Abstract—The performance of three multi-target tracking algorithms are compared under the challenging problem of bearings-only tracking in the presence of clutter and missed detections. The algorithms under consideration are the Gaussian Mixture Probability Hypothesis Density (GMPHD) filter, the Gaussian Mixture Cardinalised Probability Hypothesis Density (GMCPHD) filter and the Joint Integrated Probabilistic Data Association (JIPDA) filter. A Monte Carlo analysis is presented for a difficult bearings-only tracking scenario, in which the algorithms assume a diffuse model for target birth, such that new targets may appear at any bearing and at any time. The algorithms are evaluated in terms of the Optimal Sub-Pattern Assignment (OSPA) metric, the cardinality estimation performance, and their respective computational requirements.

I. INTRODUCTION

Bearings-only tracking of multiple targets in the presence of clutter and missed detections is a very challenging problem, which holds significant practical importance in real-world situations. Despite this, performance results for tracking algorithms applied to this problem are not often quoted in the literature. Bearings-only tracking is an important problem because sensors which operate in passive mode are usually limited to generating bearings-only measurements. Despite their limitations, passive sensors are often desirable or even essential in many situations, because they do not transmit signals that may be detected by an adversary. In other words, they allow for covert surveillance, whereas active sensors leave the observer open to detection.

The fundamental difficulties in bearings-only tracking are mainly due to the low information content of the observations, and the measurement model which has potentially high levels of non-linearity. The former leads to the observability problem, and details of this for the the idealised case in which there is no measurement origin uncertainty can be found in [1–3]. This issue essentially dictates that the observer must ‘outmanoeuvre’ the target in order to obtain accurate and reliable estimates of the target state. Furthermore, the level of observability (as characterised by the Cramer-Rao lower bound) is determined by the precise target-observer geometry, which can vary greatly between targets in a multi-target scenario.

The problem of measurement non-linearity can also dramatically affect tracking performance. This property of the observation model means that the standard linear Kalman filter cannot be applied, and suboptimal techniques must be used instead. Some of the methods proposed over the years include; the Extended Kalman Filter (EKF) [4], Unscented Kalman Filter (UKF) [5], Cubature Kalman Filter (CKF) [6], Shifted Rayleigh Filter (SRF) [7], and Particle Filter (PF) [8]. One common aspect of these algorithms is that they are all suboptimal to some extent, and have the potential to fail under certain conditions. Some examples of cases which are prone to failure include those with very close range targets (high bearing rates), and those in which the tracker is initialised with a poor starting condition.

An additional complication that can degrade performance is measurement origin uncertainty, which arises in the presence of clutter, and when multiple targets appear close to one another in the measurement space. Such interactions between targets are, due to the geometry of the observation process, more common in bearings-only tracking than for more informative measurement types. Many techniques have been proposed to handle measurement origin uncertainty, most of which fall into the following categories; Nearest Neighbour (NN) methods (including global NN), Probabilistic Data Association (PDA) methods (such as Joint PDA [9] and Joint Integrated PDA [10]), Multiple Hypothesis Tracking (MHT) [11], multi-target particle filtering [12], and more recently, Random Finite Set (RFS) based methods [13].

In this study, we consider two RFS based algorithms applied to the bearings-only multi-target tracking problem, and compare their performance to one of the more traditional tracking methods. The first algorithm considered is the Gaussian Mixture Probability Hypothesis Density (GMPHD) filter [14]. The GMPHD filter is based on propagating a Gaussian mixture representation of the first order statistical moment of the multi-target probability density, often referred to as the intensity function. The next technique is the Gaussian Mixture Cardinalised Probability Hypothesis Density (GMCPHD) filter [15, 16], which also propagates the intensity function, but in addition, propagates the probability distribution on the number of targets. This technique is designed to reduce the variance of the estimated number of targets, which is a major drawback of standard PHD filtering. The final algorithm considered is the Joint Integrated Probabilistic Data Association (JIPDA) filter [10]. This is based on the well established technique of enumerating the joint measurement association events, and marginalising over the event probabilities to obtain the
association weights for each target.

The rest of the paper is organised as follows. Section II contains a formulation of the bearings-only multi-target tracking problem. Sections III, IV and V give brief descriptions of the GMPHD, GMC-PHD, and JIPDA filters respectively. Section VI contains the results of a Monte Carlo analysis of all three methods under a bearings-only tracking scenario. Finally, some concluding remarks are given in Section VII.

II. FORMULATION OF THE BEARINGS-ONLY MULTI-TARGET TRACKING PROBLEM

The problem addressed in this paper involves a single passive sensor on board a manoeuvring platform. The position of the sensor platform is assumed to be known at all times, and its position at time index \( k \) is denoted by the Cartesian coordinates \((x_k^*, y_k^*)\). The dynamics of individual targets are assumed to follow a nearly constant velocity model, defined as follows. Consider a target \( t \), which at time \( k \) has Cartesian position coordinates of \((x_{k,t}^t, y_{k,t}^t)\) and an instantaneous velocity vector of \((\dot{x}_{k,t}^t, \dot{y}_{k,t}^t)\). The state vector for target \( t \) at time \( k \) is defined as

\[
\mathbf{x}_{k,t} = \begin{bmatrix} x_{k,t}^t & y_{k,t}^t & \dot{x}_{k,t}^t & \dot{y}_{k,t}^t \end{bmatrix}^T.
\]

For notational convenience, we also define a similar vector \( \mathbf{x}_{k,s} \) containing the true state of the sensor platform. The target state evolves according to the discrete-time transition given by

\[
\mathbf{x}_{k+1,t} = \mathbf{F}\mathbf{x}_{k,t} + \mathbf{Γ}\mathbf{v}_k
\]

where,

\[
\mathbf{F}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{Γ} = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ 0 & T \\ 0 & 0 \end{bmatrix}.
\]

\( T \) is the time between measurements, and \( \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}) \) is a \( 2 \times 1 \) i.i.d Gaussian process noise vector with \( \mathbf{Q} = \hat{q}I_{2 \times 2} \) where \( \hat{q} \) is the process noise intensity. At time \( k \), there can exist any number of targets \( N_k \), forming an RFS of true target states defined by

\[
X_k = \{\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,N_k}\}
\]

The sensor can only produce measurements of the target bearings, which for target \( t \) is modelled by the equation

\[
z_{k,t} = h(\mathbf{x}_{k,t}, \mathbf{x}_{k,s}) + w_k
\]

where

\[
h(\mathbf{x}_{k,t}, \mathbf{x}_{k,s}) = \arctan\left(\frac{x_{k,t}^t - x_{k,s}^t}{y_{k,t} - y_{k,s}}\right)
\]

and \( w_k \sim \mathcal{N}(0, \sigma_w^2) \) is zero-mean white Gaussian measurement noise with variance \( \sigma_w^2 \). For each target \( t \) in \( X_k \), the sensor generates a Bernoulli measurement RFS denoted \( D_k(\mathbf{x}_{k,t}) \), with an existence parameter \( P_{D_k} \) (the detection probability), and likelihood function \( g(z_{k,t}^t|\mathbf{x}_{k,t}, \mathbf{x}_{k,s}) = \mathcal{N}(z_{k,t}^t; h(\mathbf{x}_{k,t}, \mathbf{x}_{k,s}, \sigma_w^2)) \). Thus the RFS of true target detections at \( k \) is given by

\[
\Theta_k(X_k) = \bigcup_{\mathbf{x}_{k,t} \in X_k} D_k(\mathbf{x}_{k,t})
\]

The sensor also produces an RFS of false measurements at each time \( k \), denoted \( K_k \). This is modelled as a Poisson RFS with a known intensity \( \lambda_C \). The overall set of measurements generated by the sensor at time \( k \) can thus be expressed as

\[
Z_k = \Theta_k(X_k) \cup K_k
\]

The goal of the algorithms considered in this paper is to recursively estimate the true target sets \( X_k \) at each \( k \), conditioned on all measurement sets received up to time \( k \). For the purposes of this study, we do not consider it necessary for the trackers to estimate continuously labelled target trajectories, and instead a set of point estimates at each time is sufficient. Nonetheless, JIPDA does provide labelled tracks, so in the analysis, we consider both the case when the labels are ignored, and when they are accounted for.

III. GAUSSIAN MIXTURE PHD FILTER

Let \( \mathcal{X} \) represent the single target state space and \( \mathcal{F}(\mathcal{X}) \) the set of all finite subsets of \( \mathcal{X} \). Let \( f_{k|k-1}(\cdot|\cdot) \) denote the multi-target transition density, which describes the underlying models of target dynamics, births and deaths. Similarly, let \( g_k(\cdot|\cdot) \) denote the multi-target likelihood, which describes the multi-target sensor measurement including the underlying models of detections, false alarms, and target generated measurements. The multi-target Bayes recursion propagates the multi-target posterior density \( \pi_k(\cdot|Z_{1:k}) \) in time \([17, 18]\) according to

\[
\pi_{k|k-1}(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X)\pi_{k-1}(X|Z_{1:k-1})\mu_s(dX),
\]

\[
\pi_k(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k)\pi_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X)\pi_{k|k-1}(X|Z_{1:k-1})\mu_s(dX)},
\]

where \( \mu_s \) is an appropriate reference measure on \( \mathcal{F}(\mathcal{X}) \).

As a computationally efficient approximation to the recursion in (7)-(8), the PHD recursion was proposed by Mahler in [18]. This is a first moment approximation, and as such it propagates the posterior intensity of the RFS of targets over time, without requiring any explicit data associations to be computed. Let \( v_{k|k-1} \) and \( v_k \) denote the intensities associated with the predicted and posterior multi-target state. Then, based on the following assumptions

- All targets evolve and generate measurements independently of each other;
- The birth RFS and the surviving RFSs are independent of each other;
- The clutter RFS is Poisson and independent of the measurement RFSs;
- The predicted multi-target RFS is Poisson,
the original PHD recursion is given by

\[ v_{k|k-1}(x) = \int p_{S,k}(\zeta) f_{k|k-1}(x|\zeta) v_{k-1}(\zeta) d\zeta + \gamma_k(x) \]  
\[ v_k(x) = [1 - p_{D,k}(x)]v_k|k-1(x) + \sum_{z \in Z_k} p_{D,k}(x) g_k(z|x)v_{k|k-1}(x) \]  
\[ + \sum_{z \in Z_k} \bar{\kappa}_k(z) + \int p_{D,k}(x) g_k(z|x)v_{k|k-1}(x) d\zeta \]

where at time \( k \), \( f_{k|k-1}(\cdot|\zeta) \) is the single target transition density given previous state \( \zeta \), \( p_{S,k}(\cdot) \) is the probability of target existence given previous state \( \zeta \), \( \gamma_k(\cdot) \) is the intensity of target births, \( Z_k \) is the multi-target measurement set, \( g_k(z|\cdot|x) \) is the single target measurement likelihood given current state \( x \), \( p_{D,k}(x) \) is the probability of target detection given current state \( x \), and \( \kappa_k(\cdot) \) is the intensity of clutter.

The PHD recursion may be implemented using either a Gaussian mixture (GM) or sequential Monte Carlo (SMC) approach. Although the SMC approach naturally allows for non-linear measurement models, its application to bearings-only tracking is difficult due to problems associated with extracting estimates from the particle representation of the PHD. In bearings-only tracking, the radial components of individual target distributions are usually highly elongated compared to the cross-range components. This leads to severe difficulties when attempting to isolate clusters of particles in the estimate extraction process. For this reason, we restrict our attention to the GM version of the PHD filter in this study.

The standard form of the GMPHD recursion [14] requires the target birth model to be specified in terms of a Gaussian mixture. For the problem studied here, targets may appear anywhere in the bearing space with equal probability. Although this type of model can be approximated using a mixture of Gaussians spaced across all possible bearings, a more convenient approach is to change the form of the birth model such that it takes on a uniform distribution in the bearing space. A similar approach to this was shown in [19], and a more comprehensive derivation with application to a linear multi-target tracking scenario is given in [20].

In what follows, we review the closed-form expressions for the GM implementation of the recursion in (9)-(10), where the target birth model is uniform across the bearing space, and Gaussian in range and velocity.

Let us assume that the posterior PHD at time \( k - 1 \) is a Gaussian mixture of the form

\[ v_{k-1}(x) = \sum_{i=1}^{J_{k-1}} w_{m,k}^{(i)} N(x; m_{k-1}^{(i)}, P_{k-1}^{(i)}) \]

Let \((\theta, r, c, s)\) represent the state space in terms of bearing, range, course and speed, and let \( \Psi(x; \gamma) \) represent a function which transforms the density \( \gamma \) into the Cartesian space. Then the predicted PHD at time \( k \) is given by

\[ v_{k|k-1}(x) = v_{S,k|k-1}(x) + \Psi(x; \gamma_k(\theta, r, c, s)) \]

where,

\[ v_{S,k|k-1}(x) = P_{S,k} \sum_{i=1}^{J_{k-1}} w_{m,k}^{(i)} N(x; m_{k-1}^{(i)}, P_{k-1}^{(i)}) \]  
\[ \gamma_k(\theta, r, c, s) = U(\theta; [0, 2\pi]) N(r; \bar{r}, \sigma_r^2) N(c; \bar{c}, \sigma_c^2) \]  
\[ m_{k|k-1}^{(i)} = Fm_{k-1}^{(i)} \]  
\[ P_{k|k-1}^{(i)} = FP_{k-1}^{(i)} F^T + \Gamma Q \Gamma^T \]

\( \bar{r} \) and \( \sigma_r^2 \) are the prior range and range variance for new targets, \( \bar{c} \) and \( \sigma_c^2 \) are the prior speed and speed variance for new targets, and \( \sigma_\theta^2 \) is the prior course variance for new targets.

The posterior PHD at time \( k \) is given by

\[ v_k(x) = \sum_{i=1}^{J_k} w_{m,k}^{(i)} \]  
\[ + \sum_{i=1}^{J_k} \sum_{z \in Z_k} w_{s,k}^{(i)} N(x; \hat{m}_{k}^{(i)}(z), \hat{P}_{k}^{(i)}(z)) \]

where

\[ w_{m,k}^{(i)} = (1 - P_{D,k}) w_{k|k-1}^{(i)} \]  
\[ w_{s,k}^{(i)} = \frac{P_{D,k} q_k^{(i)}(z) w_{k|k-1}^{(i)}}{\kappa_k(z) + P_{D,k} \sum_{i=1}^{J_{k-1}} w_{k|k-1}^{(i)} q_k^{(i)}(z) + w_{k}^{(i)}/2\pi} \]  
\[ \bar{m}_k^{(i)}(z) = m_{k|k-1}^{(i)} + K_k^{(i)}(z - \hat{z}_{k|k-1}^{(i)}) \]  
\[ \bar{p}_k^{(i)} = P_{k|k-1}^{(i)} - K_k^{(i)} H_k^{(i)} P_{k|k-1}^{(i)} \]  
\[ q_k^{(i)}(z) = N(z; \hat{z}_{k|k-1}^{(i)}, S_{k|k-1}^{(i)}) \]  
\[ \hat{z}_{k|k-1}^{(i)} = h(m_{k|k-1}^{(i)}, x_{k,s}) \]  
\[ K_k^{(i)} = P_{k|k-1}^{(i)} H_k^{(i)} S_{k|k-1}^{(i)}^{-1} \]  
\[ S_{k|k-1}^{(i)} = H_k^{(i)} P_{k|k-1}^{(i)} H_k^{(i)^T} + \sigma_\theta^2 \]  
\[ H_k^{(i)} = \frac{\delta h(x, x_{k,s})}{\delta x} \bigg|_{x=m_{k|k-1}^{(i)}} \]

The means \( \hat{m}_k(z) \) and covariances \( \hat{P}_k(z) \) of the updated birth terms in (17) are calculated according to the approximation [21]

\[ \hat{m}_k(z) = \begin{bmatrix} x_{s}^* + \bar{r} \sin z \\ y_{s}^* + \bar{r} \cos z \\ \bar{s} \sin(z - \pi) \\ \bar{s} \cos(z - \pi) \end{bmatrix} \]
the main parts of the GMCPHD calculation.

In the previous section, we restrict our attention here to the sequential Monte Carlo, and for the same reasons as stated as well as the intensity function. This has the effect of reducing propagating the probability distribution of the number of targets to most likely to come into close contact with the sensor.

Note that in equations (21) to (27), we have used EKF based linearisation to handle measurement non-linearity. This can indeed be replaced with any non-linear Gaussian filter, such as the UKF, SRF or CKF. For the simulations presented in Section VI, we consider just the EKF version.

IV. GAUSSIAN MIXTURE CPHD FILTER

The CPHD filter is a generalisation of the PHD filter, as it propagates the probability distribution of the number of targets as well as the intensity function. This has the effect of reducing the variance of the estimated number of targets, resulting in fewer false alarms. Similarly to the PHD filter, the CPHD filter may be implemented using either Gaussian mixtures or sequential Monte Carlo, and for the same reasons as stated in the previous section, we restrict our attention here to the GM implementation. The standard form of the GMCPHD filter requires a Gaussian mixture birth model, but an alternative form which allows the birth model to be uniformly distributed on the measurement space is derived in [20]. We now review the main parts of the GMCPHD filter.

Assume that at time \( k - 1 \) we have a posterior cardinality distribution \( \rho_{k-1} \), and a posterior intensity function \( v_k \) of the same form as in (11). Then the predicted cardinality distribution at time \( k \) is given by

\[
\hat{P}_k(z) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x}^2 \sigma_z & 0 \\ \sigma_{yx} & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_y^2 & \sigma_{y}^2 \sigma_z \\ 0 & 0 & 0 & \sigma_y^2 \end{bmatrix} \tag{29}
\]

where \((x^*_k, y^*_k)\) is the sensor position at time \( k \), and \( \bar{r}, \sigma^2_x, \bar{s}, \sigma^2_s \) and \( \sigma^2_e \) are the parameters of the Gaussian components of the prior state for new targets as defined previously. Note that the prior course is set to \( z - \pi \) so targets are initially assumed to be travelling directly towards the sensor along the line of bearing.

This is a common assumption in bearings-only tracking, as it represents a highly threatening scenario in which the target is most likely to come into close contact with the sensor.

Finally, \( \rho_k(n) \) is defined by (28), and \( v_k(x) \) is defined in equations (21)-(27), \( \bar{m}_k(z) \) is defined by (28), and \( \hat{P}_k(z) \) is defined by (29).

V. JOINT INTEGRATED PROBABILISTIC DATA ASSOCIATION

Joint Integrated Probabilistic Data Association (JIPDA) [10] is an extension of Joint Probabilistic Data Association (JPDA) [9], both of which are based on generating soft measurement to target associations, by assigning probabilities to each measurement-target pair in a joint framework. This is done by enumerating all feasible joint association events, calculating

at time \( k \) are given by

\[
\rho_k(n) = \frac{\mathcal{Y}^n_k[v_{k|k-1}, Z_k](n)\rho_{k|k-1}(n)}{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k], \rho_{k|k-1} \rangle} \tag{31}
\]

\[
v_k(x) = \sum_{i=1}^{J_{k-1}} w_{k,m}(i) N(x; m_{ik}(z), P_{k}) + \sum_{z \in Z} w_{k,z}(z) N(x; \bar{m}_k(z), \hat{P}_k(z)) \tag{32}
\]

where,

\[
w_{k,m}(i) = (1 - P_{D,k}) w_{k-1}(i) \frac{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k], \rho_{k|k-1} \rangle}{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k], \rho_{k|k-1} \rangle} \tag{33}
\]

\[
w_{k,z}(z) = \frac{(1, \kappa_k)}{\mathcal{K}_k(z)} P_{D,k} \frac{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k - \{ z \}], \rho_{k|k-1} \rangle}{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k], \rho_{k|k-1} \rangle} \tag{34}
\]

\[
w_{k,b}(k) = \frac{(1, \kappa_k)}{\mathcal{K}_k(z)} P_{D,k} \frac{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k - \{ z \}], \rho_{k|k-1} \rangle}{\langle \mathcal{Y}^n_k[v_{k|k-1}, Z_k], \rho_{k|k-1} \rangle} \tag{35}
\]

\[
\mathcal{Y}^n_k[v_{k|k-1}, Z](n) = \sum_{j=0}^{\min(|Z|, n)} (|Z| - j)! P_k(n - j) \langle 1, w_{k|k-1} \rangle^n - (j + u) \langle 1, w_{k|k-1} + w_k \rangle^n P_{j+u} e_j (\Xi_k(v_{k|k-1}, Z)) \tag{36}
\]

\[
\Xi_k(v_{k|k-1}, Z) = \left\{ \frac{(1, \kappa_k)}{\mathcal{K}_k(z)} \left( \frac{w_b}{2\pi} + P_{D,k} \sum_{i=1}^{J_{k-1}} w_{k|i-1}(i) q_k(i)(z) \right) : z \in Z \right\} \tag{37}
\]

\[
q_k^b(j)(z) = N(z; \bar{z}_{k|k-1}^j, \bar{S}_{k|k-1}^j) \tag{38}
\]

\[
P_{j+u} = \frac{n!}{(n - (j + u))!} \tag{39}
\]

Finally, \( m_{ik}(z), P_{ik}^j, z_{k|k-1}^j \) and \( S_{k|k-1}^j \) are the same as defined in equations (21)-(27), \( \bar{m}_k(z) \) is defined by (28), and \( \hat{P}_k(z) \) is defined by (29).
their posterior probabilities, and summing over all events that contain the marginal assignment event of interest. The standard Probabilistic Data Association (PDA) technique is then employed to update each track. JIPDA extends JPD by including target existence probability in the recursion. This makes it capable of track management, i.e., tracks can be discarded when the target is unlikely to exist.

We now review the probability calculations performed by JIPDA. In what follows, \( E_{k,t} \) denotes the event of target \( t \) existing at time \( k \), and \( A_{k,t}^i \) denotes the event of measurement \( i \) being associated with target \( t \) at time \( k \), with \( i = 0 \) indicating no measurement.

Let \( \phi_i \) denote the \( i \)-th joint association/existence event. Also, let \( T_0 \) denote the set of tracks with no assigned measurements under event \( \phi_i \), and \( T_1 \) denote the set of tracks assigned one measurement under event \( \phi_i \). The posterior probability of event \( \phi_i \) is [10]

\[
P(\phi_i | Z_{1:k}) = C^{-1} \prod_{t \in T_0^i} (1 - \beta_D P_g P(E_{k,t} | Z_{1:k-1})) \prod_{t \in T_1^i} P_D P_g P(E_{k,t} | Z_{1:k-1}) \phi_i^{m(t,i)} \rho_i^{C} \tag{40}
\]

where \( P_D \) is the detection probability, \( P_g \) is the validation region probability, \( P(E_{k,t} | Z_{1:k-1}) \) is the predicted target existence probability for track \( t \) at time \( k \), \( \rho_i^C \) is the prior clutter density, \( \rho_i^C = P(Z_k^n(t,i) | Z_{1:k-1}) \) where \( m(t,i) \) denotes the measurement associated to track \( t \) under the joint event \( \phi_i \), and \( C \) is a normalising constant.

Denote by \( \Lambda(t,i) \) the set of joint events in which track \( t \) is associated with measurement \( i \), where \( i = 0 \) indicates no measurement. Then, the posterior probability that no measurement originates from track \( t \) is

\[
P(A_{k,t}^0 | Z_{1:k}) = \sum_{e \in \Lambda(t,0)} P(\phi_e | Z_{1:k}) \tag{41}
\]

and the posterior probability that the target exists, but no measurement has originated from track \( t \) is

\[
P(E_{k,t}, A_{k,t}^0 | Z_{1:k}) = \frac{(1 - P_D P_g) P(E_{k,t} | Z_{1:k-1})}{1 - P_D P_g P(E_{k,t} | Z_{1:k-1})} \times P(A_{k,t}^0 | Z_{1:k}) \tag{42}
\]

Likewise, the posterior target existence probability for track \( t \) with measurement \( i \) originating from the target is

\[
P(E_{k,t}, A_{k,t}^i | Z_{1:k}) = \sum_{e \in \Lambda(t,i)} P(\phi_e | Z_{1:k}). \tag{43}
\]

From (42) and (43), the posterior probability of target existence for track \( t \) can be computed as

\[
P(E_{k,t} | Z_{1:k}) = P(E_{k,t}, A_{k,t}^0 | Z_{1:k}) + \sum_{i \in \{j : \mu(k,t,j) > 0\}} P(E_{k,t}, A_{k,t}^i | Z_{1:k}) \tag{44}
\]

where \( \mu(k,t,j) = 1 \) if measurement \( j \) is in the validation region of track \( t \) at time index \( k \), and zero otherwise. Given the above existence probabilities, the association probabilities for track \( t \) are

\[
\beta_0^t = \frac{P(E_{k,t}, A_{k,t}^0 | Z_{1:k})}{P(E_{k,t} | Z_{1:k})} \tag{45}
\]

\[
\beta_i^t = \frac{P(E_{k,t}, A_{k,t}^i | Z_{1:k})}{P(E_{k,t} | Z_{1:k})} \tag{46}
\]

for all \( i \in \{j : \mu(k,t,j) > 0\} \).

The target state for track \( t \) is updated using the validated measurements from \( Z_k \) weighted by the above association probabilities [10]. The final step of the recursion is the computation of the predicted target existence probabilities which is accomplished using the Markov chain transition probabilities [10].

The main drawback of JIPDA is that when the target and clutter density are high, the algorithm becomes computationally intractable due to a combinatorial explosion in the number of association hypotheses. In fact, the computational complexity is exponential in the number of targets, making it unsuitable for high target densities. For this reason, it is infeasible to use JIPDA to perform track initiation in all but very low clutter scenarios. For this study we have used the single target version to initiate tracks, known as Integrated Probabilistic Data Association (IPDA) [22]. To avoid redundant tracks, IPDA acts only on measurements which are not validated during the JIPDA update. Once the existence probability of an IPDA track reaches a threshold, that track is transferred to JIPDA until the existence probability falls below a termination threshold, at which time it is deleted.

VI. SIMULATION RESULTS

Performing a rigorous numerical performance analysis on multiple tracking algorithms is a difficult task due to the possibility that a particular set of conditions may favour one algorithm over another. For this reason, the results shown here should be treated as representative only, as different scenario geometries or sensor characteristics may lead to different outcomes.

For this analysis, we have carried out Monte Carlo simulations for a challenging bearings-only multi-target tracking scenario. The scenario consists of a time-varying number of target arrivals that can be detected. Naturally, the use of such a birth model leads to the increased possibility of false tracks.

The scenario runs for a total of 2500 seconds. Five targets are present at the beginning, with a further five arriving at various times within the first 500 seconds, and three leaving in the latter part of the scenario. The sensor generates a set of measurements once every 10 seconds. The standard deviation of the measurement noise for true target detections is 1°, the detection probability is a constant 0.95 for all targets, and the number of clutter measurements per scan is Poisson with a mean of \( \lambda_C = 40 \). The clutter measurements are
distributed uniformly across the entire bearing space $[0, 2\pi]$. The sensor platform travels at a speed of 10 knots and undergoes several course changes to ensure that the target states become observable. The target bearings are initially well spaced, but they become close together and cross over as the scenario progresses, before moving apart again towards the end. The scenario geometry is shown in Figure 1, and an example of the bearing measurements seen by the trackers in Figure 2.

![Figure 1: Target-observer geometry](image1)

![Figure 2: Example bearing measurements](image2)

All algorithms assume a prior target range at the time of birth of $\bar{r} = 12 km$ with standard deviation $\sigma_r = 4 km$. The prior target course is closing on the line of bearing with standard deviation $\sigma_{\lambda} = 50^\circ$, and the prior target speed is $\bar{s} = 10 kt$ with standard deviation $\sigma_s = 4 kt$. The birth intensity is set to $\lambda_0 = 0.05$, and the true clutter density of $\lambda_C = 40$ is assumed known. For the PHD and CPHD filters, the Mahalanobis distance threshold for merging components in the intensity function is set to 4, and the weight threshold for elimination of components is $10^{-5}$.

For JIPDA, an initial existence probability of 0.03 is used for new targets, with the threshold for track confirmation set to 0.5, and the termination threshold set to 0.01. We have shown the results of using three different values for the gate probability parameter, which controls how many measurements are likely to fall in the validation region for any given target. This affects the number of joint association events that are generated by the algorithm, which in turn affects the execution time, particularly when the targets are close together. The three values used are 0.97, 0.95 and 0.90.

The performance was assessed by carrying out 100 independent Monte Carlo runs and plotting the average OSPA [23], cardinality estimates, and execution time for each tracker. For the OSPA metric, a cutoff parameter of 2km and an order parameter of 1 were used. The simulations were carried out in Matlab on a 64-bit Linux PC with an Intel Core i7 2600K CPU (quad-core, 3.4GHz) and 16GB of memory. In developing the code for all three algorithms, extensive use of the Matlab profiler was made to remove any avoidable computational bottlenecks. We also made extensive use of vectorisation and MEX functions to exploit parallel processing and obtain faster execution time.

Figure 3 shows the standard OSPA metric (where the track labels for JIPDA are ignored), and Figure 4 shows the execution time (note the use of a log scale on the vertical axis). Clearly, JIPDA is capable of giving better estimation error than both the PHD and CPHD filters when a high value for the gate probability is used. However, as seen in Figure 4, this comes with a heavy computational burden. The execution time of JIPDA is lower than PHD and CPHD when the targets are spaced far apart, however, when the targets become close together during the middle of the scenario, the execution time increases dramatically as the number of joint association events increases. With the gate probability set to 0.97, the JIPDA had a worst case execution time at least 25 times greater than the CPHD. Furthermore, although not shown here, it was observed that increasing the gate probability up to 0.99 made the JIPDA intractable for real-time processing due to excessive CPU and memory requirements. The execution time for the PHD and CPHD remain relatively constant throughout the entire scenario, which is a major advantage that this approach holds over JIPDA.

In an attempt to bring the execution time of JIPDA down to more comparable levels with the (C)PHD, the gate probability was reduced. Although this decreased the computation time, the results in Figures 3 and 4 show that the error performance was significantly affected, with the worst case execution time still much higher than the PHD and CPHD. The degradation in performance of JIPDA with reduced gate probability can also be seen in the cardinality plots in Figures 8, 9, and 10. The cardinality estimates appear to become slightly worse as the gate probability is reduced.

One advantage that JIPDA holds over PHD and CPHD is that it intrinsically maintains labelled tracks over time.
However, when targets become closely spaced, the labels may incorrectly switch to a different track. To demonstrate the labelling performance of JIPDA, Figure 5 shows the ‘OSPA for Tracks’ metric [24] for that algorithm. The OSPA distance when track labels are accounted for is significantly higher, indicating that some track switching and/or fragmentation is taking place. This shows that for this scenario, JIPDA is incapable of reliably holding the correct track labels during the period when the targets are close together.

As one would expect, the CPHD filter outperforms the PHD filter. Figure 3 shows that the OSPA distance is significantly lower for CPHD than PHD, which is due to the lower variability of the cardinality estimates. The cardinality estimation performance for these two filters is illustrated in Figures 6 and 7, which clearly show a lower standard deviation for the CPHD filter. The only drawback of CPHD is that it takes longer since target births are modeled using an i.i.d cluster RFS instead of a Poisson RFS as used by the PHD filter.

VII. CONCLUSION

We have compared the performance of the GMPHD and GMCPHD filters with the JIPDA filter on a challenging bearings-only tracking scenario consisting of ten crossing targets. The filters were not provided with any prior knowledge of the times or locations at which new targets appear, so we used a form of the PHD and CPHD filters which allows the target birth model to be uniform on the measurement space.
For the scenario tested, the Monte Carlo simulation results show that JIPDA was capable of obtaining lower estimation error than the PHD and CPHD filters, provided that the gating threshold was set high enough. However, this came at a significant cost, because the computation time was extremely variable, and significantly higher than the (C)PHD when the targets were in close contact with each other. When JIPDA was tuned to achieve execution times more comparable with that of the (C)PHD filter, it’s performance degraded significantly, both in terms of it’s estimation accuracy and track labelling ability. In fact, as the gate threshold was reduced, there came a point beyond which the CPHD filter outperformed the JIPDA, with computation time still lower. As expected, the CPHD filter outperformed the PHD filter in the steady state, but with increased delay in responding to the appearance of new targets.

REFERENCES


