Globally Optimal Distributed State Fusion White Noise Deconvolution Estimators

Xiaojun Sun, Guangming Yan and Bo Zhang
Department of Automation
Heilongjiang University
Harbin, China
sxj@hlju.edu.cn

Abstract—White noise deconvolution or input white noise estimation has a wide range of applications including oil seismic exploration, communication, signal processing, and state estimation. The globally optimal distributed state fusion white noise deconvolution estimators are presented for the multisensor linear discrete systems using the Kalman filtering method. They are derived from the centralized fusion white noise deconvolution estimators so that they are identical to the centralized fusers, i.e. they have the global optimality. Compared with the existing globally suboptimal distributed state fusion white noise estimators, the computation of complex covariance matrices is avoided. A simulation for the Bernoulli-Gaussian input white noise shows the effectiveness of the proposed results.

Keywords—multisensor information fusion; distributed state fusion; white noise deconvolution; global optimality; Kalman filter

I. INTRODUCTION

The input white noise estimation or white noise deconvolution has important applications in oil seismic exploration [1-3] and occurs in many fields including communication, signal processing and state estimation [4]. The optimal input white noise estimators based on Kalman filtering were presented in [1-3]. The general and unified white noise estimation theory based on Kalman filtering was presented in [5], which includes both the input white noise estimators and measurement white noise estimators with the results in [1-3] as a special case. A unified white noise estimation theory based on the modern time series analysis method was presented in [4].

In order to improve the estimation accuracy based on a single sensor, the multisensor information fusion has received great attention in recent years, which has been widely applied to many fields including guidance, defence, robotics, integrated navigation, target tracking, GPS positioning, and signal processing. For Kalman filtering-based fusion, the basic fusion methods can be classified as the state fusion methods and measurement fusion methods [6,7]. The state fusion methods include the centralized and distributed fusion method. The centralized state fusion method simply combines all local measurement equations to obtain an augmented measurement equation, which accompanies the state equation to yield the global optimal Kalman filter [8]. But its disadvantage is the larger computational burden is required due to the higher dimension of the augmented measurement vector. The distributed state fusion methods weight or combine local Kalman filters to obtain the fused Kalman filter. They have the advantages that a lower computation and communication cost is required, and the fault detection and isolation are facilitated. Recently, the distributed information fusion white noise deconvolution filter weighted by scalars is given in [9], and three distributed fusion white noise deconvolution estimators weighted by scalars, diagonal matrices and matrices are given in [10, 11], but they are all globally suboptimal.

In this paper, using the Kalman filtering method, the globally optimal distributed state fusion white noise deconvolution estimators are presented for the multisensor systems. They are completely equivalent to the centralized fusion white noise deconvolution estimators, so that they have the global optimality. Compared with the existing globally suboptimal distributed state fusion white noise estimators [10, 11], the computation of complex covariance matrices is avoided.

II. PROBLEM FORMULATION

Consider the multisensor linear discrete time-invariant stochastic system

\begin{equation}
    x(t+1) = \Phi x(t) + \Gamma w(t)
\end{equation}

\begin{equation}
    y_i(t) = H_i x(t) + v_i(t), \quad i = 1, \ldots, L
\end{equation}

where \( t \) is the discrete time, \( x(t) \in \mathbb{R}^n \), \( y_i(t) \in \mathbb{R}^{m_i} \), \( w(t) \in \mathbb{R}^r \), \( v_i(t) \in \mathbb{R}^{m_i} \) are the state, measurement, process and measurement noises of the \( i \)th sensor subsystem, respectively, and \( \Phi \), \( \Gamma \) and \( H_i \) are time-invariant matrices with compatible dimensions.

Assumption 1. \( w(t) \in \mathbb{R}^r \) and \( v_i(t) \in \mathbb{R}^{m_i} \) are independent white noises with zero mean and

\[
    \mathbb{E} \left[ \begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix} \mathbb{E} \left[ \begin{bmatrix} w^T(k) \\ v_i^T(k) \end{bmatrix} \right] \right] = \begin{bmatrix} Q & 0 \\ 0 & R_i \delta \end{bmatrix} \delta_k
\]

This work was supported by the National Natural Science Foundation of China under Grant No. 61104209, Outstanding Youth Science Foundation of Heilongjiang University under Grant No. JCL201103 and Key Laboratory of Electronics Engineering, College of Heilongjiang Province under Grant No. DZZD2010-5.
where \( E \) denotes the mathematical expectation, the superscript \( T \) denotes the transpose, \( \delta_{\alpha\beta} \) is the Kronecker delta function, i.e. \( \delta_{\alpha\alpha} = 1 \), \( \delta_{\alpha\beta} = 0 (\alpha \neq \beta) \).

**Assumption 2.** \( x(0) \) is uncorrelated with \( w(t) \) and \( v_i(t) \), and \( \text{Ex}(0) = \mu \), \( \text{cov}(x(0)) = P_0 \), where \( \text{cov} \) denotes covariance.

**Remark 1.** In fact, when the ARMA models with nonzero constant terms in both the autoregressive and moving average parts is transferred into the equivalent state space model, we will find that the cross-covariance matrices between the process noises and measurement noises are identical for all sensors at each time \( t \). So Assumption 1 given in this paper is not restrictive for practical application.

The problem is to find the globally optimal distributed state fusion white noise deconvolution fuser \( \hat{w}(0) (t \mid t + N) \).

### III. Globally Optimal Distributed State Fusion White Noise Deconvolution Estimators

#### A. Centralized Fusion White Noise Deconvolution Estimators

Introducing an augmented measurement vector, we combine all measurement equations into a centralized measurement fusion equation

\[
y^{(0)}(t) = H^{(0)} x(t) + v^{(0)}(t)
\]

with the definitions

\[
y^{(0)}(t) = [y_1^T(t), \cdots, y_L^T(t) ]^T
\]

\[
H^{(0)} = [H_1^T, \cdots, H_L^T]^T
\]

\[
v^{(0)}(t) = [v_1^T(t), \cdots, v_L^T(t) ]^T
\]

and the fused measurement white noise \( v^{(0)}(t) \) has the variance matrix \( R^{(0)} \) as follows

\[
R^{(0)} = diag(R_1, \cdots, R_L)
\]

For the centralized fusion system (1) and (4), applying the standard Kalman filter [12] with the initial time \( t_0 = 0 \), the centralized fusion Kalman predictor \( \hat{x}^{(0)}(t+1 \mid t) \) and Kalman filter \( \hat{x}^{(0)}(t \mid t) \) are respectively given by

\[
\hat{x}^{(0)}(t+1 \mid t) = \hat{x}^{(0)}(t+1 \mid t) + K^{(0)}(t+1) e^{(0)}(t+1)
\]

\[
e^{(0)}(t+1) = y^{(0)}(t+1) - H^{(0)} \hat{x}^{(0)}(t+1 \mid t)
\]

The filtering gain \( K^{(0)}(t+1) \) is given as

\[
K^{(0)}(t+1) = P^{(0)}(t+1 \mid t) H^{(0)} Q_e^{(0)-(1)}(t+1)
\]

\[
Q_e^{(0)}(t+1) = H^{(0)} P^{(0)}(t+1 \mid t) H^{(0)}^T + R^{(0)}
\]

\[
\hat{x}^{(0)}(t+1 \mid t) = \Psi^{(0)}(t) \hat{x}^{(0)}(t \mid t-1) + K^{(0)}(t) y^{(0)}(t)
\]

with the definitions

\[
K_p^{(0)}(t) = \Phi K_f^{(0)}(t)
\]

\[
\Psi_p^{(0)}(t) = \Phi - K_p^{(0)}(t) H^{(0)}
\]

where the prediction error variance matrix \( P^{(0)}(t+1 \mid t) \) satisfies the time-varying Riccati equation

\[
P^{(0)}(t+1 \mid t) = \Phi P^{(0)}(t \mid t) \Phi^T + \Gamma Q \Gamma^T
\]

The filtering error variance matrix \( P^{(0)}(t+1 \mid t+1) \) is given by

\[
P^{(0)}(t+1 \mid t+1) = [I_n - K_f^{(0)}(t+1) H^{(0)}] P^{(0)}(t+1 \mid t)
\]

The centralized fusion Kalman predictor and filter are globally optimal in the sense that their accuracy is higher than that of each local Kalman predictor and filter, and is higher than that of the weighted fusion Kalman predictor and filter [10,11].

For the centralized fusion system (1) and (4), applying the standard Kalman filter [12] with the initial time \( t_0 = 0 \), the centralized fusion white noise deconvolution estimator \( \hat{w}(0)(t \mid t + N) \) is given as

\[
\hat{w}(0)(t \mid t + N) = 0, \quad N \leq 0
\]

\[
\hat{w}(0)(t \mid t + N) = \sum_{j=1}^{N} M^{(0)}(t \mid t + j) e^{(0)}(t + j), \quad N > 0
\]

The estimation gain is given as

\[
M^{(0)}(t \mid t+1) = Q \Gamma^T H^{(0)T} Q_e^{(0)-(1)}(t+1)
\]

\[
M^{(0)}(t \mid t+1) = Q \Gamma^T \left[ \prod_{j=1}^{N-1} \Psi_p^{(0)}(t+j) \right] H^{(0)T} Q_e^{(0)-(1)}(t+N)
\]

\[
N > 1
\]

The estimation error variance matrix is given as

\[
P^{(0)}(t \mid t + N) = \sum_{j=0}^{N} M^{(0)}(t \mid t+j) Q^{(0)}(t+j) M^{(0)T}(t \mid t+j)
\]

\[
N > 0
\]

#### B. Globally Optimal Distributed State Fusion White Noise Deconvolution Estimators

**Lemma 1** [13]. (Matrix Inversion Lemma) Assume that \( A \), \( B \) and \( C \) are \( n \times n \), \( n \times m \) and \( n \times m \) matrices, respectively, and the inverse matrices \( A^{-1} \) and \( (I_n + C^T A^{-1} B)^{-1} \) exist, then we have

\[
(A + B C^T)^{-1} = A^{-1} - A^{-1} B (I + C^T A^{-1} B)^{-1} C^T A^{-1}
\]

where \( I_n \) is the \( m \times m \) identity matrix.
Proof. Setting $D = A + BC^T$ yields
\[
I_n = D^{-1}D = D^{-1}(A + BC^T) = D^{-1}A + D^{-1}BC^T \quad (25)
\]
\[
D^{-1} = A^{-1} - D^{-1}BC^TA^{-1} \quad (26)
\]
From (26), we find that it is required to compute $D^{-1}$ in order to obtain $D^{-1}$. Right multiplying (26) with $B$, we have
\[
D^{-1}B = A^{-1}B - D^{-1}BC^TA^{-1}B \quad (27)
\]
\[
D^{-1}B(I_n + C^TA^{-1}B) = A^{-1}B \quad (28)
\]
\[
D^{-1}B = A^{-1}B(I_n + C^TA^{-1}B)^{-1} \quad (29)
\]
Substituting (29) into (26), it follows that
\[
D^{-1} = A^{-1} - A^{-1}B(I_n + C^TA^{-1}B)^{-1}C^TA^{-1} \quad (30)
\]
which yields that (24) holds. This completes the proof.

**Theorem 1** For the multisensor linear discrete system (1) and (2), the optimal distributed state fusion Kalman predictor $\hat{x}^{(0)}(t+1|t)$ and filter $\hat{x}^{(0)}(t+1|t+1)$ are respectively given as
\[
\hat{x}^{(0)}(t+1|t+1) = \hat{x}^{(0)}(t+1|t) + \sum_{i=1}^{L} [K_{\beta}(t+1)\hat{y}_i(t+1) - H_i\hat{x}^{(0)}(t+1|t)] \quad (31)
\]
\[
\hat{x}^{(0)}(t+1|t) = \Phi \hat{x}^{(0)}(t|t) \quad (32)
\]
\[
P^{(0)}(t+1|t+1) = [I_n - \sum_{i=1}^{L} K_{\beta}(t+1)H_i]P^{(0)}(t+1|t) \quad (33)
\]
\[
K_{\beta}(t+1) = P^{(0)}(t+1|t+1)H_i^TH_i^{-1} \quad (34)
\]
with the prediction error variance matrix $P^{(0)}(t+1|t)$ given by (16).

**Proof.** From Lemma 1 we have
\[
(A + BC^T)^{-1} = A^{-1} - A^{-1}B(I + C^TA^{-1}B)^{-1}C^TA^{-1} \quad (35)
\]
Applying (11) and (17), and taking $A = P^{(t)}(t|t-1)$, $B = H_i^T$, $C^T = R_i$, $H_i = H_i^T$ at time $t$, we have
\[
(P^{(0)}(t|t-1) + H_i^TH_i^{-1}H_i^T)^{-1} = P^{(0)}(t|t-1) - P^{(0)}(t|t-1)H_i^TH_i^{-1} \quad (36)
\]
\[
\forall (I + R_i^{-1}H_i^TP^{(0)}(t|t-1)H_i^T)^{-1} = P^{(0)}(t|t-1) - P^{(0)}(t|t-1)H_i^TH_i^{-1} \quad (37)
\]
\[
[H_i^TP^{(0)}(t|t-1)H_i + R_i^{-1}H_i^TP^{(0)}(t|t-1)H_i^T] = P^{(0)}(t|t-1) - P^{(0)}(t|t-1)H_i^TH_i^{-1} \quad (38)
\]
\[
[I - K_{\beta}(t)H_i^T]P^{(0)}(t|t-1) = P^{(0)}(t|t) \quad (39)
\]
which yields
\[
P^{(0)}(t|t) = P^{(0)}(t|t-1) + H_i^TH_i^{-1}H_i^TP^{(0)}(t|t-1) \quad (40)
\]
Using (11), (12), (16) and (37) yields
\[
K_{\beta}^{(0)}(t) = P^{(0)}(t|t-1)H_i^TH_i^{-1}P^{(0)}(t|t-1)H_i^TP^{(0)}(t|t-1) \quad (41)
\]
\[
P^{(0)}(t|t-1) = P^{(0)}(t|t-1) - P^{(0)}(t|t-1)H_i^TH_i^{-1} \quad (42)
\]
\[
P^{(0)}(t|t) = P^{(0)}(t|t-1) - P^{(0)}(t|t-1)H_i^TH_i^{-1} \quad (43)
\]
From (6), (8) and (38), we have
\[
K_{\beta}^{(0)}(t) = P^{(0)}(t|t)H_i^TH_i^{-1} \quad (44)
\]
Applying (5), (6), (9) and (10) yields (31). From (9), (10), (14) and (15), we have (32). From (6), (17) and (39), we can obtain (33). The proof is completed.

**Remark 2.** For the multisensor system (1) and (2), applying the standard Kalman filter [12] with the initial time $t_0 = 0$, we can obtain the local Kalman predictor $\hat{x}_i(t+1|t)$ and Kalman filter $\hat{x}_i(t|t)$ with the forms similar to these in (9)–(17).

**Theorem 2** For the multisensor linear discrete system (1) and (2), the optimal distributed state fusion white noise deconvolution estimator $\hat{w}^{(0)}(t+1|t+N)$ is given as
\[
\hat{w}^{(0)}(t+1|t+N) = \sum_{j=1}^{N} [M_i(t|t+j) \times (y_i(t+j) - H_i\hat{x}^{(0)}(t+j|t+j-1))] \quad (45)
\]
\[
M_i(t|t+j) = Q_i^T \sum_{i=1}^{N} \sum_{j=1}^{N} [K_{\beta}(t)H_i^TH_i^{-1} \Phi] \quad (46)
\]
\[
\hat{w}^{(0)}(t+1|t+N) = \sum_{j=1}^{N} [Q_i^T \sum_{i=1}^{N} \sum_{j=1}^{N} [K_{\beta}(t)H_i^TH_i^{-1} \Phi] \times P^{(0)}(t|t-1) \quad (47)
\]
\[
P^{(0)}(t|t-1) = P^{(0)}(t|t-1) - P^{(0)}(t|t-1)H_i^TH_i^{-1} \quad (48)
\]
\[
[I - K_{\beta}(t)H_i^T]P^{(0)}(t|t-1) = P^{(0)}(t|t) \quad (49)
\]
The estimation error variance matrix is given as
\[
P^{(0)}(t|t+N) = Q_i^T \sum_{i=1}^{N} \sum_{j=1}^{N} [K_{\beta}(t)H_i^TH_i^{-1} \Phi] \quad (50)
\]
\[
\sum_{j=1}^{N} \prod_{i=1}^{L} M_i(t+j) H_j(t+j) P^{(0)}(t+j | t+j-1) \\
\times \left( \sum_{j=1}^{N} H_j^T M_j^T(t+j) + \sum_{i=1}^{L} M_i(t+j) R_i M_j^T(t+j) \right)
\]

(45)

**Proof.** From (38), we have

\[
H^{(0)T} Q_{e}^{(0)-1}(t) = H^{(0)T} \left[ R^{(0)} + H^{(0)T} P^{(0)}(t) H^{(0)T} \right]^{-1} = P^{(0)-1}(t | t-1) P^{(0)}(t | t) H^{(0)T} R^{(0)-1}
\]

(46)

Applying (20) and (46), it follows that

\[
M^{(0)}(t | t+1) = Q^{T} P^{(0)-1}(t+1 | t) P^{(0)}(t+1 | t+1) H^{(0)T} R^{(0)-1} =
\]

\[
Q^{T} P^{(0)-1}(t+1 | t) P^{(0)}(t+1 | t+1)
\]

\[
\times \left[ H_1^T R_1^{-1} \quad H_2^T R_2^{-1} \quad \cdots \quad H_L^T R_L^{-1} \right]
\]

\[
= [M_1(t | t+1) \quad M_2(t | t+1) \quad \cdots \quad M_L(t | t+1)]
\]

(47)

with the definition of \( M_i(t | t+1) \) given by (42).

From (6), (14), (15), (21), (39) and (46), we have

\[
M^{(0)}(t | t+N) = Q^{T} \left\{ \prod_{j=1}^{N+1} \prod_{i=1}^{L} M_i(t+j) H_j(t+j) \right\}
\]

\[
\times P^{(0)-1}(t+N | t+N-1) P^{(0)}(t+N | t+N) H^{(0)T} R^{(0)-1} =
\]

\[
Q^{T} \left\{ \prod_{j=1}^{N+1} \left[ \left( I - \sum_{i=1}^{L} K_i(t+j) H_j \right) \Phi \right] \right\}
\]

\[
\times P^{(0)-1}(t+N | t+N-1) P^{(0)}(t+N | t+N)
\]

\[
\times \left[ H_1^T R_1^{-1} \quad H_2^T R_2^{-1} \quad \cdots \quad H_L^T R_L^{-1} \right]
\]

\[
= [M_1(t | t+N) \quad M_2(t | t+N) \quad \cdots \quad M_L(t | t+N)]
\]

(48)

with the definition of \( M_i(t | t+N) \) given by (43).

From (19), (47) and (48), we can obtain (41). Applying (6), (8), (12), (47) and (48) yields (45). The proof is completed.

**Remark 3.** For the multisensor system (1) and (2), applying the standard Kalman filter [12] with the initial time \( t_0 = 0 \), we can obtain the local white noise estimators \( \hat{w}_i(t | t+3) \) with the form similar to these in (18)–(23).

**Remark 4.** The distributed state fusion white noise deconvolution estimators given by Theorem 2 are derived from the centralized fusion white noise deconvolution estimators so that they are identical to the centralized fusers, i.e. they have the global optimality.

IV. SIMULATION EXAMPLE

In oil seismic exploration [1-3], the seismic waves are reflected in different geological layers. The oil exploration is performed via the reflection coefficient sequence. The reflection coefficient sequence contains the important information for finding and discovering the oil field and determining its geometry shape, which can be described by Bernoulli-Gaussian white noise [1-3]. Therefore, estimating the Bernoulli-Gaussian input white noise becomes a key technical problem for oil seismic exploration. But the proposed white noise fuser is not limited to handle the Gaussian signals.

Consider the multisensor linear discrete system with white measurement noises and 3 sensors

\[
x(t+1) = \Phi x(t) + \Gamma w(t) \quad \text{(49)}
\]

\[
y_i(t) = H_i x(t) + v_i(t), \quad i = 1, 2, 3 \quad \text{(50)}
\]

where \( x(t) \in R^3 \) is the state, \( y(t) \in R \) is the measurement, \( v_i(t) \in R \) is a white measurement noise with zero mean and variance \( R_i \). \( w(t) = b(t)g(t) \) is a Bernoulli-Gaussian white noise, where \( b(t) \) is the Bernoulli white noise with the value as 1 or 0 and the probability \( P(b(t) = 1) = \lambda \), \( P(b(t) = 0) = 1-\lambda \), and \( g(t) \) is a Gaussian white noise with zero mean and variance \( Q_e(t) \) and independent of \( b(t) \). Obviously, there is the relation \( Q = \lambda Q_e \). The objectives are to find local and the globally optimal distributed state fusion white noise deconvolution smoothers \( \hat{w}_i(t | t+3) \), \( i = 1, 2, 3 \) and \( \hat{w}_i^{(0)}(t | t+3) \) for the input white noise based on the measurements \( y(t+3), y(t+2), \cdots, y(1) \).

In simulation, we take

\[
\Phi = \begin{bmatrix} 0.1 & 0.25 \\ 0.1 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0.15 \\ 0.25 \end{bmatrix}, \quad H_1 = [1 \quad 0],
\]

\[
H_2 = [0 \quad 1], \quad H_3 = [1 \quad 1],
\]

\[
\lambda = 0.32, \quad Q_e = 1,
\]

\[
R_1 = 0.0015, \quad R_2 = 0.0025, \quad R_3 = 0.0035
\]

(51)

The simulation results are shown in Fig.1, Fig. 2, Table 1 and Table 2. Fig. 1 gives the input noise \( w(t) \) and the local and distributed fusion white noise smoothers, respectively, where the vertical coordinates ate the endpoints of solid lines denote the true values, and the vertical coordinates of the dots denote the estimates. Fig. 2 shows the comparison of curves of the accumulated error squares for the local and distributed state fusion white noise smoothers. Table 1 gives the comparison of values for the centralized and distributed state fused white noise smoothers. The comparison of variances for the local, centralized and distributed state fusion white noise smoothers is given in Table 2. All of the above results show that the accuracy of the distributed state fusion white noise smoother is higher than that of the local white noise smoothers and the distributed state fusion white noise deconvolution estimator is numerically identical to the centralized fusion white noise deconvolution estimator, i.e. it has the global optimality.
(a) $w(t)$ and $\hat{w}_1(t \mid t+3)$

(b) $w(t)$ and $\hat{w}_2(t \mid t+3)$

(c) $w(t)$ and $\hat{w}_3(t \mid t+3)$

(d) $w(t)$ and $\hat{w}^{(0)}(t \mid t+3)$

Figure 1. $w(t)$ and local and optimal distributed state fusion white noise deconvolution smoothers

Figure 2. The comparison of curves of the accumulated error squares for the local and distributed state fusion white noise smoothers

TABLE I. COMPARISON OF VALUES FOR THE CENTRALIZED AND DISTRIBUTED STATE FUSION WHITE NOISE SMOOTHERS

<table>
<thead>
<tr>
<th>$t$</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{w}_{1}^{(0)}(t \mid t+3)$</td>
<td>-0.038842</td>
<td>0.092907</td>
<td>-0.105030</td>
<td>0.066221</td>
</tr>
<tr>
<td>$\hat{w}_{2}^{(0)}(t \mid t+3)$</td>
<td>-0.038842</td>
<td>0.092907</td>
<td>-0.105030</td>
<td>0.066221</td>
</tr>
</tbody>
</table>

a. Subscript D denotes the Distributed fuser
b. Subscript C denotes the centralized fuser

TABLE II. COMPARISON OF VARIANCES FOR THE LOCAL, CENTRALIZED AND DISTRIBUTED STATE FUSION WHITE NOISE SMOOTHERS

<table>
<thead>
<tr>
<th>$t$</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}^{(0)}(t \mid t+3)$</td>
<td>0.062922</td>
<td>0.035664</td>
<td>0.021366</td>
<td>0.011525</td>
</tr>
<tr>
<td>$P_{22}^{(0)}(t \mid t+3)$</td>
<td>0.062922</td>
<td>0.035664</td>
<td>0.021366</td>
<td>0.011525</td>
</tr>
<tr>
<td>$P_{33}^{(0)}(t \mid t+3)$</td>
<td>0.062922</td>
<td>0.035664</td>
<td>0.021366</td>
<td>0.011525</td>
</tr>
</tbody>
</table>

a. Superscript $i$, $i=1,2,3$ denotes the local estimator
b. Subscript D denotes the Distributed fuser
c. Subscript C denotes the centralized fuser
V. CONCLUSION

Based on the Riccati equation, the optimal distributed state fusion white noise deconvolution estimators are presented using the Kalman filtering method. They are completely equivalent to the centralized fusion white noise deconvolution estimators, so that they have the global optimality. The proposed results can be applied to oil seismic exploration, communication, signal processing, and state estimation.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 61104209, Outstanding Youth Science Foundation of Heilongjiang University under Grant No. JCL201103 and Key Laboratory of Electronics Engineering, College of Heilongjiang Province under Grant No. DZZD2010-5. The authors wish to thank the reviewers for their constructive comments.

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