Mobile Emitter Geolocation and Tracking using Correlated Time Difference of Arrival Measurements

Woo Chan Kim and Taek Lyul Song
Department of Electronic Systems Engineering
Hanyang University, Ansan
Republic of Korea
Email: wcquim@gmail.com, tsong@hanyang.ac.kr

Darko Mušicki
Department of Electrical Engineering
University of Melbourne
Australia
Email: darko.musicki@gmail.com

Abstract—This paper presents an effective recursive tracking method for the single mobile emitter using correlated TDOA measurements. In multiple-sensor environment the time difference of arrival (TDOA) measurements are correlated. We consider only the case of single emitter tracking without data association issues, e.g., no missed detection or false measurements. We describe a scenario of three static receivers with synchronous TDOA measurements. In this situation, the TDOA measurements are correlated. We propose a de-correlated TDOA measurement structure for optimal estimation. Each TDOA measurement defines a hyperbola-like region, representing possible emitter locations. This likelihood function is approximated via Gaussian mixture, making the algorithm a dynamic bank of variable number of Kalman filters. The performance is evaluated using Monte Carlo simulations, and compared with the Cramer-Rao Lower Bound (CRLB).

I. INTRODUCTION

Geolocation, an emitter localization on the earth surface, is used in important applications including the military surveillance and civilian law enforcement. For improved visibility of emitters, sensors are often mounted on unmanned aerial vehicles (UAVs). UAVs’ limited size and weight restricts sensor types and their performances.

Emitter geolocation consists of two parts. Measurement/sensor selection and estimation/information fusion/processing of measurements provided by sensors.

Starting with the bearing-only emitter localization, passive geolocation has a long history. [1] is one of the first “modern” papers in this field. For some time the passive geolocation using Time Difference of Arrival (TDOA) [2], [3] gained the research attention. Using additional sensors (additional TDOA measurements), the emitter geolocation can be evaluated at the intersection of two or more hyperbolae curves. With only two sensors (one TDOA measurement), the sensors need to move to obtain observability [4]. Using both the TDOA and Frequency Difference of Arrival (FDOA) further improves the geolocation [5] performance.

The proposed nonlinear estimation filter is based on Gaussian Mixture presentation of Measurements—Integrated Track Splitting (GMM-ITS) filter first published in [6]. GMM-ITS is a recursive procedure, derived using the Bayes formula. The nonlinear measurement likelihood is approximated by a Gaussian mixture. In addition, the emitter state probability density function (pdf) is approximated by a Gaussian mixture using a dynamic (track splitting) mixture of linear Kalman filters. GMM-ITS is used in bearing only problems [7] and also with the TDOA[5]. Furthermore, the tracking algorithm can be extended for data association environments [8].

We consider geolocation using the TDOA measurements. A TDOA measurement is calculated by the signal arrival time difference between two sensors. If \( n \) sensors exist in surveillance space, the available number of sensor pairs equals \( n(n-1)/2 \). These measurements are correlated due to the common receiver time measurement noise. Ignoring the measurement correlation reduces the geolocation accuracy. In this paper we present the GMM-ITS using correlated measurements with the TDOA measurements providing a case study.

The problem statement is presented in Section II. De-correlation stage of TDOA measurements is described in Section III. Section IV presents the GMM presentation of TDOA measurements in observation space. The approach is evaluated using simulations in Section V followed by concluding remarks in Section VI.

II. PROBLEM STATEMENT

A. State vector

A moving emitter exists on the earth surface. We assume that the sensors are also located in a flat earth plane, simplifying the geometry. The earth curvature, geographical relief, and the sensor heights can be accommodated in a straightforward manner. The emitter kinematic state which is composed of the position and the velocity at time \( k \) is
\[ e_k^T = \begin{bmatrix} x_k^T & \dot{x}_k^T \end{bmatrix}, \]  
\[ e_k = F_{k-1} e_{k-1} + w_{k-1}, \]  
where \( x_k \), \( \dot{x}_k \), and \( T \) denote the emitter position, the emitter velocity, and the matrix transpose symbol, respectively. The “position projection” matrix \( H \) is defined by \( x_k = H e_k \). The emitter trajectory at time \( k \) propagates by

\[ e_k = F_{k-1} e_{k-1} + w_{k-1}, \]  
where \( F_k \) denotes the state transition matrix. The plant noise sequence \( w_k \) is assumed to be a zero mean and white Gaussian sequence with covariance matrix \( Q \), which is not correlated with any measurement noise sequence.

The emitter is observed by the sensors, with the state vectors at time \( k \) of

\[ s_k^{(i)} = \begin{bmatrix} x_k^{(i)} & \dot{x}_k^{(i)} \end{bmatrix}, \]  
where \( x_k^{(i)} \) and \( \dot{x}_k^{(i)} \) denote known position and velocity vectors of sensor \( i \) respectively.

### B. Time Difference of Arrival

The range difference vector between the emitter and sensor \( i \) at time \( k \) is given by

\[ r_k^{(i)} = x_k - x_k^{(i)}, \]  
with the unit vector \( i_{k,i} \) equal to

\[ i_{k,i} = \sqrt{r_k^{(i)}} \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \end{bmatrix}. \]  

The time delay of signal received at sensor \( i \) is \( \left\| r_k^{(i)} \right\| / V_c \) with \( V_c \) denoting the signal propagation speed. The received signal time delay measurement \( y_k^{(i)} \) is

\[ y_k^{(i)} = \left\| r_k^{(i)} \right\| / V_c + v_k^{(i)}, \]  
where the signal time delay measurement noise at sensor \( i \) is denoted by \( v_k^{(i)} \) and is assumed to be a zero mean and white Gaussian sequence with covariance \( \sigma_v^2 \). We also assume that the time delay noise sequences are not correlated with each other,

\[ E[v_k^{(i)} v_k^{(j)}] = \begin{cases} \sigma_v^2, & i = j \\ 0, & \text{otherwise} \end{cases} \]  

The TDOA is the difference between corresponding time intervals and is given by

\[ h(e_k; s_k^{(i)}, s_k^{(j)}) = \frac{\left\| r_k^{(i)} \right\| - \left\| r_k^{(j)} \right\|}{V_c}, \quad i \neq j. \]  

\[ h^{(i,j)}(e_k) \]  

![Figure 1. TDOA scenario and constant TDOA emitter location curves](image)

In Fig. 1, the points on each dashed curve have the same range difference to a pair of sensors, and thus have the same time difference of arrival. The TDOA measurement between sensor \( i \) and sensor \( j \) at time \( k \) is

\[ z_k^{(i,j)} = y_k^{(i)} - y_k^{(j)} = h^{(i,j)}(e_k) + v_k^{(i)} - v_k^{(j)}, \quad i \neq j. \]  

The Jacobian matrix of \( h^{(i,j)}(e_k) \) with respect to \( x_k \) is given by

\[ H^{(i,j)}(e_k) = \frac{1}{V_c} \begin{bmatrix} i_{k,i} - i_{k,j} \end{bmatrix}^T, \quad i \neq j. \]  

More than three sensors may be used. In this paper we consider three stationary sensors, creating a combination of three sensor pairs. At time \( k \), each pair of sensors provides a TDOA measurement.

The sensor pair measurements are \( z_k^{(1,2)} \), \( z_k^{(1,3)} \), and \( z_k^{(2,3)} \). For convenience, the pair \( p \) which is composed with sensor \( i \) and sensor \( j \) is denoted by \( \langle p \rangle = (i, j) \): \( \langle 1 \rangle = (1,2) \), \( \langle 2 \rangle = (1,3) \), and \( \langle 3 \rangle = (2,3) \). Then (9) is identical to
where \( h^{(p)}(e_k) \) is a noiseless time difference of arrival between the sensors, and the TDOA measurement noise \( \Delta v_k^{(p)} \) equals \( v_k^{(i)} - v_k^{(j)} \).

For the three-sensor case, the TDOA measurements are used to generate a measurement vector \( z_k \)

\[
z_k = \begin{bmatrix} z_k^{(1)} \\ z_k^{(2)} \\ z_k^{(3)} \end{bmatrix},
\]

(12)

where the covariance matrix of the TDOA measurement noise \( R_k \) is

\[
R_k = \begin{bmatrix}
R_k^{(1,1)} & R_k^{(1,2)} & R_k^{(1,3)} \\
R_k^{(2,1)} & R_k^{(2,2)} & R_k^{(2,3)} \\
R_k^{(3,1)} & R_k^{(3,2)} & R_k^{(3,3)}
\end{bmatrix} = 
\begin{bmatrix}
\sigma_{k,1}^2 & \rho_{k,2}\sigma_{k,1}\sigma_{k,2} & \rho_{k,3}\sigma_{k,1}\sigma_{k,3} \\
\rho_{k,2}\sigma_{k,1}\sigma_{k,2} & \sigma_{k,2}^2 & \rho_{k,3}\sigma_{k,2}\sigma_{k,3} \\
\rho_{k,3}\sigma_{k,1}\sigma_{k,3} & \rho_{k,3}\sigma_{k,2}\sigma_{k,3} & \sigma_{k,3}^2
\end{bmatrix},
\]

(13)

where \( \sigma_{k,p}^2 = E\left\{ (\Delta v_k^{(p)})^2 \right\} \) for \( p = 1,2,3 \) and \( \rho_{k,p,q}\sigma_{k,p}\sigma_{k,q} = E\left\{\Delta v_k^{(p)}\Delta v_k^{(q)}\right\} \) for \( p,q = 1,2,3 \) with \( p \neq q \).

The diagonal elements of \( R_k \) equal to \( 2\sigma_i^2 \) and the non-diagonal elements can be calculated by using (7). Then, \( R_k \) is a matrix which is related to the covariance of signal received noise.

\[
R_k = \sigma_v^2 \begin{bmatrix}
2 & 1 & -1 \\
1 & 2 & 1 \\
-1 & 1 & 2
\end{bmatrix}.
\]

(14)

The \( R_k \) is not a diagonal matrix which also means that using the sensors in this manner leads to correlated TDOA measurements.

III. DECORRELATED TDOA MEASUREMENTS

Ignoring the measurement correlation causes suboptimal estimation and inaccurate estimation covariance matrix. Thus correlation removal process is essential for geolocation accuracy. We de-correlate the measurements by using pseudo-measurement approach (in other words by applying the Gram-Charlier orthogonalization procedure).

Decorrelating the measurements leads to the instrumental measurement vector

\[
u_k = \begin{bmatrix} u_k^{(1)} \\ u_k^{(2)} \end{bmatrix} = \begin{bmatrix} z_k^{(1)} \\ z_k^{(2)} - \frac{1}{2} z_k^{(3)} \end{bmatrix}.
\]

(15)

where \( u_k^{(2)} \) is the function of \( z_k^{(2)} \) and \( z_k^{(3)} \). Also \( u_k^{(2)} \) can be expressed by \( z_k^{(2)} \) and \( z_k^{(3)} \) as follows.

\[
u_k^{(2)} = z_k^{(2)} - \frac{1}{2} z_k^{(3)} = \frac{1}{2} y_k^{(3)} + \frac{1}{2} y_k^{(2)} - y_k^{(3)}.
\]

(16)

From (15) and (16), the vector \( u_k \) is identical to

\[
u_k = \begin{bmatrix} u_k^{(1)} \\ u_k^{(2)} \end{bmatrix} = \begin{bmatrix} z_k^{(1)} \\ \frac{1}{2} z_k^{(2)} + \frac{1}{2} z_k^{(3)} \end{bmatrix}.
\]

(17)

and the covariance matrix \( R_k^u \) is

\[
R_k^u = \sigma_v^2 \begin{bmatrix}
2 & 0 \\
0 & 1.5
\end{bmatrix}.
\]

(18)

Also note that the Gram-Charlier orthogonalization results in only two instrumental measurements (starting from three in measurements in \( z_k \)); the redundancy is automatically removed.

IV. NONLINEAR MEASUREMENT FUSION

The passive measurements are often non-linear as they have non-Gaussian likelihood (uncertainty in the observation space). Given the measurement, TDOA likelihood is Gaussian in the measurement space (11). In the surveillance space (the two-dimensional Cartesian plane), the TDOA measurement projection results in non-Gaussian measurement likelihood as depicted in Fig. 1. For estimation in non-linear systems, we use Gaussian Mixture Measurements (GMM) algorithm of [6].

The GMM—Integrated Track Splitting (GMM-ITS) filter is based on the idea that any density may be modeled by a Gaussian mixture [9]. The estimated state pdf of an emitter is non-Gaussian since we use non-linear measurements. Therefore, both state estimate pdf and the observation space measurement likelihoods are modeled by Gaussian mixtures. A “component” is defined as an element of the Gaussian mixture (the measurement component or the track component) while the state estimate is termed as “track”.

702
A. GMM ITS tracking

Denote by \( Z^k \) the sequence of all measurement vectors \( z_k \) taken up to and including time \( k ; \ Z^k = \{ z_k, Z^{k+1} \} \) with \( Z^0 = \{ \} \).

A posteriori pdf of emitter state \( e \) at time \( k-1 \) is a Gaussian mixture, given by

\[
p(e_{k-1} | Z^{k-1}) = \sum_{c=1}^{C_{k-1}} \xi(c) N(e_{k-1} ; \hat{e}_{k-1}(c), P_{k-1}(c)). \tag{19}
\]

where \( c, \ C_{k-1} \) \( N(\mu, \Sigma) \) denote the track component index, the number of track components, and the Gaussian pdf with mean value \( \mu \) and covariance matrix \( \Sigma \), respectively. Also, \( \hat{e}_{k-1}(c) \) and \( P_{k-1}(c) \) are the mean and the covariance of track component \( c \), with the relative probability \( \xi(c) \), where

\[
\sum_{c=1}^{C_{k-1}} \xi(c) = 1; \ \xi(c) \geq 0. \tag{20}
\]

The track state prediction is calculated by applying the Chapman-Kolmogorov equation. As the equation is linear, a prior track pdf at time \( k \) can be expressed by a Gaussian mixture.

\[
p(e_k | Z^{k-1}) = \sum_{c=1}^{C_k} \xi(c) N(e_k ; \hat{e}_{k-1}(c), P_{k-1}(c)). \tag{21}
\]

where each track component \( c \) is propagated using the standard Kalman filter prediction.

The TDOA measurement likelihood in the observation space at time \( k \) is also approximated by a Gaussian mixture

\[
p(w_k) = \sum_{g=1}^{G_k} \gamma^{(g)}(g) N(w_k ; \hat{w}_k(g), R_k(g)). \tag{22}
\]

where \( g \) is the measurement component index, \( G_k \) is the number of measurement components, and \( p \) is the pair of sensors index. \( \hat{w}_k^{(g)}(g) \) and \( R_k^{(g)}(g) \) denote the mean and covariance of measurement component \( g \) with the constraint of

\[
\sum_{g=1}^{G_k} \gamma^{(g)}(g) = 1; \ \gamma^{(g)}(g) \geq 0. \tag{23}
\]

Detailed GMM representation of a TDOA measurement is described in the next subsection. Each track component \( c \) updates its state pdf by using each measurement component \( g \). Thus each track component is replaced by \( g \) new track components. As both the track and the measurement components are Gaussian, the track update uses standard Kalman filter update. The number of new track components at time \( k \) is \( C_{k-1} \cdot G_k \) and the a posteriori track pdf becomes

\[
p(e_k | Z^k) = \sum_{c^*=1}^{C_k G_k} \xi^{(c^*)} N(e_k ; \hat{e}_{k|k-1}^{(c^*)}, P_{k|k-1}^{(c^*)}). \tag{24}
\]

where \( c^* = \{ g, c \} \) is an index of new a posteriori track component. The \( \hat{e}_{k|k-1}^{(c^*)} \) and \( P_{k|k-1}^{(c^*)} \) are the mean and covariance of track component \( c^* \). Relative probabilities \( \xi^{(c^*)} \) are calculated by

\[
\xi^{(c^*)} \propto \xi(c) \gamma^{(g)}(g) N(\hat{w}_k^{(g)}(g) ; H \hat{e}_{k|k-1}(c), S_k(c^*)) \tag{25}
\]

with

\[
S_k(c^*) = H P_{k|k-1}(c) H^T + R_k^{(g)}(g). \tag{26}
\]

The track output is the mean and the covariance of the pdf presented by equation (24). This output is not part of the next cycle algorithm recursion.

\[
\hat{e}_{k|k} = \sum_{c^*=1}^{C_k G_k} \xi^{(c^*)} \hat{e}_{k|k}^{(c^*)} \tag{27}
\]

\[
P_{k|k} = \sum_{c^*=1}^{C_k G_k} \xi^{(c^*)} P_{k|k}^{(c^*)} \tag{28}
\]

The number of track components grows exponentially in time which must be controlled in order to prevent saturation of computational resources. Hence, track component management [10], [11] in the form of pruning and merging is essential for all practical implementations.

B. TDOA measurement GMM presentation

The first step for GMM presentation of TDOA measurement is drawing two curves defined by \( |r_1^{(g)}| = |r_2^{(g)}| = \sqrt{V_e \Delta t^2 \pm \sigma_{e,g}^2} \) in the observation space. The area between the curves is filled with a set of non-overlapping ellipsoids. Each ellipsoid corresponds to one measurement component in (22), with the ellipsoid center equal to \( \hat{w}_k^{(g)}(g) \), and the ellipsoid size and the shape defining \( R_k^{(g)}(g) \).
The procedure starts with dividing each uncertainty curve by a set of points, where both sets have the same cardinality. Consider the situation in Fig. 2 where \( D \) denotes the distance between the sensors, and \( \Delta r \) denotes the range difference between target and the two sensors. The angle \( \alpha \) is parameter; as the \( \alpha \) changes the TDOA curve is drawn.

![Figure 2. TDOA geometry](image)

Given \( \alpha \), \( \|x^{(i)}\| \) can be calculated by

\[
\|x^{(i)}(\alpha, \Delta r)\| = \frac{D^2 - (\Delta r)^2}{2(\Delta r - D\cos(\alpha))},
\]

(29)

and then the emitter location as

\[
He(\alpha, \Delta r) = x^{(i)} - \|x^{(i)}(\alpha, \Delta r)\| \begin{bmatrix} \cos(\alpha - \alpha_0) \\ \sin(\alpha - \alpha_0) \end{bmatrix},
\]

(30)

where \( \alpha_0 \) denotes the slant angle of sensors baseline. We are segmenting each curve at a limited set of angles denoted here by \( \alpha = [\alpha_1, \ldots] \). Also we want to define the measurement component \( g \) whose outline is the inscribed ellipse. To describe the non-overlapping ellipsoids, we assume that points \( x_1, x_2 \) are on one curve and points \( x_1, x_4 \) are on the other curve as described in Fig. 3.

Each ellipsoid can be determined by the points and angle \( \alpha \). The measurement component is defined by the ellipsoid mean \( \hat{w}_i^*(\rho)(g) \) and its covariance \( R_i(\rho)(g) \).

The end point of one semi axis of the ellipsoid’s one axis are given by

\[
x_{i1} = (x_1 + x_i)/2
\]

\[
x_{i2} = (x_2 + x_i)/2.
\]

(31)

The length and the angle of one semi-axis of the ellipse are calculated by

\[
\Delta x_i = x_{i1} - x_{i2}
\]

\[
D_c = \frac{\|\Delta x_i\|}{2}
\]

\[
i(\alpha_i) = [\cos(\alpha_i) \sin(\alpha_i)]^T = \Delta x_i / \|\Delta x_i\|
\]

(32)

The length of the other semi-axis is given by

\[
D_r = i(\alpha_i + \pi/2)^T (x_1 + x_2 - x_1 - x_4)/4.
\]

(33)

The center of the inscribed ellipse is

\[
\hat{w}_i^*(\rho)(g) = (x_{i1} + x_{i2})/2,
\]

(34)

which is also the mean of the measurement component \( g \) corresponding to the ellipse. The covariance matrix of the measurement component \( g \) is given by

\[
R_i(\rho)(g) = T(\alpha_i) \begin{bmatrix} D_c^2 & 0 \\ 0 & D_r^2 \end{bmatrix} T(\alpha_i)^T.
\]

(35)

where the rotation matrix is \( T(\alpha_i) = \begin{bmatrix} i(\alpha_i) & i(\alpha_i + \pi/2) \end{bmatrix} \). The segmented ellipses achieved by using ‘sensor 1’ and ‘sensor 2’ are shown in Fig. 4.

![Figure 3. GMM presentation by using TDOA ± σ uncertainty](image)

![Figure 4. Non-overlapping segmented ellipsoid](image)
The relative probability \( \gamma^{(2)}(g) \) that the emitter is within ellipse \( g \) is proportional to the footprint area

\[
\gamma^{(2)}(g) \propto \sqrt{R^{||^2}(g)} 
\]

(36)

where \( | | \) denotes the determinant.

C. Measurement component fusion

This subsection presents the measurement fusion method when using multiple TDOA measurements which are obtained by three sensors simultaneously.

We first generate the set of measurement components from the measurement \( z_k^{(i)} \) (the first element of (15)). The instrumental measurement \( u_k^{(2)} \) from (15) is then applied sequentially to each measurement component using the extended Kalman filter (EKF).

By using the instrumental measurement \( u_k^{(2)} \) for the fusion, ‘fused’ measurement component \( g \) is defined by

\[
\hat{w}_k^{(1,2)}(g) = \hat{w}_k^{(1)}(g) + K_k^{(1,2)}(g)u_k^{(2)} - h^{(2)}(\hat{w}_k^{(1)}(g)) \\
R_k^{(1,2)}(g) = [I - K_k^{(1,2)}(g)H^{(2)}(\hat{w}_k^{(1)}(g))]R_k^{(1)}(g) 
\]

(37)

where \( h^{(2)}(\hat{w}_k^{(1)}(g)) \) and its Jacobian matrix \( H^{(2)}(\hat{w}_k^{(1)}(g)) \) are calculated by

\[
h^{(2)}(x_k) = \left[ 0.5\|r_1^{(2)}(x_k)\| + 0.5\|r_3^{(2)}(x_k)\| - \|r_2^{(2)}(x_k)\| \right] / V_c \\
H^{(2)}(x_k) = \frac{1}{V_c} \left[ 0.5i_{k,1} + 0.5i_{k,2} - i_{k,3} \right]^T 
\]

(38)

The Kalman filter gain and the innovations covariance matrix are

\[
K_k^{(1,2)}(g) = R_k^{(1)}(g)H^{(2)}(\hat{w}_k^{(1)}(g))^T \left[ H^{(2)}(\hat{w}_k^{(1)}(g))R_k^{(1)}(g)H^{(2)}(\hat{w}_k^{(1)}(g))^T \right]^{-1} + \sigma_{k,2}^2 \\
S_k^{(1,2)}(g) = H^{(2)}(\hat{w}_k^{(1)}(g))R_k^{(1)}(g)H^{(2)}(\hat{w}_k^{(1)}(g))^T + \sigma_{k,2}^2 
\]

(39)

where \( \sigma_{k,2}^2 \) is the covariance of the instrumental measurement \( u_k^{(2)} \) that is equal to \( 1.5\sigma_e^2 \) from (18).

The ‘fused’ relative probability is given by

\[
\gamma^{(1,2)}(g) \propto \gamma^{(1)}(g)N[u_k^{(2)}; h^{(2)}(\hat{w}_k^{(1)}(g)), S_k^{(1,2)}(g)] \\
\sum_{g \in G} \gamma^{(1,2)}(g) = 1; \, \gamma^{(1,2)}(g) \geq 0 
\]

(40)

If the TDOA measurement \( z_k^{(2)} \) is used for fusion without considering correlation, some variables of (37), (39), and (40) are changed as follows: (a) \( u_k^{(2)} \) is replaced with \( z_k^{(2)} \), (b) \( h^{(2)}(x_k) \) and \( H^{(2)}(x_k) \) is calculated by using (8)-(10), (c) the measurement covariance \( \sigma_{k,2}^2 \) is equal to \( 2\sigma_e^2 \) rather than \( 1.5\sigma_e^2 \).

If another measurement such as \( z_k^{(3)} \) is used for fusion in addition to \( z_k^{(1)} \) and \( z_k^{(2)} \), \( \hat{w}_k^{(1,2,3)}(g) \), \( R_k^{(1,2,3)}(g) \) and \( \gamma^{(1,2,3)}(g) \) are obtained by applying (37)-(40) consecutively.

Resulting Gaussian Mixture is used to update the track in the standard GMM manner described in Section IV.A.

![Figure 5. Geolocation Scenario](image_url)

V. SIMULATION STUDY

An emitter geolocation scenario using three stationary sensors is simulated. The simulation scenario is depicted in Fig. 5.

The sensors and the emitter are assumed to be on the flat Earth surface. The emitter moves with a uniform motion of \( 8.5m/s \). The TDOA measurements are obtained every \( 10s \). The time delay measurement noise standard deviation projects to \( 20m/V_c \).

The performance of the proposed GMM-ITS is compared with the Cramér-Rao lower bound (CRLB). Three geolocation approaches are compared:

- Using two correlated TDOA measurements; \( z_k^{(1)} \) and \( z_k^{(2)} \) (curves labeled “GMM\_ITS\_CASE\_1”), and ignoring their correlation.
- Using three correlated TDOA measurements; \( z_k^{(1)} \), \( z_k^{(2)} \), and \( z_k^{(3)} \) (curves labeled “GMM\_ITS\_CASE\_2”) again ignoring their correlation.
- Using the de-correlated measurements; \( u_k^{(1)} \) and \( u_k^{(2)} \) described in (17)-(18) (curves labeled “GMM\_ITS\_CASE\_3”)
The “GMM_ITS_CASE_1” and “GMM_ITS_CASE_2” ignore the correlation between the measurements. The number of measurement components $G_i$ is 100 with $30\text{km}$ maximum distance and assumed maximum target velocity of $15\text{m/s}$. The number of track components $C_i$ is limited to 100. The track components with relative probabilities less than $10^{-6}$ are pruned. The CRLB is evaluated by zero-process noise assumption [5]. Each simulation run simulates a 500s interval, and the statistics are accumulated over 500 runs.

Fig. 6 and Fig. 7 present the rms estimation errors. We conclude that in this scenario the proposed “GMM_ITS_CASE_3” results have smallest estimation errors. Comparing “GMM_ITS_CASE_1” with “GMM_ITS_CASE_2” means that the estimator which uses more TDOA information can bring worse results due to measurement correlation. The advantages of proposed approach in this situation are small but definite. This outcome may be due to the specific simulated geometry, as the angles of TDOA curve intersection is very small (Figure 1.).

VI. CONCLUSIONS

This paper presents an approach to using correlated measurements with GMM-ITS nonlinear estimator, applied to the case of geolocation using multi-sensor TDOA measurements. Ignoring the measurement correlation decreases the accuracy of the emitter geolocation. We consider the case of three sensor TDOA geolocation. The simulation results show small estimation improvements based on this approach. Our research will examine whether the dependency of sensor/target positioning influence the results.

REFERENCES