Passive 3D Multitarget Tracking Using Multiple Heterogeneous Sensors

T. Sathyan
School of Computer Science
The University of Adelaide
Adelaide, SA, Australia.
thuraiappah.sathyan@adelaide.edu.au

S. Arulampalam
Maritime Operations Division
Defence Science and Technology Organisation
Edinburgh, SA, Australia.
sanjeev.arulampalam@dsto.defence.gov.au

Abstract—In this paper we investigate a multi-hypothesis algorithm for tracking multiple underwater targets in the three dimensional (3D) space using two acoustic sensor arrays attached to the same underwater platform. One of the arrays measures the bearing to the target and the other array, consisting of three spatially distributed hydrophones, measures two time difference of arrivals of the acoustic wave from the target. The scenario we consider consists of an unknown number of targets with measurement origin uncertainty due to missed detections and false alarms. The objective is to estimate the number of targets in the surveillance region and their kinematic states, where the challenge is to accurately estimate the depths of the targets. The algorithm that we propose is a multiple hypothesis tracker based on the multidimensional assignment algorithm. The simulation results demonstrate that even when only vague prior information is available, the proposed algorithm accurately estimates the number of targets and their states including the depth.

Keywords: Maritime tracking, multitarget tracking, passive arrays, data association, multidimensional assignment (MDA), time difference of arrival, bearing.

I. INTRODUCTION

Multiple target tracking using passive sensors is of great importance in a number of maritime surveillance applications. In practice, the number of targets in the surveillance region is unknown and the tracker needs to estimate both the number of targets and their states, often consisting of the position and velocity of the targets. Most publications in the literature on passive tracking only consider the problem of target motion analysis (TMA), i.e., estimating the state of a single target, rather than the multitarget tracking problem, which involves the challenging data association problem.

Passive TMA that has received much attention is the bearings-only TMA, where the sensor array measures the angle of arrival of the acoustic wave emitted from the targets. One can identify two cases of bearings-only TMA studied in the literature. In one case it is assumed that there are multiple spatially separated sensor arrays all measuring the bearing to the target. In this case the arrays can measure the bearings either synchronously or asynchronously, and two or more arrays are required for tracking the targets. In the other case only one sensor array is used and in this case it has been shown that the ownership that hosts the array must out-maneuver the target to achieve observability. Furthermore, most publications on bearings-only TMA only assume a two dimensional (2D) surveillance space.

In this paper we consider the problem of multitarget tracking using two heterogeneous passive sensor arrays mounted on a single platform. One of the arrays is assumed to be a circular array that measures the bearing to the target and the other array consists of three spatially separated hydrophones that measure the pseudo time of arrival of the acoustic wave, i.e., they are subjected to an unknown constant time offset. The three measured pseudo time of arrivals, however, can be converted to two time difference of arrival (TDOA) measurements.

The localization accuracy achievable using the two arrays on their own is very poor even in the 2D surveillance space. By fusing the measurements from the two sensor arrays, one can expect improved localization performance in the 2D space. Further, it may also become possible to estimate the depth of the target through this sensor fusion. Reference [4] is the first publication to consider such a fusion problem in which a number of nonlinear filtering algorithms were evaluated for fusing the bearing and TDOA measurements. As reported in [4], the filtering problem itself is challenging due to the highly peaked likelihoods of the TDOA measurements.

As in [4] we consider 3D tracking using a bearing and two TDOA sensors. Unlike [4], we address multiple target tracking with measurement origin uncertainty due to both missed detections and false alarms. In [4], while estimating different filtering algorithms it was assumed that the accurate prior information about the initial target horizontal range and depth were available. Although this allowed a fair comparison between different filtering algorithms, in practice, only a vague prior information can be expected. We only assume the prior knowledge of minimum and maximum values for the horizontal range and depth. This information is often readily available in practice through sensor dynamic range and ocean depth charts.

The algorithm that we propose uses range parametrization for initializing the tracks from the bearing measurements, and uses a multi-hypothesis unscented Kalman filter (UKF) [5] for state estimation. An important and challenging problem in multitarget tracking with measurement origin uncertainty is the data association problem, i.e., deciding from which target, if any, a particular measurement has originated. A number of
algorithms have been proposed for solving the data association problem. The multiple hypothesis tracker (MHT) [12] and joint probabilistic data association (JPDA) [1] algorithm are two well established solutions. The probabilistic multiple hypothesis (PMHT) tracker [14], another well-known solution for the data association problem, although extensively studied, in general, has not been shown to be superior to MHT or JPDA. The often cited reason for the inferiority of the PMHT algorithm is that it (incorrectly) relaxes the one-to-one constraint between targets and measurements for algorithmic simplicity. In this paper we choose the multidimensional assignment (MDA) algorithm [7] [10] for solving the data association problem. The MDA algorithm formulates the data association as a discrete optimization problem and provides a suboptimal solution with quantifiable accuracy. It has been successfully used to solve the data association problems that arise in the context of measurement-to-measurement or measurement-to-track association.

The rest of this paper is organized as follows. Section II provides details of the problem considered in this paper and introduces the notations used. In Section III we describe the proposed algorithm for the problem under consideration. Simulation results that demonstrate the performance of the proposed algorithm and the feasibility of 3D tracking using a bearing and two TDOA sensors are presented in Section IV and concluding remarks are given in Section V.

II. Problem Formulation

We consider three dimensional (3D) tracking of multiple targets using two passive arrays. One of the arrays is a circular array that measures the bearing to the target and the other is a linear array consisting of three spatially separated hydrophones that measure two TDOAs. The number of targets in the surveillance region is not known a priori and it is required to estimate the kinematic parameters of all the targets in the region.

We model the target motion in the horizontal plane using a nearly constant velocity (NCV) model. The target motion in the vertical direction is modeled using a random walk (or nearly constant position) model. Let \( x(k) = [p(k)^T, v(k)^T]^T \) denote the state of a target at scan \( k \), where \( p(k) = [\eta(k), \xi(k), \zeta(k)]^T \) is the 3D position of the target and \( v(k) = [\hat{\eta}(k), \hat{\xi}(k)]^T \) is the velocity of the target in the horizontal plane. Note that we do not attempt to estimate the target speed in the vertical direction. The dynamics of the target is then modeled as

\[
x(k) = F(k, k-1)x(k-1) + w(k-1)
\]

where \( F(k, k-1) \) is the state transition matrix, \( w(k-1) \) is a zero-mean white Gaussian process noise sequence with covariance matrix \( Q(k-1) \) that models gentle maneuvers of the target in all three dimensions. \( F(k, k-1) \) and \( Q(k) \) are given by

\[
F(k, k-1) = \begin{bmatrix}
1 & 0 & 0 & T_k & 0 \\
0 & 1 & 0 & 0 & T_k \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
Q(k) = \begin{bmatrix}
q_1 T_k^2/3 & 0 & 0 & q_1 T_k^2/2 & 0 \\
0 & q_2 T_k^3/3 & 0 & 0 & q_2 T_k^2/2 \\
0 & 0 & q_1 T_k^2/2 & 0 & q_1 T_k \\
0 & 0 & q_2 T_k^2/2 & 0 & q_2 T_k
\end{bmatrix}
\]

where \( t_k \) is the elapsed time between scans \( k - 1 \) and \( k \), \( q_i \), \( i = 1,2,3 \) is the power spectral density corresponding to the \( x, y \) and \( z \) components of the process noise.

We use \( s = 1 \) to denote the bearing sensor and \( s = 2,3 \) to denote the two TDOA sensors. The \( j \)th measurement reported by the bearing sensor at scan \( k \) is either target originated or a false alarm. Hence we can write the bearing measurement equation as

\[
z_s^j(k) = \begin{cases} 
  h_\theta(p(k), p_s(k)) + v_s^\theta(k) & \text{if target originated} \\
  \text{if false alarm}
\end{cases}
\]

where \( s = 1 \), \( p(k) \) denotes the unknown target position, and \( p_s(k) \) denotes the known position of the circular array. The measurement noise \( v_s^\theta(k) \) is Gaussian distributed with zero mean and variance \( \sigma_s^\theta \). The measurement function is given by

\[
h_\theta(p(k), p_s(k)) = \arctan\left(\frac{\eta(k) - \eta_s(k)}{\xi(k) - \xi_s(k)}\right)
\]

where it is assumed that the \( \arctan \) function returns bearings in the range \([0, 2\pi]\) rad, measured from the positive \( y \)-axis. Furthermore, the false measurement \( \tilde{z}_s \) in (4) is assumed to be uniformly distributed in the bearing space and the number of such measurements is modeled as a Poisson random variable.

Let the locations of the three hydrophones that measure the TDOAs be denoted by \( p_s(k) \). Without loss of generality we assume that the first hydrophone is used as the reference and it is colocated with the circular array. Hence, we can write the \( j \)th TDOA measurement at scan \( k \) from sensor \( s \) (where \( s = 2,3 \)) as

\[
z_s^c(k) = \begin{cases} 
  h_c(p(k), p_s(k)) + v_s^c(k) & \text{if target originated} \\
  \text{if false alarm}
\end{cases}
\]

where the measurement function is given by

\[
h_c(p(k), p_s(k)) = \frac{||p(k) - p_s(k)|| - ||p(k) - p_1(k)||}{c}
\]

The false measurement \( \tilde{z}_s \) is modeled in a similar manner to that corresponding to the bearing sensor except that it is uniformly distributed in the field of view of the TDOA sensor.

Note that measurements from the two TDOA sensors are correlated due to the presence of reference hydrophone in the two measurements. When the two TDOA measurements are
considered together, we model the covariance matrix of the joint measurement as
\[
R^\tau = \begin{bmatrix}
\sigma_1^2 + \sigma_2^2 & \sigma_1^2 \\
\sigma_1^2 & \sigma_1^2 + \sigma_2^2
\end{bmatrix}
\]  
(8)

where \( \sigma_i \) represents the standard deviation of the time of arrival measurement of the noise process at hydrophone \( l \).

Let \( Z_s(k) = \{ z_j^s(k) \}_{j=1}^{M_s(k)} \) denote all the measurements reported by sensor \( s \) at scan \( k \), where \( M_s(k) \) is the total number of measurements in this scan. Also let \( Z(k) = \{ Z_s(k) \}_{s=1}^{3} \) denote the measurements from all three sensors at scan \( k \). The objective now is to estimate the number of targets in the surveillance region and track their 3D states using the measurements \( Z(k), k = 1, 2, \ldots \) reported by the three sensors. The number of targets may be time varying and the association between the measurements and the targets is not known a priori. In the following we assumed that the sensor measurements are synchronous, which is practically feasible since both arrays are fitted to the same platform.

III. PROPOSED ALGORITHM

A Type 3 track initialization and maintenance system is defined as tracking systems that employ multiple synchronous sensors, each measuring partial target position information [3]. Clearly, with the assumption that the measurements from the circular and linear arrays are synchronous, the 3D tracking system considered in this paper is a Type 3 system.

As noted in [3] Type 3 systems present the most challenging data association scenario. In such systems, typically, at every scan, the data association problem is solved in two steps: 1) measurement-to-measurement or static association and 2) measurement-to-track or dynamic association. The static association step groups the measurements from different sensors that could have originated from the same target and the dynamic association assigns the grouped (combined or composite) measurements to the tracks from the previous scan. An assignment-based solution [11] formulates the static and dynamic associations as multidimensional and two dimensional assignment problems, respectively.

Typical passive sensor measurements are functions of target position and to calculate the static association cost in multidimensional assignment (MDA) technique requires accurate localization of the target from the sensor measurements. For the problem under consideration, however, the 3D localization accuracy is rather poor (even when all three measurements from the targets are present), which in turn leads to inaccurate data association. We use a direct track to measurement association formulation using the MDA framework presented in [13].

Since the number of targets in the surveillance region is not known to the tracker, we attempt to initialize tracks from each unassociated bearing measurement at every scan. Although it is possible to initialize tracks from the TDOA measurements as well, we avoid doing this to reduce the number of false tracks that are created. Measurements from the three sensors are associated directly to the tracks at every scan, and the track state is updated with the associated measurements. We use a multi-hypothesis UKF for state estimation. Moreover, we use a simple logic based track confirmation and deletion procedure. It is, however, possible to use a more sophisticated track score-based approach for track maintenance [3].

We now describe the initialization and data association procedures used in the proposed algorithm.

A. Initialization

The first scan consists of three lists of measurements (one bearing and two TDOA) and it is required to find the measurements originating from the same target in the three lists before a track can be initialized. As in the previous works [11] [13] one may consider performing a MDA between the three lists to find the measurements from the same targets. Since the target localization is rather poor with one bearing and two TDOA measurements, we found that this approach has not resulted in good initial estimates of the tracks.

The approach we took initializes a track for each of the bearing measurements in the first scan using the range-parametrization (RP) technique [6] [9]. Then we associate these initial RP tracks with the two lists of TDOA measurements through a 3-D assignment. Tracks that are assigned to one or more TDOA measurements are updated with these measurements and carried forward to the next scan. Further, tracks that are not associated to any TDOA measurement are assumed false tracks and hence, deleted.

In the RP track initialization, for each bearing measurement \( \theta \), a number of range hypotheses are generated assuming that the horizontal range interval \((r_{min}, r_{max})\) and the height interval \((\zeta_{min}, \zeta_{max})\) within which the target may lie are known. The range interval is divided into a number of sub-intervals and for each sub-interval a track hypothesis is generated. Typically the sub-interval boundaries are chosen as a geometric progression with a common ratio [6]
\[
\rho = \left( \frac{r_{max}}{r_{min}} \right)^{1/N_r}
\]  
(9)

where \( N_r \) is the number of sub-intervals. The height interval is divided into a number of (say \( N_h \)) sub-intervals with equal spacing.

Corresponding to each of the \( N_r \) horizontal range hypotheses a track component is generated first in the horizontal plane. Then for each horizontal track component \( N_h \) vertical track components are generated. Therefore for each bearing measurement a track with \( N_r N_h \) components is generated.

If \( r_i \) denotes the \( i \)th horizontal range hypothesis and \( \zeta_j \) denotes the \( j \)th height hypothesis, then the state of the corresponding track component is initialized to
\[
x = [\eta_i(k) + r_i \sin \theta, \xi_i(k) + r_i \cos \theta, \zeta_j, 0, 0]^T
\]  
(10)

where \((\eta_i(k), \xi_i(k))\) is the horizontal location components of the circular array at scan \( k \). The covariance matrix corresponding to the initial state hypothesis is given by
where

\[
P_{s,k,l} = \begin{bmatrix}
P_{\eta\eta} & P_{\eta\xi} & 0 & 0 & 0 \\
P_{\eta\xi} & P_{\xi\xi} & 0 & 0 & 0 \\
0 & 0 & P_{\xi\xi} & 0 & 0 \\
0 & 0 & 0 & P_{\eta\eta} & P_{\eta\xi} \\
0 & 0 & 0 & 0 & P_{\xi\xi}
\end{bmatrix}
\]  

(11)

The initial target velocity components are assumed to be zero in (10), while it is assumed that the maximum speed \(v_{\max}\) of the targets in the surveillance region is known. Initial variance of speed components were set with the assumption that the target speed is in the range \([-v_{\max}, v_{\max}]\) m/s. A multi-hypothesis UKF is initialized with mean and covariance defined above for each bearing measurement \(\phi\).

After initializing the RP tracks from the bearing measurements, a 3-D assignment is performed between these tracks and the two lists of TDOA measurements. Each component hypothesis of a track that is associated with valid TDOA measurements is updated by executing only the update step of the UKF. Tracks that are not assigned to valid TDOA measurements from either list are deleted.

### B. Data Association

We associate the tracks directly to the three lists of measurements, i.e., at scan \(k\) tracks from scan \(k-1\) are associated directly with the measurements from scan \(k\). This is formulated as a 4-D assignment problem.\(^1\)

The MDA algorithm formulates the data association as a constrained global optimization problem. The objective is to find the best set of assignments between the track list and measurement lists that minimizes the total assignment cost. At scan \(k\) the cost of associating a particular triplet of measurements \(\{z_{j,s}\}_{s=1}^{3}\), consisting of at most one measurement from each sensor, to track \(i\) at scan \(k-1\) is defined as

\[
c_k(i, \{j_s\}_{s=1}^{3}) = -\ln \frac{\phi_k(i, \{j_s\}_{s=1}^{3})}{\phi_k(0, \{j_s\}_{s=1}^{3})}
\]  

(19)

where \(\phi_k(i, \{j_s\}_{s=1}^{3})\) is the joint likelihood that the triplet of measurements originated from the target that is being followed by track \(i\) and \(\phi_k(0, \{j_s\}_{s=1}^{3})\) is the likelihood that the same three measurements are false, i.e., it corresponds to a non-existing “dummy” track. The measurement index \(j_s\) can be equal to zero as well, in which case it denotes a “dummy” measurement and is used to account for missed detections.

The association likelihoods are given by

\[
\phi_k(i, \{j_s\}_{s=1}^{3}) = \begin{cases}
\prod_{s=1}^{3} [1 - PD]^{1-u(j_s)}[PD]^{u(j_s)}\Lambda_s(i, \{j_s\}_{s=1}^{3}) & n > 0 \\
\prod_{s=1}^{3} V_s^{u(j_s)} & n = 0
\end{cases}
\]  

(20)

where \(\Lambda_s(i, \{j_s\}_{s=1}^{3})\) is the likelihood of track \(i\) when associated to triplet of measurements \(\{z_{j,s}\}_{s=1}^{3}\). \(V_s\) is the spatial false alarm density at scan \(s\), and \(u(j_s)\) is a binary indicator function defined as

\[
u(j_s) = \begin{cases}
0 & \text{if } j_s = 0 \\
1 & \text{otherwise}
\end{cases}
\]  

(21)

Note that the filter calculated likelihood must be evaluated jointly due to the correlation between the two TDOA measurements. Clearly a dummy measurement in the 3-tuple is removed before calculating the measurement likelihood.

In view of the multiple hypothesis filter used for each track, the measurement likelihood \(\Lambda_s(i, \{z_{j,s}\}_{s=1}^{3})\) is calculated using the total probability theorem as

\[
\Lambda_s(i, \{z_{j,s}\}_{s=1}^{3}) = \sum_{i=1}^{N} \Lambda_s(i_l, \{z_{j,s}\}_{s=1}^{3})w_{i,t}
\]  

(22)

subject to one-to-one correspondence constraints that each measurement in a measurement list is assigned at most to one track, and each track is associated at most to one validated measurement in each measurement list. These constraints can be written as

\[
\sum_{i=1}^{M} \sum_{j_1=0}^{M_1} \sum_{j_2=0}^{M_2} \sum_{j_3=0}^{M_3} c_k(i, \{j_s\}_{s=1}^{3}) \rho(i, \{j_s\}_{s=1}^{3}) = 1, \quad i = 1, 2, \ldots, N_t
\]

(24)

Note that if a sensor has no detections in a given scan it will not be included in the association step, hence, only a 3-D assignment is required to be solved.
where $\rho(i, \{j_s\}_{s=1}^3)$ is a binary variable such that

$$
\rho(i, \{j_s\}_{s=1}^3) = \\
\begin{cases} 
1 & \text{if track } n \text{ is assigned to } \{z_{j_s}\}_{s=1}^3 \\
0 & \text{otherwise}
\end{cases} 
$$

(25)

It can be shown that the global optimization problem defined above is NP-hard even under unity detection probability and no false alarms, when the track list is associated with two or more lists of measurements [8]. Therefore, finding the optimal solution using polynomial time complexity algorithms is impractical, for all but very small problems. For applications such as target tracking that require real-time performance, algorithms that can give suboptimal solutions in pseudo-polynomial time are preferred.

In this paper, the Lagrangian relaxation-based suboptimal algorithm [8], which can give a near optimal solution that is quantifiable through the duality gap, is used to solve the MDA problem. In this algorithm, two sets of constraints are simultaneously relaxed using Lagrangian multipliers and the resulting 2-D assignment problem is solved optimally using a quasi-polynomial time complexity algorithm such as the Auction algorithm [2]. Then the Lagrange multipliers are updated individually, which enforces the relaxed constraints. See [8] for details.

Note that after associating the tracks to measurements, all the unassociated measurements are grouped and an attempt is made to initialize tracks as explained previously. If there are no unassociated bearing measurements, then we do not attempt to initialize new tracks.

The same data association procedure is used during the initialization stage to associate TDOA measurements to the RP tracks initiated from the bearing measurements. In this case the association dimension is three, since there are two measurement lists in addition to the track list.

IV. PERFORMANCE EVALUATION

Simulations were carried out to evaluate the performance of the proposed algorithm. Fig. 1 depicts the scenario that was used for simulations. The scenario consists of three targets ($T_1$, $T_2$, and $T_3$) maintaining a parallel NCV trajectory for the entire duration of the simulation. The power spectral density of the process noise components were assumed to be $q_1 = q_2 = 10^{-3}$ m$^2$/s$^3$ and $q_3 = 10^{-2}$ m$^2$/s, where $T$ = 15s is the fixed sampling duration. Target $T_1$ starts at (4,1) km in the horizontal plane and all three targets were at a depth of 0.17 km initially. The initial separation distance between the targets is $d$ km and we considered three different values (0.5, 1, and 2) for $d$.

The ownship is initially at (0,0,0.1) km and moves with a constant velocity of (0,5,0) m/s for the first 21 scans. It then performs a coordinated turn in the horizontal plane for 9 scans with a turn rate of -1 deg/s and then maintains a constant velocity until the end of the simulation, which consisted of 50 scans. The ownship motion was assumed to be deterministic.

Measurements were generated for all three sensors with a probability of detection of 0.9. Furthermore, measurement noise standard deviations for bearing and TDOA were assumed to be 2 deg and 20 µs, respectively. The number of false measurements were assumed to be Poisson distributed with an average of five per scan per sensor and they were generated uniformly in the interval [0, 2π] rad for the bearing and [-10, 10] ms for TDOA.

The prior information used in the tracker was the minimum and maximum values of horizontal range ($r_{min}, r_{max}$) and the depth ($\zeta_{min}, \zeta_{max}$), and the maximum speed $v_{max}$ of the target. The following values were used: $r_{min} = 0.6$ km, $r_{max} = 15$ km, $\zeta_{min} = 0$ km, $\zeta_{max} = 1$ km, and $v_{max} = 15$ m/s. Ten range parametrized components were generated and for each component five depth components were generated as described in Section III-A, i.e., $N_r = 10$ and $N_h = 5$. All component hypotheses of a track are updated with the measurement that is assigned to the track. Furthermore, the hypotheses with updated weights below 0.01 are deleted, and the remaining weights are re-normalized. A cascaded 2/2 followed by 3/5 logic was used for track confirmation. This means that a track that is initialized in a particular scan must be associated with a measurement in the very next scan and that it must be associated with a measurement in at least three of the first five scans. A confirmed track is deleted if it were not associated with a measurement in at least three scans in a sliding window of five scans.

A. Performance metrics calculation

We only consider the confirmed tracks in the performance metrics calculation. At each scan a 2-D assignment is performed to determine the association between targets and the confirmed tracks. The track that is associated to a target in a majority of the scans in a particular run is assumed to be the one that corresponds to the target.

For each of the three targets, once a corresponding track is found in each Monte Carlo (MC) run, the position and velocity root mean squared error (RMSE) are found straightforwardly.
It is possible that in a particular scan a track corresponding to a target may not be found, which we report as a separate measure. We define average track purity as the percentage of scans in which a track that is decided as following a target is associated with that target. We also report association accuracy, which is found by comparing the target originated measurements and the measurements associated to the track that is following this target. Note that if a target is not detected by a sensor in a particular scan and if the track that is following the target is assigned to the dummy measurement corresponding to that sensor, then we consider it as a correct assignment. We also report the average number of valid tracks held by the algorithm at any given scan.

B. Results

A total of 100 MC runs were performed to generate the results reported in this section. After identifying the tracks for each target in every MC run, we found that in eight runs a track was not held for one target for the case \( d = 2 \text{ km} \). The same measure for the other two separation distances was found to be ten. From the associations found between targets
and tracks, the RMSE was calculated. Figures 2, 3, and 4 show the RMSE in position, velocity, and depth, respectively, for the three targets for the case \( d = 2 \) km. Target \( T_3 \) shows higher RMSE compared to the other two targets.

Figures 5 and 6 show the 3D position and velocity RMSE calculated over 100 MC runs and averaged over the three targets for three different target separation distances \( d = 0.5 \) km, 1 km, and 2 km are shown. In all three cases the position RMSE towards the end of the simulation time is around 1.3 km. Hence, except in the case of \( d = 2 \) km, one may not be able to clearly specify the relative geometry between the targets, although their position RMSE is reasonable. The velocity RMSE corresponding to the case of \( d = 2 \) km shows a slight decrease in the RMSE due to improved data association.

We also plot the RMSE in depth estimation in Figure 7, where the error when the separation is 0.5 km is at least 10 m higher compared to the other two separation distances.

Table I lists the average values of track purity and average track lengths for the three separation distances considered. The track purity, as one would expect, increases with the increase in target separation. The track length, however, was nearly the same for the three targets. This again suggests that at smaller distances although there is additional track swaps the algorithm was able to hold tracks for all three targets in almost all scans.

Table I also lists the average association accuracy obtained from the MDA algorithm for the three separation distances considered. When \( d = 2 \) km, on average nearly 85% of the measurements are associated correctly. While this figure declines with decrease in the distance, it is still nearly 74% at the separation of 0.5 km. Note that the 12% reduction in the association accuracy when \( d = 0.5 \) km compared to when \( d = 2 \) km does not directly translate to the tracking accuracy.

We also found, on average, 1.8 extra tracks per MC run with average track length of 6.4 scans for the case \( d = 2 \) km. For the other two separation distances there were two extra tracks per run with the average track length of 7.2 scans when \( d = 1 \) km and 8.1 scans when \( d = 0.5 \) km.

The program for the simulations was coded in Matlab, except for the computationally intensive MDA algorithm which was coded in C++. The execution time, on average, for a single MC run was found to be 3.24 s. This translates, on average, to 0.19 s per scan whose duration is assumed to be 15 s, which suggests that the algorithm is real-time feasible. An optimized implementation of the proposed algorithm in C++ would lead to further reduction in the execution time.

### V. Conclusions

In this paper we studied the problem of 3D tracking of multiple underwater targets using two acoustic sensor arrays attached to a single platform. The arrays measure the bearing and TDOA of the arriving acoustic wavefront. It was assumed that the two sensor arrays are synchronized. The measurements were subject to non-unity detection probability and false alarms. The number of targets in the surveillance region was unknown a priori.

The algorithm that we proposed initializes tracks using a parametrization technique for each (unassociated) bearing measurement and uses the multidimensional assignment (MDA) to solve the data association problem. The MDA algorithm we used associates the measurements directly to the tracks. A multi-hypothesis unscented Kalman filter was used for state estimation. Simulations were carried out to demonstrate the performance of the proposed algorithm. The results indicated the effectiveness of the proposed algorithm in terms of tracking accuracy and track maintenance capability. Furthermore, the algorithm was found to be robust against target separation, which illustrates the power of the MDA data association algorithm in the context of this problem. It was, however, also observed that for smaller values of target separation, one may not be able to correctly identify the relative geometry as the position errors for these cases were found to be higher than the separation distance. Despite this, the algorithm achieved good RMS accuracy in both position and velocity and notably provided reliable depth estimates in the tested scenarios.

### References


