Real-Time Kinematic Positioning Using Fused Data from Multiple GNSS Antennas

Bofeng Li  
GNSS Research Centre  
Department of Spatial Sciences  
Curtin University, Australia  
Email: bofeng.Li@curtin.edu.au

Peter J. G. Teunissen  
GNSS Research Centre  
Department of Spatial Sciences  
Curtin University, Australia  
Email: p.Teunissen@curtin.edu.au

Abstract—Array-aided precise point positioning (A-PPP) is a nonlinear estimation concept that uses Global Navigation Satellite System (GNSS) data, from multiple antennas of known local geometry, to realize improved integer ambiguity resolution thus enabling faster and improved positioning. In this contribution, we demonstrate the performance of A-PPP for long-baseline Real-Time Kinematic (RTK) positioning using dual-frequency GPS data collected on a 80 km baseline and we compare the results with conventional RTK. We formulate the underlying multivariate, integer and orthonormality constrained, GNSS model and show how the array-antenna data is fused to enable improved long-baseline RTK positioning.

I. INTRODUCTION

Integer ambiguity resolution (IAR) is the key to high precision GNSS applications. It transforms the ambiguous carrier phases into ultra-precise pseudoranges thus enabling very precise positioning. Of the several different ambiguity resolution methods that exist, the LAMBDA method is most popular due to its numerical efficiency and statistical optimality [1]–[4]. The method maximizes the probability of correct integer ambiguity estimation [5] and its integer search is efficient [6], [7].

The success of IAR depends on the strength of the underlying GNSS model. The weaker the model, the more data need to be accumulated before IAR can be successful and then the longer time it therefore takes before one can profit from the ultra-precise carrier signals. Clearly, the aim is to have short time-to-convergence, preferably zero, thereby enabling truly instantaneous GNSS positioning.

The presence of unknown biases makes the GNSS model weaker, as is the case with long baselines. The ionospheric effects are the dominant systematic errors in long-baseline positioning.

Although traditionally only one antenna is used per baseline station, the array-aided precise point positioning (A-PPP) concept has the potential of improving the situation significantly. A-PPP, proposed by [8], is a measurement concept that uses GNSS data, from multiple antennas on a platform, to realize fused data with improved precision so as to perform improved GNSS parameter estimation. This measurement concept is also applicable to other GNSS models, such as those of GNSS attitude determination and formation flying [9] and CORS (Continuously Operating Reference Station) network ambiguity resolution [10].

Fig. 1 shows two A-PPP equipped platform stations, \( b_0 \) and \( u_0 \). In A-PPP, multiple antennas with known antenna geometry, \( b_1, \ldots, b_r \) and \( u_1, \ldots, u_r \), are mounted on the platforms. The known antenna-array geometry allows one to fuse the observations from all array antennas to a set of observations that can be understood as coming from one single virtual antenna. Since the fused observations can be seen as being the result of a least-squares adjustment, their precision is better than that of the original observations from the individual antennas. Therefore improved IAR and positioning for the \( b_0 - u_0 \) baseline can be expected based on these fused observations.

In this paper, we explore the potential benefits of A-PPP for long-baseline real-time kinematic (RTK) positioning. An 80 km baseline experiment was conducted for which both stations were equipped with a 4-antenna array platform. The newly formed model for observation fusion with multiple antennas on platform was solved using the principle of multivariate mixed integer least squares estimation. Then the fused data was processed to demonstrate the A-PPP RTK (ARTK) performance in comparison with conventional RTK (CRTK) which uses 1-antenna equipped baseline stations. The rest of this paper is organized as follows. In Sect. II, we formulate the antenna-antenna observation equations as a multivariate model. In Sect. III, we show how the observations from multiple antennas are fused and used in the long-baseline RTK model. The 80 km baseline experiment and its results are presented in Sect. IV and the research findings are summarized in Sect. V.

In the following, \( I_n \) denotes the unit matrix of order \( n \) and \( e_n \) the \( n \)-column vector with 1s. \( e_1 = [1, 0, \ldots, 0]^T \) is a unit vector with its 1 in the first slot. The Kronecker product and vec-operator are denoted as \( \otimes \) and vec. For the Kronecker product, the following properties \((AB) \otimes (CD) = (A \otimes C)(B \otimes D)\) and vec\((ABC) = (C^T \otimes A)\text{vec}(B)\) will be frequently applied.

II. MULTIVARIATE ANTENNA-ARRAY GNSS MODEL

A. The multivariate observation equations

For a total of \( s \) simultaneously tracked satellites, the single-epoch, single-frequency undifferenced (UD) observation equa-
This matrix transforms single-frequency UD observations into between-satellite single-differences (SD). Hence, pre-multiplying (2) with \((I_{2f} \otimes D_s^T)\) yields

\[
E(y_{fr}) = M_s x_r + e_{2f} \otimes x_r + e_{fr} + N \tilde{\alpha}_r - \theta_r
\]

where \(y_{fr} = [y_{fr,1}, \ldots, y_{fr,f}]^T\) with \(y_{fr} = (I_f \otimes DT)\phi_r = [y_{\phi_{fr,1}}, \ldots, y_{\phi_{fr,f}}]^T\) and \(y_{fr} = D_s^T \phi_{fr}\), having a similar structure; \(\tilde{\tau}_r = D_s^T \tau_r\), \(\tilde{\alpha}_r = [(\alpha_{fr}^1)^T, \ldots, (\alpha_{fr}^f)^T]\) and \(\theta_r = [\delta \tilde{t}_r, (I_f \otimes D_s), dt_r(I_f \otimes D_s)]^T\); \(M_r = e_{fr} \otimes (D_s^T G_r)\), \(N = [\Lambda, 0]^T \otimes I_{s-1}\).

Since the distance between the antennas on the platform is about 1 m in our case (see also Fig. 2), the between-satellite SD atmospheric delays can be considered the same for different antennas. Also the same receiver-satellite geometry matrix can then be used for each antenna on the platform. Hence, \(M_i = M, \tilde{\tau}_i = \tilde{\tau}, \tilde{e}_i = \tilde{e}_r\). The multivariate set of SD observation equations for the \(r\) antennas reads therefore

\[
E(Y) = MX + e_{2f} e_r^T \tilde{\tau} + \nu \tilde{e}_r + N \tilde{\alpha} - \Theta
\]

where \(Y = [y_{1r}, \ldots, y_{nr}]\), \(X = [x_1, \ldots, x_r]\), \(\Theta = [\theta_1, \ldots, \theta_r]\) and \(\tilde{\alpha} = [\tilde{\alpha}_1, \ldots, \tilde{\alpha}_r]\).

If we post-multiply the above system (5) with the invertible matrix \(R_r = [c_1, D_r]\), \(D_r = [-c_{r-1}, I_{r-1}]\), we obtain

\[
E\left[\begin{array}{c}
{y_r} \\
\tilde{Y}
\end{array}\right] = \left[\begin{array}{c}
M X + N Z \\
M \tilde{X} + N Z
\end{array}\right]
\]

where \([y_{1r}, \tilde{Y}] = Y R_r = [Y_{r1}, Y D_r]\) and \(\tilde{Y} = [y_{1r}, \ldots, y_{nr}]\) is the transformed double difference (DD) observation matrix. \(\tilde{X} = XD_r = [x_{r1}, \ldots, x_{rr}]\) is the relative coordinate (baseline) matrix. \(Z = AD_r = [z_{r1}, \ldots, z_{rr}]\) is the DD ambiguity matrix of which all entries are integer.

**B. Multivariate stochastic model**

In this subsection, we describe the stochastic model of (6). We start with the stochastic model of the single-epoch, single-frequency UD observational vector (1),

\[
D\begin{bmatrix} \phi_{fr} \\
\alpha_{fr} \\
\end{bmatrix} = \begin{bmatrix} 0 \\
\sigma_{\phi_{fr}}^2 \\
\end{bmatrix} \otimes Q_0
\]

where \(Q_0\) is the elevation-dependent cofactor matrix of the single-epoch, single-frequency UD observations, and \(\sigma_{\phi_{fr}}^2\) are the variances of the UD phase and code on frequency \(j\), respectively. Here the cross correlation between phase and code is assumed nonexistent. The stochastic model for the SD model (4) follows then as

\[
D(y_{fr}) = Q_{yr} = S \otimes D_s^T Q_0 D_s
\]

where \(S = \text{blkdiag}(S_\phi, \Sigma_{\phi,fr}), S_\phi = \text{diag}(\sigma_{\phi_{fr}}^2, \ldots, \sigma_{\phi_{fr}}^2)\) and \(\Sigma_{\phi,fr} = \text{diag}(\sigma_{\phi_{fr}}^2, \ldots, \sigma_{\phi_{fr}}^2)\). If we assume to have the same type of antennas on the platform, then \(Q_{yr} = Q_y f = I_{1}, \ldots, r\) (in [8] the model is given for the case the antennas have a different quality). Then the stochastic model of (5)
follows as

\[ D(\text{vec}(Y)) = I_r \otimes Q_y = I_r \otimes (S \otimes D^2_{\tau} Q_0 D_s) \]  (9)

Finally, the stochastic model of the multivariate model (6) follows then as

\[ D \begin{bmatrix} y_1 \\
\text{vec}(Y) \end{bmatrix} = R^T_{y,r} R_r \otimes Q_y = \begin{bmatrix} 1 - c_1^T D_r (D^T_r D_r)^{-1} \\
0 
\end{bmatrix} \otimes I_{2f(s-1)} \]  (10)

Note, although the two equations in (6) have no parameters in common, their data are correlated, i.e., (10).

III. LONG BASELINE, IONOSPHERE-WEIGHTED MODEL USING FUSED ANTENNA-ARRAY DATA

A. Data fusion from array antennas

From (10) it follows that \( y_1 \) and \( \bar{Y} \) of (6) are correlated. As shown in the A-PPP concept of [8], application of the one-to-one decorrelation matrix

\[ \begin{bmatrix} \bar{y} \\
\text{vec}(\bar{Y}) \end{bmatrix} = \begin{bmatrix} 1 - \gamma \end{bmatrix} \begin{bmatrix} 1 \\
0 \end{bmatrix} \otimes I_{2f(s-1)} \]

to the observation equations of (6), results in an equivalent but decorrelated system of observation equations, namely

\[ E \begin{bmatrix} \bar{y} \\
\text{vec}(\bar{Y}) \end{bmatrix} = \begin{bmatrix} \bar{X} + e_2f \otimes \tau + \nu \otimes \tilde{\iota} + N(\bar{a}_1 + \bar{\varepsilon}) - \theta_1 \\
\bar{M} \bar{X} + NZ \end{bmatrix} \]  (11)

where \( \bar{y} = \sum_{i=1}^r y_i/r \), \( \bar{x} = \sum_{i=1}^r x_i/r \) and \( \bar{\varepsilon} = \sum_{i=2}^r z_{1i}/r \). The dispersion of \( \bar{y} \) and \( \bar{Y} \) is given as

\[ D \begin{bmatrix} \bar{y} \\
\text{vec}(\bar{Y}) \end{bmatrix} = \begin{bmatrix} 1/r \\
0 \end{bmatrix} \begin{bmatrix} 1 \\
0 \end{bmatrix} \otimes D^T_r D_r \otimes Q_y \]  (12)

Note that \( \bar{y} \) is \( r \)-times more precise than \( y_1 \). Furthermore, as is shown in [8], the given geometry of the antenna configuration allows for the integer matrix estimator \( \hat{Z} \) of \( Z \) to be determined with a very high success-rate from the second equation of (11). Hence, for all practical purposes one may assume \( Z \) known and then also \( \varepsilon = \frac{1}{r} \bar{Z} e_{r-1} \) known in the first equation of (11). The known geometry of the array antennas on the platform also allows one to express \( \bar{x} \) as function of the coordinate vector \( x_r \) of an arbitrary antenna. Since in our case the array antennas are mounted symmetrically on the platform, the barycentre \( \bar{x} = x_0 \) corresponds to the station point that we would like to solve for. Therefore the fused observation equations of a platform-equipped station can now be written as

\[ E(\bar{y} - N\bar{\varepsilon}) = \bar{M} x_0 + e_2f \otimes \tau + \nu \otimes \tilde{\iota} + N\bar{a}_1 - \theta_1 \]  (13)

B. Ionosphere-weighted RTK model between-platform based on the fused data

For two platform-equipped stations, say \( b \) and \( u \), the between-platform model of observation equations can be written as

\[ E(\bar{y}_{bu}) = \bar{M} x_{bu} + e_2f \otimes \bar{\tau}_{bu} + \nu \otimes \bar{\iota}_{bu} + Nz_{bu} \]  (14)

where \( \bar{y}_{bu} = \bar{y}_u - \bar{y}_b - N(\bar{z}_u - \bar{z}_b) \), \( x_{bu} = x_u - x_b \) and \( z_{bu} = \bar{a}_{1,u} - \bar{a}_{1,b} \) is the between-platform DD integer ambiguity vector. The variance matrix is given as

\[ D(\bar{y}_{bu}) = \frac{1}{r} (Q_{y_b} + Q_{y_u}) \]  (15)

where \( Q_{y_b} \) and \( Q_{y_u} \) are the variance matrices of the original data of platforms \( b \) and \( u \), respectively. This shows how between-platform relative positioning can benefit from the antenna-array and in particular from \( r \), the number of antennas in the array. The tropospheric delay is modelled by parameterizing it as function of the relative zenith tropospheric delay \( \tau^z \) and mapping function \( m_{bu} \), i.e., \( \tilde{\tau}_{bu} = m_{bu}(\tau^z) \).

The ionospheric delays are modeled by estimating all DD ionospheric delays as parameters with stochastic constraints. With (14), the long-baseline model becomes therefore

\[ E(\bar{y}_{bu}) = M x_{bu} + (c_2f \otimes m_{bu}) \tau^z + \nu \otimes \bar{\iota}_{bu} + Nz_{bu} \]
\[ E(\bar{\iota}_{bu}) = \bar{\iota}_{bu}^0, \text{ with variance } D(\bar{\iota}_{bu}) = \sigma_{\bar{\iota}_{bu}}^2 \]  (16)

This model is referred to as the ionosphere-weighted model. Usually \( \bar{\iota}_{bu}^0 = 0 \) is taken as sample value for the a-priori DD ionospheric delays. The ambiguity vector \( \bar{z}_{bu} \) is assumed constant in time and \( \tau^z \) can usually also be considered constant for certain time spans.

C. Implementation steps of observation reduction

We summarize the steps that need to be followed for fusing the antenna-array data.

1) Collect all the single-epoch, undifferenced phase and code observations, \( \phi^{\phi}_{r,j} \) and \( p^{s}_{r,j} \), on all frequencies of all antennas in an array.

2) Formulate the multivariate DD observation equations

\[ E(\bar{Y}) = \bar{M} \bar{X} + NZ \]  (17)

and resolve for the integer matrix \( Z \). Since it is crucial to estimate \( Z \) with a sufficiently high success-rate (also in the single-frequency case), special precautions need to be taken. High success-rates can be achieved when the known array geometry is used. If this known geometry is in the body-frame, then the MC-LAMBDA method can be used to achieve high success-rates, see e.g. [8], [11]. In case \( \bar{X} \) is completely known, then even the standard LAMBDA method can be used to integer estimate \( Z \) with high success-rate, see e.g. [6]. In that case one may even opt for the easier-to-apply integer bootstrapping or integer rounding methods, provided they are applied to the decorrelated ambiguities.

3) With the integer DD ambiguity solution \( \hat{Z} = [\hat{z}_{12}, \cdots, \hat{z}_{1r}] \), the UD phase \( \hat{\phi}^{\phi}_{r,j} \) and code \( \hat{p}^{s}_{r,j} \) observations can now be fused as

\[ \hat{\phi}^{\phi}_{r,j} = \sum_{i=1}^r (\phi^{\phi}_{r,j} - \lambda_j \hat{z}^{s}_{i1,j})/r \], \[ \hat{p}^{s}_{r,j} = \sum_{i=1}^r p^{s}_{r,j}/r \]  (18)

This is done for all tracked satellites and all observed frequencies, \( j = 1, \cdots, f \).
IV. EXPERIMENT AND ANALYSIS

A. Experiment set-up

The platforms that we used for our experiment are of the type shown in Fig. 2. The side length of the platform is 1 meter. The platforms were equipped with 4 Sokkia (receiver type: GSR2700ISX, antenna type: Internal Pinwheel™) and 4 Javad (receiver type: Javad Delta, antenna type: GrAnt-G3T) receivers, respectively.

We collected dual-frequency GPS data in the Perth area (Australia) over a period of 3 hours, on a 80 km baseline, with sampling interval of 1 s. The sky-plot of all tracked satellites in the 3-hour observation span is shown in Fig. 3. The satellite PRN 11 with highest elevation at the first epoch is taken as reference satellite to form 12 pairs of DD observation series. In the computations, the cut-off elevation was set to $10^\circ$.

We made sure that code-smoothing was switched off for all receivers to cancel the time correlation in observations. After fixing the platform integer ambiguities, we first analysed the noise of the data [12]. The standard deviations of all observation types for the two types of receivers are shown in Table I.

<table>
<thead>
<tr>
<th>Antenna</th>
<th>L1 (mm)</th>
<th>L2 (mm)</th>
<th>C1 (cm)</th>
<th>P2(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sokkia</td>
<td>2.3</td>
<td>3.2</td>
<td>30</td>
<td>42</td>
</tr>
<tr>
<td>Javad</td>
<td>2.1</td>
<td>3.3</td>
<td>25</td>
<td>22</td>
</tr>
</tbody>
</table>

We will now study the performs of the A-PPP concept for long-baseline positioning. First we consider the ambiguity resolution performance, then the position accuracy. The results will be compared with the conventional approach of using a single antenna per platform. In the following, the results of A-PPP RTK baseline positioning will be referred to as ARTK, while the conventional approach will be referred to as CRTK.

B. Full and Partial Ambiguity Resolution

The 80-km baseline dual-frequency GPS data was processed using the ionosphere-weighted model presented in section III-B. The standard deviation of the DD ionospheric constraint was set to $\sigma_{\tilde{\eta}} = 15$cm. Kalman filtering was applied with the standard deviation of the dynamic noise for the ZTD set at 1 cm/s. The standard deviations for the dynamic noise of the baseline coordinates was set at infinity. Hence, it was a truly kinematic analysis in which the baseline coordinates were not linked in time.
First we evaluate the IAR performance of ARTK by comparing it with CRTK using the empirical success rate $P_E$ defined as

$$P_E = \frac{\text{# correct integer solutions}}{\text{# total integer solutions}}$$  \hspace{1cm} (19)$$

where the “correct integer solution” is obtained by comparing it with the “true” integer solution computed using all the available 3-hours data.

Due to the length of the baseline, one cannot expect instantaneous full ambiguity resolution to be successful. This is also clear from Fig. 4. One can expect ambiguity resolution to be successful if the ADOP is 0.12 cycles or smaller. In the present case, however, the ADOP is always larger than 0.25 cycles, in case of CRTK, even larger than 0.4 cycles.

Since fast full ambiguity resolution can not be expected to be successful for such a long baseline, we compared the ARTK and CRTK partial ambiguity resolution (PAR) performances. This was done for two common and simple PAR scenarios. In the first one, only the widelane ambiguities are fixed, while in the other one only ambiguities are fixed for data collected with elevations larger than a given threshold $\theta_0$, like e.g. 20°.

C. Positioning Accuracy

Now we compare the positional accuracy of both RTK modes. With the ambiguities assumed known, the single-epoch, horizontal solutions of ARTK and CRTK are shown in the scatter-plots of Fig. 7. This clearly shows the improvement of ARTK over CRTK.

The statistics of the two 80-km baseline RTK solutions are listed in Table II. The means of ARTK are much closer to 0 than those of CRTK for the three coordinate components. This result shows that the standard deviations of the RTK solution is improved in ARTK mode by factors of 2.2, 1.7 and 2.0 respectively for the north, east and up components.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>North</th>
<th>East</th>
<th>Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARTK</td>
<td>0.11</td>
<td>-0.38</td>
<td>1.25</td>
</tr>
<tr>
<td>STD</td>
<td>0.54</td>
<td>0.59</td>
<td>3.89</td>
</tr>
<tr>
<td>CRTK</td>
<td>0.24</td>
<td>-0.71</td>
<td>3.27</td>
</tr>
<tr>
<td>STD</td>
<td>1.20</td>
<td>0.98</td>
<td>7.87</td>
</tr>
</tbody>
</table>

We also compared the 95% horizontal and vertical positional limits (HPL and VPL) of ARTK and CRTK as computed from the Kalman filter outputted baseline coordinate covariance matrices (Here HPL is defined as the radius of a circle over which the probability of positional error reaches 95%). Fig. 8 shows the HPL and VPL of the two RTK solutions. At the beginning, when convergence still needs to settle in, the HPL and VPL are relatively large. The convergence speed of ARTK is faster than that of CRTK. As the time series show, the HPL and VPL of ARTK are both smaller than their CRTK counterparts. Their means are presented in Table III for the
TABLE III
HPL AND VPL STATISTICS FOR ARTK AND CRTK ("PART" RefERS TO THE CONVERGED PART OF THE TIME SERIES)

<table>
<thead>
<tr>
<th></th>
<th>whole (cm)</th>
<th>part (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HPL VPL</td>
<td>HPL VPL</td>
</tr>
<tr>
<td>ARTK</td>
<td>1.67 3.44</td>
<td>1.46 2.77</td>
</tr>
<tr>
<td>CRTK</td>
<td>3.26 6.67</td>
<td>2.91 5.51</td>
</tr>
</tbody>
</table>

Fig. 8. The HPL and VPL of ARTK and CRTK

whole time series and also for the part after convergence, i.e. after 10 minutes. From Table III, both HPLs and VPLs of ARTK are basically half the counterparts of CRTK.

V. CONCLUSION

A-PPP provides a new concept that uses GNSS measurements, from an array of antennas on a platform, to realize a strengthened GNSS model with improved positioning capabilities. In this contribution, the benefits of A-PPP have been explored for long-baseline RTK positioning using antenna-array equipped baseline stations. The underlying theory was formulated and the results of a real-data experiment was presented. The results from an 80 km baseline experiment, with 4-antenna equipped stations, suggests that conventional RTK can indeed be improved by means of the A-PPP concept. This holds true for the speed with which successful IAR can be done, as well as for the accuracy of positioning.

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