Attitude Determination of LEO Satellites Using an Array of GNSS Sensors

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Abstract—The proliferation of Global Navigation Satellite Systems (GNSS) paves the way for an increasing number of applications in positioning, guidance and navigation. GNSS-based attitude determination is one of such an important and promising application. In this contribution we explore the potential of GNSS-based attitude determination for Low Earth orbiting (LEO) formation flying Synthetic Aperture Radar (SAR) satellites.

Precise attitude determination using multiple GNSS receivers/antennas relies on a successful resolution of the integer carrier phase ambiguities. For a set of rigidly satellite-mounted GNSS antennas, a number of nonlinear geometrical constraints can be exploited for the purpose of increasing the probability of correct integer ambiguity estimation. In this contribution, we make use of the Multivariate Constrained Least-squares AMBiguity Decorrelation Adjustment (MC-LAMBDA) method. By incorporating the known antenna geometry into its ambiguity objective function, this method is shown to demonstrate reliable and instantaneous single-frequency integer ambiguity resolution. The resulting attitude estimates are further improved using nonlinear filtering. Our hardware-in-the-loop experiment with the University of New South Wales “Namuru” GNSS receivers shows the potential of stand-alone, unaided, single-frequency attitude determination of LEO satellites.

I. INTRODUCTION

In this contribution we explore Global Navigation Satellite Systems (GNSS-) based attitude determination for formation flying Synthetic Aperture Radar (SAR) satellites. This study is part of the Australian Space Research Program project (“Garada”) \textsuperscript{1}. The proposed mission will use multiple satellites in formation to collect Earth observations. For bistatic SAR each satellite carries transmitter and/or receiver antennas. Hence, positioning and attitude determination is an important issue to guarantee an overlapping of transmitter receiver antenna footprints during the mission. For this purpose, each satellite will be equipped with multiple multi-GNSS “Namuru” receivers. The Namuru system is based on a reconfigurable field-programmable gate array allowing the baseband processing hardware to be completely customized \textsuperscript{2}, \textsuperscript{3}. In this work, we consider attitude determination for a Low Earth Orbiting (LEO) satellite using an array of Namuru receivers mounted on it.

Multiple GNSS receivers/antennas rigidly mounted on a platform can be used to determine platform orientation (attitude), see e.g., \textsuperscript{4}–\textsuperscript{10}. GNSS-based attitude determination is attractive, because it is driftless compared with traditional approaches. However, one has to resolve the integer carrier phase ambiguities in order to profit from the high precision of the carrier phase observables. The Least squares AMBiguity Decorrelation Adjustment (LAMBDA) method \textsuperscript{11} is currently the standard method for solving unconstrained GNSS ambiguity resolution problems, see, e.g., \textsuperscript{12}–\textsuperscript{18}. This method is computationally efficient and it has proven to be optimal in the sense that it provides integer ambiguity solutions with the highest possible success-rates for unconstrained and linearly constrained GNSS models \textsuperscript{13}, \textsuperscript{19}, \textsuperscript{20}. However, this method may not yield an optimal solution in case of nonlinearly constrained GNSS models. Therefore, to exploit the known nonlinear antenna geometry in the local body frame, we make use of the Multivariate Constrained (MC-) LAMBDA method \textsuperscript{21}–\textsuperscript{27}.

We consider attitude determination with ambiguity resolution for the challenging single-epoch, single-frequency case. Single-epoch ambiguity resolution has the advantage that it is immune to cycle-slips and satellite changes, the latter of which is common in fast moving receivers such as receivers on a LEO satellite. To improve the attitude estimation accuracy, the epoch-by-epoch estimates from the MC-LAMBDA method are further filtered using an Unscented Kalman Filter (UKF), which uses a sampling approach to propagate the uncertainty of the attitude state vector \textsuperscript{28}, \textsuperscript{29}. The proposed approach is tested using data collected in a hardware-in-the-loop experiment with a LEO satellite scenario. The results confirm the potential of GNSS-only attitude determination for LEO satellites.

II. GNSS-BASED ATTITUDE DETERMINATION

This section describes epoch-by-epoch GNSS-only attitude determination using multiple GNSS-antennas/receivers and their known body-frame array geometry. Let us consider a set of \( r + 1 \) antennas/receivers firmly mounted on a platform of interest and simultaneously tracking \( m + 1 \) GNSS satellites. The set of linearized Double Difference (DD) GNSS phase and code observations obtained on the \( r \) baselines formed by these antennas at an observation epoch forms a multivariate Gauss-Markov model \textsuperscript{22}:

\[
E(Y) = AZ + GB \\
D(\text{vec}(Y)) = Q_{yy} = P \otimes Q_{yy} \tag{1}
\]

where \( E(\cdot) \) and \( D(\cdot) \) denote the expectation and dispersion operator, \( \otimes \) denotes the Kronecker product, \( Y = [y_1, \ldots, y_r] \) is
the $2m \times r$ matrix of $r$ linearized (observed-minus-computed) DD observation vectors, $y_i = [p_i^T \phi_i^T]^T$ consists of DD code and phase observations, $Z = [z_1, \ldots, z_r]$ is the $m \times r$ matrix of $r$ unknown DD integer ambiguity vectors $z_i$, $B = [b_1, \ldots, b_r]$ the $3 \times r$ matrix of $r$ unknown baseline vectors $b_i$, $G$ is the $2m \times 3$ geometry matrix that contains the unit line-of-sight vectors, $A$ is the $2m \times m$ matrix that links the DD data to the integer ambiguities, and $P$ and $Q_{yy}$ are known matrices of order $r \times r$ and $2m \times 2m$, respectively. Here, $\text{vec}(\cdot)$ denotes the vector-operator, which transforms a matrix into a vector by stacking the columns of the matrix one underneath the other. Matrix $P$ takes care of the correlation that follows from the fact that the $r$ baselines have one antenna in common and matrix $Q_{yy}$ takes care of the precision of the phase and code data. Phase observation $\phi_i$ is more precise than code observation $p_i$, but it is ambiguous by integer cycles ($z_i$). To make use of precise phase observations, one should resolve these ambiguities. In the following, we describe an integrated method for attitude determination and ambiguity resolution with the knowledge of antenna array geometry.

The known body-frame antenna-geometry is included through the parametrization

$$B = RB_0$$

with the unknown $3 \times q$ orthogonal matrix $R$ ($R^T R = I_q$) and the known $q \times r$ matrix $B_0$ describing the known geometry of the antenna configuration in the body frame ($q$ is the dimension of the span of the $r$ baselines). The rotation matrix $R$ represents the rotation from the body frame coordinate system to the local North-East-Down (NED) coordinate system. For $q = 3$, $R$ is related to attitude (the Euler) angles for the 3-2-1 sequence of rotations as follows:

$$R = \begin{bmatrix} c_\theta c_\phi & -c_\phi s_\theta + s_\phi s_\theta c_\phi & s_\phi s_\theta c_\phi \\ c_\theta s_\phi & c_\phi c_\theta + s_\phi s_\theta s_\phi & -s_\phi c_\theta + s_\phi s_\theta s_\phi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}$$

where $s_\alpha = \sin(\alpha)$ and $c_\alpha = \cos(\alpha)$. Hence, the attitude angles are given as

$$\text{Heading} : \phi(R) = \tan^{-1}\left(\frac{R_{2,1}}{R_{1,1}}\right)$$

$$\text{Elevation} : \theta(R) = -\sin^{-1}(R_{2,1})$$

$$\text{Bank} : \psi(R) = \sin^{-1}\left(\frac{R_{3,2}}{\sqrt{R_{1,1}^2 + R_{2,1}^2}}\right)$$

Note that for $q < 3$, only first $q$ columns of $R$ are estimable. For example, for a linear antenna array ($q = 1$) only first column is estimable and hence only heading and elevation are estimable.

Substitution of (3) into (1), leads to the GNSS attitude model ([27], [30])

$$E(Y) = AZ + GRB_0 \quad Z \in \mathbb{Z}^{m \times r}, R \in \mathbb{O}^{3 \times q}$$

$$D(\text{vec}(Y)) = Q_{YY} = P \otimes Q_{yy}$$

Our objective is to solve for the attitude matrix $R$ in a least-squares sense, thereby taking the integer constraints on matrix $Z$ and the orthonormality constraints on matrix $R$ into account. Hence, the minimization problem that will be solved reads

$$\min_{Z \in \mathbb{Z}^{m \times r}, R \in \mathbb{O}^{3 \times q}} \| \text{vec}(Y - AZ - GRB_0) \|^2_{Q_{YY}}$$

with $\| \cdot \|_2 = (\cdot)^T Q_2^{-1}(\cdot)$. The above problem does not admit a closed-form solution. In the following, we describe a three-step procedure for solving (10).

### A. Float solutions:

The float solution is defined as the solution of (10) without the constraints. When we ignore the integer constraints on $Z$ and the orthonormality constraints on $R$, the so-called float solutions $\hat{Z}$ and $\hat{R}$, and their variance-covariance matrices are obtained as follows:

$$N \cdot \begin{bmatrix} \text{vec}(\hat{Z}) \\ \text{vec}(\hat{R}) \end{bmatrix} = \begin{bmatrix} I_s \otimes A^T \\ B_0 \otimes G^T \end{bmatrix} Q_{YY}^{-1} \text{vec}(Y)$$

and

$$Q_{ZZ} = Q_{\hat{R}Z}$$

The $Z$-constrained solution of $R$ and its variance-covariance matrix can be obtained from the float solution as follows

$$\text{vec} \left( \hat{R}(Z) \right) = \text{vec}(\hat{R}) - Q_{\hat{R}Z} Q_{ZZ}^{-1} \text{vec} \left( \hat{Z} - Z \right)$$

$$Q_{\hat{R}(Z)\hat{R}(Z)} = Q_{\hat{R}R} - Q_{\hat{R}Z} Q_{ZZ} Q_{\hat{Z}Z} Q_{Z\hat{R}}$$

Using the above estimators, the original problem in (10) can be decomposed as

$$\min_{Z \in \mathbb{Z}^{m \times r}, R \in \mathbb{O}^{3 \times q}} \| \text{vec}(Y - AZ - GRB_0) \|^2_{Q_{YY}}$$

$$= \| \text{vec}(\hat{E}) \|^2_{Q_{YY}} + \min_{Z \in \mathbb{Z}^{m \times r}} \| \text{vec}(\hat{Z} - Z) \|^2_{Q_{ZZ}}$$

$$+ \min_{R \in \mathbb{O}^{3 \times q}} \| \text{vec} \left( \hat{R}(Z) - R \right) \|^2_{Q_{\hat{R}(Z)\hat{R}(Z)}}$$

with $\hat{E}$ the matrix of least-squares residuals.

### B. Ambiguity resolution:

In this step, the matrix of integer ambiguities is determined. Based on the orthogonal decomposition in (16), the multivariate constrained integer minimization can be formulated as:

$$\hat{Z} = \arg \min_{Z \in \mathbb{Z}^{m \times r}} C(Z)$$

where

$$C(Z) = \| \text{vec}(\hat{Z} - Z) \|^2_{Q_{ZZ}}$$

$$+ \| \text{vec} \left( \hat{R}(Z) - R \right) \|^2_{Q_{\hat{R}(Z)\hat{R}(Z)}}$$
TABLE I
ORBIT SPECIFICATION FOR THE SIMULATED LEO SATELLITES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi major axis</td>
<td>7,058.14 km</td>
</tr>
<tr>
<td>Inclination</td>
<td>98.0443°</td>
</tr>
<tr>
<td>Right ascension</td>
<td>-90.046°</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0°</td>
</tr>
<tr>
<td>Mean anomaly</td>
<td>0°</td>
</tr>
<tr>
<td>Argument of perigee</td>
<td>0°</td>
</tr>
</tbody>
</table>

The cost function $C(Z)$ is the sum of two coupled terms: the first weighs the distance from the float ambiguity matrix $\tilde{Z}$ to the nearest integer matrix $Z$ in the metric of $Q_{\tilde{Z}Z}$, while the second weighs the distance from the conditional float solution $\tilde{R}(\tilde{Z})$ to the nearest orthonormal matrix $R$ in the metric of $Q_{\tilde{R}(\tilde{Z})\tilde{R}(\tilde{Z})}$. Unlike with the standard LAMBDA method, the search space of the above optimization is non-ellipsoidal due to the rigorous application of the orthonormal constraints. Moreover, its solution requires the computation of a nonlinear constrained least-squares problem (19) for every integer matrix in the search space. In the MC-LAMBDA method, this problem is mitigated through the use of easy-to-evaluate bounding functions [31]. Using these bounding functions, two strategies, namely the Expansion and the Search and Shrink strategies, were developed, see e.g. [21], [25]. These techniques avoid the computation of (19) for every integer matrix in the search space, and compute the integer minimizer $\tilde{Z}$ efficiently.

C. Fixed solution:

In this final step, we obtain the attitude solution by substituting $\tilde{Z}$ into (14), thus giving $\tilde{R}(\tilde{Z})$. This solution has a much better accuracy than $\tilde{R}$ (cf. 15), but it is, in general, still non-orthogonal. The orthogonal attitude matrix $\hat{R}(\hat{Z})$ is then estimated by solving the nonlinear least-squares problem (19) for $Z = \hat{Z}$. The sought-for attitude angles $[\phi(\hat{R}(\hat{Z})) \ \theta(\hat{R}(\hat{Z})) \ \psi(\hat{R}(\hat{Z}))]^{T}$ are then finally obtained from (5) - (7) based on $R = \hat{R}(\hat{Z})$.

III. HARDWARE-IN-THE-LOOP EXPERIMENT

This section describes the hardware-in-the-loop experiment with the Namuru receiver conducted at UNSW and it presents the ambiguity resolution results using both the MC-LAMBDA and standard LAMBDA method. Furthermore, recursive filtering is discussed in Section III-A. In the experiment, the NamuruV2Rx receiver was connected to a Spirent GSS6560 simulator tracking GPS signals. The simulated scenario is based on orbital parameters of a LEO satellite as shown in Table I. Figure 1 shows the true trajectory of the satellite. This nadir looking satellite is equipped with three antennas/receivers forming the following antenna geometry

$$B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{[m]}$$

(20)
kinematic equations can be reduced to the following model

For a nadir looking satellite (i.e., for small

Assuming constant angular rates, the discrete time attitude
generated using a UKF filter. For the 3-2-1 sequence

A. Recursive Filtering

As shown, the use of the known antenna geometry significantly
enhances the integer resolution capability.

A. Recursive Filtering

The epoch-by-epoch MC-LAMBDA attitude solution is
further processed using a UKF filter. For the 3-2-1 sequence
of rotations, the kinematic equations of the attitude angles are
given as [32]

\[
\begin{align*}
\dot{\phi} &= \left(\omega_3 \cos(\psi) + \omega_2 \sin(\psi)\right) \sec(\theta) \\
\dot{\theta} &= \left(\omega_2 \cos(\psi) - \omega_3 \sin(\psi)\right) \\
\dot{\psi} &= \omega_1 + \left(\omega_1 \cos(\psi) + \omega_2 \sin(\psi)\right) \tan(\theta)
\end{align*}
\]

where \(\omega_i\) is the angular velocity about the \(i\)th rotation axis. For a nadir looking satellite (i.e., for small \(\theta\) and \(\psi\)), the above kinematic equations can be reduced to the following model

\[
\begin{align*}
\dot{\phi} &= \omega_3 \\
\dot{\theta} &= \omega_2 \\
\dot{\psi} &= \omega_1
\end{align*}
\]

Assuming constant angular rates, the discrete time attitude
kinematic model is then given as [33]

\[
\xi_k = F\xi_{k-1} + v_{k-1}
\]

where the state vector \(\xi_k \equiv \left[\phi_k \ \dot{\phi}_k \ \theta_k \ \dot{\theta}_k \ \psi_k \ \dot{\psi}_k\right]^T\) consists
of attitude angles and angular rates, and the state transition
matrix \(F\) is given as

\[
F = I_3 \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}
\]

with \(T\) the sampling interval. The process noise \(\nu_{k-1}\) has a
zero mean normal distribution with variance-covariance matrix
\(Q_{v\nu,k-1}\), which is given as

\[
Q_{v\nu,k-1} = \begin{bmatrix} \sigma^2_\phi & 0 & 0 \\ 0 & \sigma^2_\theta & 0 \\ 0 & 0 & \sigma^2_\psi \end{bmatrix} \otimes \begin{bmatrix} T^3/3 \\ T^2/2 \\ T \end{bmatrix}
\]

Full GPS constellation with L1 frequency was simulated for
about eight hours resulting in 27962 epochs of data. Figure 2
shows the satellites availability (tracked by all three receivers)
for a cutoff elevation angle of \(0^\circ\). Intermittent poor satellite
geometry (high PDOP value) can be observed that may be due
to changing satellite visibility for fast moving receivers and
poor satellite availability at regions close to polar (Figure 1(c)).

Table II reports the single-frequency, single-epoch success rate
of MC-LAMBDA compared with that of standard LAMBDA.
As shown, the use of the known antenna geometry significantly
enhances the integer resolution capability.

<table>
<thead>
<tr>
<th></th>
<th>LAMBDA</th>
<th>MC-LAMBDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success (%)</td>
<td>19.0</td>
<td>96.4</td>
</tr>
</tbody>
</table>

Table II

SINGLE EPOCH INTEGER RESOLUTION SUCCESS RATE

for a nadir looking satellite and satellite availability for a elevation cutoff angle of \(0^\circ\). Table II reports the single-frequency, single-epoch success rate
of MC-LAMBDA compared with that of standard LAMBDA. As shown, the use of the known antenna geometry significantly
to changing satellite visibility for fast moving receivers and
to regions close to polar (Figure 1(c)).

As to the observations, we may choose among three
options based on the final epoch-by-epoch estimates from
Section II-C: the first option is to use the angular estimates
\(\left[\phi \right(\bar{R}(\bar{Z}) \right) \theta \left(\bar{R}(\bar{Z}) \right) \psi \left(\bar{R}(\bar{Z}) \right)]^T\), the second option is to use
the orthonormalized rotation matrix \(\bar{R}(\bar{Z})\), and the third option is to use the fixed solution \(\bar{R}(\bar{Z})\). For the first option (linear observation model), the standard Kalman filter can be used, while the other two options require nonlinear filtering. The first
two options, however, involve a nonlinear optimization in (19)
and nonlinear transformations in (5) - (7). This would therefore
require a first-order approximation to obtain the variance-
covariance matrix. To avoid such approximation, we choose
the third option, for which the observational model is defined as

\[
\zeta_k = h(\xi_k) + w_k
\]

with \(\zeta_k\) given by \(\text{vec}\left(\bar{R}(\bar{Z})\right)\) at epoch \(k\). The nonlinear
observational function \(h(\xi_k)\) is defined by (4), and the observation
noise \(w_k\) is assumed to have a zero mean normal distribution
with covariance matrix \(Q_{w\nu,k}\), which is given by \(Q_{\bar{R}(\bar{Z})\bar{R}(\bar{Z})}\)
at epoch \(k\).

The recursive filter is initialized using attitude angular
solutions from the first two epochs (two point initialization [33])
giving the initial state \(\xi_{0|0}\) and its variance-covariance matrix
\(Q_{0|0}\). Based on the above dynamic and observational models,
the recursive filter proceeds as follows. In the time update step, the predicted state and its variance-covariance matrix at epoch
\(k\) are given as

\[
\begin{align*}
\xi_{k|k-1} &= F\xi_{k-1|k-1} \\
Q_{k|k-1} &= FQ_{k-1|k-1}F^T + Q_{\nu\nu,k-1}
\end{align*}
\]

In the measurement update step with the nonlinear observa-
tional model, \(2n + 1\) deterministic samples (sigma points),
representing a normal distribution with mean \(\xi_{k|k-1}\) and
variance-covariance matrix \(Q_{k|k-1}\), are generated as, [28],
[29],
\[ \xi^i \] = \xi_{k|k-1} + \sqrt{(n + \kappa)Q_{\xi,k|k-1}} \]  
\[ \omega^i \] = \frac{\kappa}{2(n+\kappa)} 
\[ \xi^{i+n} \] = \xi_{k|k-1} - \sqrt{(n + \kappa)Q_{\xi,k|k-1}} \]  
\[ \omega^{i+n} \] = \frac{\kappa}{2(n+\kappa)} \]  
(33)

where \( n \) is the dimension of \( \xi_{k|k-1} \), \( \kappa \in \mathbb{R} \), 
\( (\sqrt{(n + \kappa)Q_{\xi,k|k-1}}) \) is the \( i \)th row or column of the 
matrix square root of \((n + \kappa)Q_{\xi,k|k-1}\), \( \omega^i \) is the weight, which 
is associated with the \( i \)th sigma point. In this work, we 
take tuning parameter \( \kappa = 0.5 \). These sigma points are then 
propagated using the nonlinear observation model
\[ \zeta^i = h(\xi^i), \quad i = 0, \ldots, 2n \]  
(34)

Based on these transformed sigma points, the predicted ob-
servational vector and its variance-covariance matrices are 
computed as
\[ \zeta_{k|k-1} = \sum_{i=0}^{2n} \omega_i \zeta^i \]  
(35)
\[ Q_{\zeta\zeta,k|k-1} = \sum_{i=0}^{2n} \omega_i (\zeta^i - \zeta_{k|k-1}) (\zeta^i - \zeta_{k|k-1})^T \]  
(36)
\[ Q_{\zeta\zeta,k|k-1} = \sum_{i=0}^{2n} \omega_i (\zeta^i - \zeta_{k|k-1}) (\zeta^i - \zeta_{k|k-1})^T \]  
(37)

Finally, the updated states and its variance-covariance matrix 
at epoch \( k \) are given as
\[ \xi_{k|k} = \xi_{k-1|k-1} + W_k \nu_k \]  
(38)
\[ Q_{k|k} = Q_{k|k-1} - W_k S_k W_k^T \]  
(39)

with Kalman gain matrix \( W_k = Q_{\xi\zeta,k|k-1} S_k^{-1} \), innovation 
vector \( \nu_k = \xi_k - \xi_{k|k-1} \), and innovation variance-covariance 
matrix \( S_k = Q_{\zeta\zeta,k|k-1} + Q_{\zeta\zeta,k}\). 

The attitude angular estimates obtained from the MC-
LAMBDA epoch-by-epoch processing and the UKF filtering 
are compared with the truth values obtained from the Spirent 
simulator. Figures 3, 4, and 5 show the estimates for the 
attitude angles as a function of simulation time. The red and 
blue lines correspond to the epoch-by-epoch MC-LAMBDA 
attitude solution and the filtered attitude solution, respectively, 
while the true values from the Spirent simulator are given in 
black. The angular error is defined as the estimate-minus-true 
value. It is observed that the spikes in the attitude angular 
errors and the failed epochs in the epoch-by-epoch solution 
correspond to the poor GPS satellite availabilities (high PDOP 
values). Figure 6 shows the effect of GPS satellite geometry 
(PDOP value) on the epoch-by-epoch integer ambiguity reso-
lution and on the angular accuracy. For a small PDOP value 
(good GPS satellite geometry), we have a very low failure 
fraction and also a small angular error. A large PDOP value, 
which occurs less frequently, results in a high failure fraction 
and in a large angular error, and hence affects the overall 
performance. Table III reports the mean values of the angular 
errors, while Table IV reports the root mean squared errors 
(RMSEs) for the estimates. The heading angle is estimated 
with highest precision. Filtering significantly improves the 
accuracy of the elevation and bank estimates. This is due to 
the use of the a priori knowledge of the attitude dynamics.

**IV. Conclusions**

In this contribution we studied GNSS-based attitude deter-
mination of LEO satellites in the context of the Australian 
Space Research Program Garada project. The Multivariate 
Constrained (MC-)LAMBDA method was used for epoch-
by-epoch integer ambiguity resolution. This method exploits 
the a priori knowledge of the antenna geometry and hence 
significantly improves the capacity of correctly fixing the 
integer ambiguities. We demonstrated the effectiveness of the 
MC-LAMBDA method and the nonlinear UKF filtering using a 
hardware-in-the-loop experiment with Namuru receiver. This
Fig. 4. Estimated elevation angle and estimation error (estimate-minus-true value)

(a) Estimate

(b) Error

Fig. 5. Estimated bank angle and estimation error (estimate-minus-true value)

(a) Estimate

(b) Error

the unconstrained LAMBDA method clearly demonstrated the effectiveness of being able to rigorously include the nonlinear constraints of the antenna geometry. Furthermore, we used nonlinear filtering to improve the attitude angular accuracy. Since the Garada mission is proposed to use multi-GNSS, multi-frequency Namuru receivers, further improved performance, compared with the current, single-frequency GPS-only study, can be expected. The Namuru receiver used in this study is a prototype of a space-borne receiver. Hence both hardware and software are suitable for attitude determination of LEO satellites as long as they have adequate GNSS satellite visibility [34].

ACKNOWLEDGMENT

The second author P.J.G. Teunissen is the recipient of an Australian Research Council Federation Fellowship (project number FF0883188). This work is supported by the Australian Space Research Program GARADA project on SAR Formation Flying. All these supports are gratefully acknowledged. The authors also would like to thank Mr Mohammad Choudhury from the University of New South Wales for providing the data set used in this work.

REFERENCES


