D_{y_{k,i}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta_{y_{k,i}} = \begin{bmatrix} \delta_{i,i} \\ 0 \end{bmatrix},
\]

\[
R_{B_{k,i}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad i = 1, 2,
\]

The scaling subspaces \( \mathcal{P}(\Omega_{x_k}), \mathcal{P}(\Omega_{y_{k,i}}), i = 1, 2, \ldots, L \) in (5)-(6) are in this case described by \( S = T = \lambda \) (a scalar), \( G = 0 \) and \( S_i = T_i = \text{Diag}(\lambda_{1,i}, \lambda_{2,i}) \), \( G_i = 0, i = 1, 2 \) respectively. The uniform distribution in \([-1, 1]\) for uncertain parameter and noises is used. The following simulation results run under Matlab R2007b with YALMIP/SeDuMi (see [34], [35]). The parameters are \( q=10 \text{ m/s}^{3/2}, r=400 \text{ m}, b_s = 0.1 \) and \( b_m = 0.001 \). The objectives of minimizing each side of a box (i.e., \( \omega_j = 1 \) and \( \omega_i = 0, i \neq j \) are used, which is called multi-algorithm fusion in [1]. The state bounding boxes at \( 2k + 1, k = 1, 2, \ldots \) are plotted in figures. All figures are placed at the last page.

Figures 1 present the two target tracking by the single sensor MEEDA Algorithm 1 and distributed fusion MEEDA Algorithm 2. Figures 2–3 present the two target tracking with 30% probability of spurious and missing measurements respectively. Here, a spurious measurement is generated randomly in the validate gate with a uniform distribution.

Figure 4 presents three target tracking for the single MEEDA Algorithm 1 and distributed fusion MEEDA Algorithm 2. Targets 1 through 3 travel with (normalized) speed \( 450\sqrt{2} \text{ m/s} \) and initial state \([0 \text{ m} \ 225\sqrt{2} \text{ m/s} \ 9000\sqrt{2} \text{ m} \ -225\sqrt{2} \text{ m/s}] \), \([0 \text{ m} \ 225\sqrt{2} \text{ m/s} \ 0 \text{ m} \ 225\sqrt{2} \text{ m/s}] \) and \([3500\sqrt{2} \text{ m} \ -450\sqrt{2} \text{ m/s} \ 4500\sqrt{2} \text{ m} \ 0 \text{ m/s}] \) respectively. From Figures 1–4, the following observations can be seen:

1) The comparison of Figure 1 and Figure 2 shows that when there is 30% probability of spurious/false measurements, the single sensor MEEDA Algorithm 1 has been affected, since the target 2 is lost. The distributed fusion MEEDA Algorithm 2 works well, but the Euclidean error/the size of the bounding box has an obvious increase when a spurious/false measurement occurs. Thus, the spurious/false measurement is a major factor influencing the performance of MEEDA Algorithm. Similarly, from comparison of Figure 1 and Figure 3, the missing measurement is also a major factor influencing the performance of MEEDA Algorithm.

2) Figure 1 and Figure 4 present the tracking figures of two targets and three targets with same parameters \( q=10 \text{ m/s}^{3/2}, r=400 \text{ m}, b_s = 0.1 \) and \( b_m = 0.001 \) respectively. For the case of two targets in Figure 1, the single sensor MEEDA Algorithm 1 can correctly associate data with the target trajectories, however, it cannot work well for the case of three targets in Figure 4. Therefore, the number of targets may be another important factor for the algorithm performance.

3) Performance of the two sensor distributed fusion MEEDA Algorithm 2 is much better than that of the single sensor MEEDA Algorithm 1 from Figures 1 through
4. Thus, when the single sensor MEEDA Algorithm 1 cannot work well, one would better minimize the sizes of validate gates and state bounding boxes by using the complementary advantage of multisensor to improve the performance of data association and tracking algorithms.

V. CONCLUSION

Data association and tracking problems for multi-Target and multisensor uncertain systems have been considered simultaneously. The predictive measurement bounding boxes, i.e., validation gates of measurements are derived, which are guaranteed to contain the measurement vectors to be predicted respectively. The corresponding optimization problem is a semidefinite program which can be efficiently solved in polynomial-time. Moreover, based on the single sensor and distributed fusion state bounding box estimation, two effective methods—single sensor MEEDA algorithm and distributed fusion MEEDA algorithm are proposed respectively. Simulations show that the number of the targets, spurious/false measurements and missing measurements are important factors of the algorithm performance. It is also shown that, to improve the performance of data association algorithms, one would better try to minimize the sizes of validate gates and state bounding boxes by sufficient use of complementary advantages of multisensor and multi-algorithm fusion.

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Figure 1. The target tracking figures of the single sensor MEEDA Algorithm 1 and distributed fusion MEEDA Algorithm 2 with parameters $q=10 \text{ m/s}^{3/2}$, $r=400 \text{ m}$, $b_s = 0.1$ and $b_m = 0.001$.

Figure 2. The target tracking figures with 30% probability of false/spurious measurements by the single sensor MEEDA Algorithm 1 and distributed fusion MEEDA Algorithm 2 with parameters $q=10 \text{ m/s}^{3/2}$, $r=400 \text{ m}$, $b_s = 0.1$ and $b_m = 0.001$.

Figure 3. The target tracking figures with 30% probability of missing measurements by the single sensor MEEDA Algorithm 1 and distributed fusion MEEDA Algorithm 2 with parameters $q=10 \text{ m/s}^{3/2}$, $r=400 \text{ m}$, $b_s = 0.1$ and $b_m = 0.001$.

Figure 4. The three target tracking figures of the single sensor MEEDA Algorithm 1 and distributed fusion MEEDA Algorithm 2 with parameters $q=10 \text{ m/s}^{3/2}$, $r=400 \text{ m}$, $b_s = 0.1$ and $b_m = 0.001$.