The Shifted Rayleigh Filter for 3D Bearings-only Measurements with Clutter

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Abstract—Bearings-only tracking concerns the estimation of the position of a moving target from noisy measurements of the direction of the target relative to the sensor platform. We present an algorithm for bearings-only tracking, in which the measurements are in 3D space and are corrupted by clutter. In common with the earlier proposed shifted Rayleigh filter (SRF), the algorithm is based on exact calculations of the updated conditional mean and covariance of the state variable, under the assumption of a Gaussian prior. The presence of clutter on bearings measurements in 3D space adds considerably to the difficulty of performing these exact calculations. We report on simulations in a challenging scenario, aimed at comparing the performance of various trackers. In these simulations, the proposed tracking algorithm consistently outperforms other moment matching filters and the mean square tracking error compares favorably with the posterior Cramér-Rao lower bound (CRLB).

I. INTRODUCTION

The bearings-only tracking problem considered in this paper is that of estimating the position of a moving target from noisy measurements of its direction, relative to the sensor location. It is of both practical and theoretical interest. The need to process bearings measurements arises in submarine tracking using passive sonar, in aircraft surveillance using radar in passive mode, and, directly, in applications involving IR sensors (see [9], [15]). On the other hand, bearings-only tracking is a severely nonlinear estimation problem, owing to the ‘arctan’ nonlinearity in the measurement equation, which, in certain challenging scenarios, exposes shortcomings of traditional filters such as the extended Kalman filter, based on linearizing the system and measurement equations about the current state estimate, and is a testing ground for alternative tracking algorithms. Many have been proposed in this area, including moment matching filters ([3], [2], [18], [1], [19], [21]) and particle filters ([6]). Recently, continuous-discrete filters in modified spherical and log spherical coordinates have also been introduced [17].

The shifted Rayleigh filter (SRF) proposed in [10] is a highly competitive algorithm for bearings-only tracking problems, covering situations in which the target moves in either the plane or in 3D space. The distinctive feature of the SRF is that it is constructed from exact formulae for the mean and covariance of the posterior probability density of the state, under the assumption that the prior probability density is Gaussian. Exact calculations of these two moments is made possible through a slight modification of the standard measurement equations, which typically has little effect on the probability distribution of the measured bearings. The SRF differs from other moment matching algorithms such as the extended Kalman filter (EKF) and the unscented Kalman filter (UKF) [16] which only approximate the updated mean and covariance. The robustness of the SRF to the initial choice of state distribution and its ability to deal with rapidly varying bearings measurements, as arise when the target passes across the sensor platform, have been documented in a number of papers ([11], [10], [4]). All the simulations in these earlier studies involve bearings measurements in the plane.

The purpose of this paper is to present, and assess through simulations, a new version of the SRF for bearings-only measurements, in 3D, to take account of clutter. The algorithm is labeled SRF3C (‘shifted Rayleigh filter for 3D bearings measurements with clutter’). A mode representation of clutter, which is a simplified version of the one described in [8], is employed. When clutter is present, the construction of the SRF requires not only exact moment calculations of the known SRF filter for clutter-free measurements, but also an additional exact calculation of the non-Gaussian predicted bearings density. This calculation has been previously carried out only for 2D measurements [11]. A notable feature of the SRF3C is that it incorporates an exact formula for the 3D predictive bearings density, whose derivation presents a significantly greater technical challenge than that of the 2D formula.

The scenario chosen for simulations, involving bearings measurements in 3D space and clutter, is a variant on the 2D clutter-free scenario considered in [4] which is used to compare the SRF with other tracking algorithms, under circumstances when the bearings measurement changes rapidly and the EKF often fails to hold track. As reported in Section 3, the SRF3C consistently outperforms the EKF, the range-parameterized EKF and the UKF in the simulations, as regards both mean square tracking error and frequency of
tracking error divergence. Comparisons are also made with a particle filter. The mean square tracking error is shown to approach the Cramér-Rao lower bound.

**Notation.** Vector variables are denoted by boldface or by underlining in the case of Greek characters. The probability density of a random variable \( x \) is written \( p(x) \), i.e. we identify the probability density of a random variable by using the same symbol (in this case \( x \)) for the random variable and also for the dummy variable in the probability density. We follow a similar convention with conditional probability densities. Thus, for given random variables \( x \) and \( y \), \( p(x|y) \) denotes the probability density of \( x \) given the random variable \( y \). Given a sequence of random variables \( z_1, z_2, \ldots, z_t \), the finite subsequence \( \{z_1, z_2, \ldots, z_t\} \) is written \( z_{1:t} \). The multivariate Gaussian density with mean \( \bar{x} \) and covariance \( P \) is written \( N(\bar{x}, P) \).

II. The Shifted Rayleigh Filter for 3D Bearings Measurements with Clutter

**Formulation of the Tracking Problem:** Consider an \( n \)-state process \( \{x_t\} \) describing (in discrete time) the motion of a point target in 3D space, and a 2-vector measurement process \( \{y_t\} \) describing the bearings of target position relative to that of the sensor platform in the presence of clutter, governed by the equations

\[
x_t = F_t x_{t-1} + u_t^x + (Q_t^x)^{1/2} v_t, \quad (1)
\]
\[
d_t = H_t x_t + u_{t}^n, \quad (2)
\]
\[
\psi_t = \chi(d_t + (Q_t^n)^{1/2} w_t), \quad (3)
\]
\[
y_t = (1 - D_t) \psi_t + D_t c_t, \quad (4)
\]

expressed in terms of the following \((t\)-dependent\) matrices and vectors:

- \( F_t \): the \( n \times n \) system matrix,
- \( u_t^x \): the exogenous state input
- \( Q_t^x \): the \( n \times n \) state covariance matrix,
- \( H_t \): the \( 3 \times n \) output matrix,
- \( u_t^n \): the exogenous measurement input
- \( Q_t^n \): the measurement noise covariance matrix

All of these matrices and vectors (at time \( t \)) may depend on past values of the measurements \( y_{1:t-1} \). \( \{v_t\} \) and \( \{w_t\} \) are Gaussian noise sequences

\[
v_t \sim N(0, I_{n \times n}) \quad \text{and} \quad w_t \sim N(0, I_{3 \times 3}).
\]

The 2-vector measurement process \( \{y_t\} \) is defined via intermediate processes \( \{d_t\}, \{\psi_t\} \{c_t\} \) and \( \{D_t\} \), which have the following interpretations:

The 3-vector process \( \{d_t\} \), called the ‘displacement’, describes the relative \( xyz \) position coordinates of the target with respect to the sensor platform, whose known position is itself described by the 3-vector process \( \{u_t^n\} \).

\( \psi_t \) describes the idealized measurement process, which would result if there were no clutter. The equation for \( \psi_t \) involves the nonlinear function \( \chi(.) : \mathbb{R}^3 \to \mathbb{R}^2 \)

\[
\chi(d) = \chi(d_1, d_2, d_3) = \left( \arctan2(d_1, d_2), \arctan \frac{d_3}{\sqrt{d_1^2 + d_2^2}} \right), \quad (5)
\]

where \( \arctan2(d_1, d_2) \) denotes the ‘four-quadrant’ inverse tangent of a vector with components \( d_1, d_2 \) in the \( x \) and \( y \) directions, respectively. The two components of \( \chi(d) \) will be recognized as the azimuthal and elevation angles \( (\theta, \alpha) \in ([-\pi, \pi], [-\pi/2, \pi/2]) \) of \( d \) expressed in spherical coordinates \(|d|, \theta, \alpha\) as represented in Figure 1.

Fig. 1. Azimuth and elevation angles \( \theta \) and \( \alpha \) in the 3D measurement frame

The 2-vector process \( c_t \) is the false measurement of the azimuthal and elevation bearings that arises from clutter. It is assumed that the clutter is homogeneous, i.e. uniformly distributed in all directions in 3D space. Consequently, the probability density of \( c_t \) is assumed to be

\[
p(c_t = (\theta, \alpha)) = (4\pi)^{-1} \cos \alpha \quad (6)
\]

\( \{D_t\} \) is a binary process, taking the value 0 if the measurement is a ‘true’ measurement and 1 if it is due to clutter. We define

\[
P[D_t = 1] = p_c, \quad P[D_t = 0] = 1 - p_c, \quad (7)
\]

where \( p_c \) is the clutter probability.

It is assumed that the processes \( v_t, w_t, c_t \) and \( D_t \), and the initial state \( x_0 \), a random variable with probability density \( x_0 \sim N(x_0, P_0) \), are independent.

The purpose is to determine recursive formulae providing approximations to the conditional mean and covariance of \( x_t \), given \( y_{1:t} \), for \( t = 1, 2, \ldots \). We briefly comment on the elements of this formulation.

**The Measurement Model:** Suppose \( D_t = 0 \) (‘no clutter’). Then the measurement is modeled by the equation

\[
y_t = \chi(d_t + (Q_t^n)^{1/2} w_t), \quad (8)
\]
in which the function $\chi(\cdot)$, defined by (5), converts the ‘augmented displacement’, that is the displacement with additive noise, into its azimuthal and elevation angles. It contrasts with the conventional measurement model in which noise, in the form of a 2-dimensional Gaussian density, is added to the ‘true’ azimuthal and elevation angles, thus:

$$y_t = \chi(d_t) + (\sigma^2)^{1/2} w_t.$$

Here, $\sigma^2$ is a variance parameter and $w_t$ is a 2D white noise process with normal distribution $w_t \sim N(0, I_{2\times2})$, whose first and second components are respectively ‘wrapped’ onto $[-\pi, +\pi]$ and ‘folded’ onto $[-\pi/2, +\pi/2]$.

The unconventional noise model is chosen to simplify calculations involved in constructing our tracking algorithm. Faced with a scenario based on the conventional measurement noise model (9), we may fit it to our framework, by taking as measurement equation

$$y_t = \chi(d_t) + (\sigma^2 I_{3\times3})^{1/2} w_t,$$

which is a special case of (8).

Equation (10) itself can be seen as an approximation to the measurement process (9) \{\hat{y}_t\}:

$$\hat{y}_t = \chi(d_t) + d_t((\sigma^2 I_{3\times3})^{1/2} w_t).$$

Notice that the ‘noise term’ $E[d_t^2 | y_{1:t-1}]$ contributes to the ‘wrapped’ density $N(0, \sigma^2)$ and the ‘folded’ density $N(0, \sigma^2)$. We see that for $\sigma \leq 0.3$ they are virtually indistinguishable. Comparison of the joint densities, although harder to visualize, also confirms this fact.

**The Clutter Model:** Clutter is described by a ‘mode’ model (see equation (4)), as used in [11], which is a simplification of the representation used in [8] in the context of probabilistic data association (PDA).

**Propagation of Conditional Probability Densities:** We shall construct an algorithm that propagates the mean and covariance, $\hat{x}_t$ and $P_t$ respectively, of Gaussian approximations to the conditional probability density of $x_t$ given $y_{1:t}$. Take any time $t \geq 1$. As a first step we develop formulae for the posterior probability density $p(x_t | y_{1:t})$ in terms of the prior density $p(x_{t-1} | y_{1:t-1})$ and the new measurement $y_t$.

Application of the decomposition rules of conditional probability densities and Bayes rule leads to the following representation of $p(x_t | y_{1:t})$:

$$p(x_t | y_{1:t}) = w_0 p(x_t | y_t = \psi_t, y_{1:t-1}) + w_1 p(x_t | y_{1:t-1})$$

in which

$$w_0 = c^{-1}(1 - p_c)p(\psi_t | y_t, y_{1:t-1})$$

and $c$ is a normalization constant, chosen to ensure $w_0 + w_1 = 1$.

Equations (13) and (14), arising in PDA, decompose the updated conditional probability density as the weighted sum of two densities, namely the updated density conditioned on “no clutter”, and the predicted density. The weightings involve the probability density of $\psi_t$ given $y_{1:t-1}$, and also the clutter probability density $p_c(\psi_t = (\theta, \alpha)) = (4\pi)^{-1}\cos(\alpha)$. We now approximate the prior conditioned probability density $p(x_{t-1} | y_{1:t-1})$ as a Gaussian probability density, i.e. we assume

$$p(x_{t-1} | y_{1:t-1}) = N(\hat{x}_{t-1}, P_{t-1})$$

with parameters $\hat{x}_{t-1}, P_{t-1}$. Formulae then become available for the means and covariances of the constituent probability densities in (13) and (14):

Write $\hat{x}_t^u$ and $P_t^u$ for the mean and covariance of $p(x_t | y_t = \psi_t, y_{1:t-1})$. Then $\hat{x}_t^u$ and $P_t^u$ are expressed in terms of the shifted Rayleigh filter equations for 3D bearings-only measurements, derived in [10]:
Prediction Step:

\[
\dot{x}_{t|t-1} = F_t \hat{x}_{t-1|t-1} + u_{t|t-1}^n \\
P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q_t^s \\
V_t = H_t P_{t|t-1} H_t^T + Q_t^{na}
\]  
(15) (16) (17)

Correction Step:

\[
K_t = P_{t|t-1} H_t^T V^{-1}_t \\
Z_t = (b_t^T V^{-1}_t b_t)^{-\frac{1}{2}} b_t^T V^{-1}_t (H_t \hat{x}_{t|t-1} + u_{t|t}^n) \\
\gamma_t = (b_t^T V^{-1}_t b_t)^{-\frac{1}{2}} \rho(z_t) \\
\delta_t = (b_t^T V^{-1}_t b_t)^{-1} (3 + \rho(z_t) - \rho^2(z_t)) \\
\hat{x}_{t|t} = (I - K_t H_t) \hat{x}_{t|t-1} - K_t u_{t|t}^n + \gamma_t K_t b_t \\
P_{t|t} = (I - K_t H_t) P_{t|t-1} + \delta_t K_t b_t b_t^T K_t^T
\]
(18) (19) (20) (21) (22) (23)
in which \( b_t \) is the 3-vector obtained from \( y_t = (\theta, \alpha) \) by means of the formula

\[
b_t = (\sin(\theta) \cos(\alpha), \cos(\theta) \cos(\alpha), \sin(\alpha))^T.
\]
The function \( \rho(z) \) in these formulae is

\[
\rho(z) = z + 2 \left( \frac{e^{-\frac{z^2}{2}} + \sqrt{2\pi}(z^2 + 1) F_N(z)}{e^{-\frac{z^2}{2}} + \sqrt{2\pi} z F_N(z)} \right)^{-1},
\]
with \( F_N(\cdot) \) being the cumulative distribution function of a normal \( N(0,1) \) variable. Concerning the other probability density appearing in the decomposition (12), we note that \( p(x_t \mid y_{1:t-1}) \) has mean \( \hat{x}_{t|t-1} \) and covariance \( P_{t|t-1} \), as given by (15) and (16) respectively.

The final calculation that is required to translate the update formula (12) into an algorithm is that of \( p(\psi_t \mid y_{1:t-1}) \); that is, the probability density of the true current measurement \( \psi_t \) given the previous measurements.

Write \( \mu = H_t \hat{x}_{t|t-1}, \; V = V_t \) and

\[
p(\theta, \alpha) = p(\psi_t = (\theta, \alpha) \mid y_{1:t-1}).
\]
Then

\[
p(\theta, \alpha) = \frac{\cos \alpha}{(2\pi)^{3/2}(\det V)^{1/2}} e^{\frac{\mu^2}{2V}} F_N \left( \frac{-b}{2\alpha^2} \right),
\]
(24)

where

\[
\begin{align*}
a &= a_{11} \sin^2(\theta) \cos^2(\alpha) + a_{22} \cos^2(\theta) \cos^2(\alpha) + a_{33} \sin^2(\alpha) + (a_{12} + a_{21}) \cos(\theta) \sin(\theta) \cos^2(\alpha) + (a_{23} + a_{32}) \cos(\theta) \cos(\alpha) \sin(\alpha) + (a_{13} + a_{31}) \sin(\theta) \cos(\alpha) \sin(\alpha), \\
b &= -[2a_{11} \mu_1 + (a_{12} + a_{21}) \mu_2 + (a_{13} + a_{31}) \mu_3] \sin(\theta) \cos(\alpha) - [2a_{22} \mu_2 + (a_{12} + a_{21}) \mu_1 + (a_{23} + a_{32}) \mu_3] \cos(\theta) \cos(\alpha) - [2a_{33} \mu_3 + (a_{23} + a_{32}) \mu_2 + (a_{13} + a_{31}) \mu_1] \sin(\alpha), \\
c &= \mu_1^2 a_{11} + \mu_2^2 a_{22} + \mu_3^2 a_{33} + \mu_1 \mu_2 (a_{12} + a_{21}) + \mu_2 \mu_3 (a_{23} + a_{32}) + \mu_1 \mu_3 (a_{13} + a_{31}),
\end{align*}
\]
with \( a_{ij} \) denoting the elements of the precision matrix \( V^{-1} \).

The Algorithm: The algorithm can be summarized as follows: the updated probability density is approximated as a Gaussian density with parameters \( \hat{x}_t \) and \( P_t \), where \( \hat{x}_t \) and \( P_t \) are the mean and covariance of \( p(x_t \mid y_{1:t}) \) calculated under the assumption \( p(x_{t|t-1} \mid y_{1:t-1}) = N(\hat{x}_{t|t-1}, P_{t|t-1}) (x_{t|t-1}) \). A full set of formulae for \( \hat{x}_t \) and \( P_t \), given \( \hat{x}_{t-1} \) and \( P_{t-1} \) can be assembled from the preceding equations:

1. Calculate \( \hat{x}_{t|t-1}, P_{t|t-1}, \hat{x}_t^{na} \) and \( P_t^{na} \) from equations (15) - (23).
2. Evaluate the weights \( w_0 \) and \( w_1 \) from equations (13) and (14) using (24) to (27).
3. Take \( \hat{x}_t \) and \( P_t \) to be

\[
\hat{x}_t = w_0 \hat{x}_t^{na} + w_1 \hat{x}_{t|t-1} \\
P_t = w_0 \left[ P_t^{na} + (\hat{x}_t^{na} - \hat{x}_t)(\hat{x}_t^{na} - \hat{x}_t)^T \right] + w_1 \left[ P_{t|t-1} + (\hat{x}_{t|t-1} - \hat{x}_t)(\hat{x}_{t|t-1} - \hat{x}_t)^T \right]
\]
(28) (29)

(We recognize the formulae for \( \hat{x}_t \) and \( P_t \) as the standard equations for the mean and covariance of a mixture of densities, expressed in terms of the weights and constituent means and covariances).

III. Simulations

In this section we compare the performance of the SRF3C, for state estimation from 3D bearings-only measurements in the presence of clutter, with that of the EKF, its range-parameterized variant (RPEKF) [20], the UKF and also the sampling-importance-resampling particle filter with local EKF linearization (EKPF) [12], [5]. The EKF, RPEKF and UKF make use of PDA to take clutter into account, as described by equations (12)-(14), while the EKPF is also similarly modified. Finally, we include the posterior Cramér-Rao lower bound.
bound for the problem at hand, calculated as in [13], where we make the optimistic assumption that a measurement originating from clutter is known to be false. The EKF in modified spherical coordinates was also implemented, but results are not included here as they showed no discernible improvement over the Cartesian coordinate formulation.

The tracking problem is a variant on the one presented in [4], in which the target moves in 3D space and the measurements are corrupted by clutter. We consider a ‘high bearings-rate’ scenario which challenges tracker robustness and accuracy. The sensor platform sets off 4 km above the origin and travels parallel to the \( xy \) plane at a constant speed of 130 m/s and a course of \(-80^\circ\) for 14 seconds. It then executes a maneuver and adopts a new course of 153\(^\circ\), which it maintains for the remainder of the simulation period. Meanwhile, the target, initially at an altitude of 6.5 km, a range \( r_0 \) of approximately 10.6 km from the sensor platform and a bearing of 29\(^\circ\), executes a descent at a constant speed of 470 m/s, a course of \(-149.5^\circ\) and pitch of \(-12.3^\circ\), eventually traveling past the sensor platform. Figure 3 illustrates the target/sensor platform geometry, projected onto the \( xy \) plane. The 6-vector target state comprises the components of the Cartesian position and velocity along the \( xyz \) axes,

\[
x_t = \begin{bmatrix} x_t & \dot{x}_t & y_t & \dot{y}_t & z_t & \dot{z}_t \end{bmatrix}^T,
\]

and dynamics are those of a ‘nearly’ constant velocity discrete time model [7], described by equation (1) in which

\[
F = \text{diag} \left( \begin{bmatrix} 1 & h & 0 \hfill & 0 & 1 \hfill & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \hfill & 1 \hfill & 0 \hfill & 1 \end{bmatrix}, \begin{bmatrix} 1 & h \hfill & 0 \hfill & 0 \end{bmatrix} \right)
\]

\((h)\) is the sampling period in seconds. The deterministic signal \( u^s_t \) is taken to be zero while the movement of the sensor platform is absorbed into equation (2) via the signal \( u^w_t \). The covariance matrix of the discretized zero-mean white Gaussian noise process \( w_t \) is expressed as:

\[
Q^t = q \text{ diag} \left( \begin{bmatrix} h^2 & h^2 & \frac{h^2}{2} \hfill & \frac{h^2}{2} \hfill & \frac{h^2}{2} \hfill & \frac{h^2}{2} \end{bmatrix}, \begin{bmatrix} \frac{h^2}{2} & h^2 & \frac{h^2}{2} \hfill & \frac{h^2}{2} \hfill & \frac{h^2}{2} \hfill & \frac{h^2}{2} \end{bmatrix}, \begin{bmatrix} \frac{h^2}{2} & \frac{h^2}{2} & \frac{h^2}{2} \hfill & h^2 \end{bmatrix} \right).
\]

Noisy angle measurements of zero mean and standard deviation \( \sigma = 3 \text{ mrad} \) (\( \sim 0.17^{\circ} \)) are received at intervals of \( h = 1 \) s and the system noise parameter \( q \) is set to \( 9.92 \times 10^{-5} \) \( \text{m}^2/\text{s}^3 \). The simulation horizon is \( T = 30 \) s. Figure 4 shows the clutter-free azimuth bearings rate, which approaches \(-2\pi \) \( \text{rad/s} \) as the target moves past the sensor platform at \( t = 27 \) s.

We perform 200 Monte Carlo runs of this scenario over different realizations of measurement noise and clutter processes. The initial range prior is taken to be \( r_0 \sim N(r_0, \sigma_{r_0}) \), with \( \sigma_{r_0} = 5 \) km, while the initial velocity priors along the \( xyz \) axes are given by \( \dot{r}_0 \sim N(0, \sigma_{\dot{r}_0}) \), where \( \sigma_{\dot{r}_0} = 500 \) m/s for the \( x \) and \( y \) coordinates and \( \sigma_{\dot{r}_0} = 166 \) m/s for the \( z \) coordinate. Initialization of the filters is then achieved through a linearized transformation about the first bearings measurement, as done in [20], which we allow to be clutter-free.

The RPEKF is implemented with 6 EKFs (covering initial mean range estimates from 1 km to 30 km), while the EKPF is set to propagate 3000 samples drawn from EKF generated importance densities.

Two performance metrics are used in the comparison of the different filters: the RMS range error and the percentage of diverged tracks. Track divergence is assessed by evaluating the normalized estimation error squared \( \epsilon_t^2 \) given by

\[
\epsilon_t^2 = (x_t - \hat{x}_{it})^T P_{it}^{-1} (x_t - \hat{x}_{it}). \tag{30}
\]

Under the hypothesis of a consistent filter, \( \epsilon_t^2 \) should have a \( \chi^2 \) distribution with degrees of freedom equal to the dimension of \( x \) (6 in this case). We deem a track to have diverged if either

- \( \epsilon_t^2 \) exceeds, after maneuver of the sensor platform, the 99% probability concentration region of a \( \chi_6^2 \) variable for more than 6 time steps, or
- the RMS range error, after maneuver of the sensor platform, exceeds the threshold of 6 km for two consecutive time steps.

Figure 5 shows the RMS errors of the range estimates for \( p_c = 0.5 \). The average performances of the EKF and UKF are clearly unacceptable, while the RPEKF shows only slightly improved robustness and accuracy. The overall low precision and frequent track divergence reflected in their RMS range errors point to the inadequacies of tracking algorithms based on simple linearization techniques (whether analytical or statistical, as with the UKF), faced with a challenging scenario involving high clutter probability, high bearings rate and large initial state uncertainty. The SRF3C, on the other hand, comes close on average to achieving the CRLB, with computation times comparable to that of the EKF. The RMS error of the EKPF, meanwhile, is comparable to that of the SRF3C but with computational requirements nearly 4 orders of magnitude
higher. Note that the CRLB curve lies above the RMS errors of the EKPF for the initial phase of the track during which transient effects are significant, owing to a lack of observability before the abrupt maneuver of the sensor platform at $t = 14$ s.

Figure 6 illustrates the percentage of divergent tracks produced by the various filters over the 200 MC runs, with varying levels of clutter probability. The SRF3C shows high robustness to clutter with practically no divergent tracks up to a clutter probability of 0.6. On the other hand, the EKF and UKF exhibit frequent breakdowns. The relative robustness of the RPEKF, owing to the use of multiple filters, can also be seen. Finally, the EKPF achieves a level of resilience to clutter comparable to that of the SRF3C.

IV. CONCLUSIONS

In this paper we proposed a new moment matching algorithm denoted SRF3C for the problem of 3D bearings-only tracking in the presence of clutter. The algorithm takes the form of the shifted Rayleigh filter with some significant modifications that take into account the possibility of clutter in the 3D bearings measurements. It has the distinctive feature of generating the exact first and second order moments of the posterior target state estimate, assuming that the prior is Gaussian. Simulations in a challenging tracking scenario reveal its potential in relation to the EKF, the UKF, the RPEKF and the EKPF. It demonstrates high tracking accuracy despite the extreme bearings rate, high probability of clutter and poor target initialization and also shows great robustness with very rare track divergence over Monte Carlo runs of increasing clutter probability. Its computational requirements, meanwhile, are similar to those of the EKF.

REFERENCES


