Abstract—This paper deals with the antenna allocation problem in Multiple-Input Multiple-Output (MIMO) radar systems with collocated antennas. After deriving the Cramer-Rao Lower Bound (CRLB) as the cost function, the optimal distribution of antennas is found by applying the relevant operators to the CRLB. A convex optimization algorithm is then proposed to find the optimal distribution of antennas that achieves the optimal CRLB. It is also shown that the optimization problem can be simplified to the well-known Semi-definite Programming (SDP) for a single target scenario. Using a number of simulations, it is shown that the localization algorithm also leads to superior results when the optimal antenna configuration is used.

I. INTRODUCTION

Multiple-input Multiple-output (MIMO) radars with collocated antennas have been recently introduced in the literature [11] as an alternative to the traditional phased-array radar systems [14]. Transmitting orthogonal signals by a closely-spaced array of antennas, collocated MIMO radars provide a number of benefits over the phased-array systems such as the diversity in the paths [5], virtual aperture extension [2], beam pattern improvement [2], and higher probability of detection [2]. Consequently, there has been a lot of interest among researchers to analyze different aspects of MIMO radars such as waveform selection [6] [12] [13], and range compression [12] and MIMO radar applications in target detection, localization, and tracking [9] [17].

The Cramer-Rao Lower Bound (CRLB) of collocated MIMO radars has been derived by many people [9] [12] [2]. The Direction-of-Arrival (DOA) of the target was defined as the parameter of the problem in [2] and, then, the CRLB was derived according to the received complex signals. When multiple targets fall inside the same resolution cell of the MIMO radar, [12] also found the CRLB and analyzed how the number of targets occupying the same cell might affect the CRLB. While all previous works only derived the CRLB for the DOA estimation, it was shown in [9] that the range information of the target can be also included in the received measurements. Therefore, a novel measurement model was proposed in [9] and the CRLB was found for both range and DOA of the target. It was also shown that the CRLB is seriously affected by the number and location of targets falling inside the same resolution cell.

Antenna allocation has been always a critical concern in MIMO array systems. An optimal antenna placement algorithm was proposed in [3] when an array of closely-spaced antennas received the Time-of-Arrival (TOA) data from a single target. It was also shown in [16] that the Posterior CRLB (PCRLB) can be used in order to find the number and optimal locations of multiple sensors while there is no restriction on the closeness of inter-sensor distances. The CRLB was also employed in [7] for antenna placement in widely-separated MIMO radars. It was shown that the trace of the CRLB matrix can be written as a convex function of the location of antennas. Then, convex optimization techniques were applied to find the optimal placement of antennas. The CRLB was also used as a performance metric in [8] for antenna selection in widely-separated MIMO radars where a subset of antennas has to be chosen out of a large number of widely-separated antennas.

Although the antenna allocation problem has been sufficiently dealt with for widely-separated MIMO radars, there is a lack in the corresponding development for collocated MIMO radars. Unlike phased-array radar systems, MIMO radars send orthogonal signals, which provides multiple independent signal paths. Therefore, the location of antennas affects the performance of estimation. The CRLB of the DOA estimation was used in [12] in order to find the optimal cross-correlation matrix of the transmitted signals. It was shown that the CRLB is a convex function of the cross-correlation matrix of the transmitted signals. In addition, it was shown in [9] that the CRLB of a collocated MIMO radar is a function of the location of antennas. Simulations in [9] show that the performance of localization is affected by the geometry of antennas. Therefore, it is of interest to find an optimal distribution of antennas that provides the best localization performance.

In this paper, the antenna allocation problem is discussed for collocated MIMO radar systems. To the best of our knowledge, there is no comprehensive work on the design and analysis of an optimal antenna allocation procedure for collocated MIMO radars. The main contributions of this paper are as follows:

- Analyzing the effect of the antenna locations on the CRLB:
  First, the CRLB is derived for a collocated MIMO radar where both DOA and range information are included in the signal model. Then, it will be shown how the location of antennas affects the CRLB. In addition, the localization algorithm is applied to the MIMO radar with different geometries of antennas, and it will be shown how influential the geometry of antennas might be on the performance of localization.

- A convex cost function:
  It is shown that applying suitable operators to the CRLB, a convex cost function can be derived in terms of the location of antennas. By maximizing the determinant of the Fisher Information Matrix (FIM), the final cost

424
function can be written in a convex form with respect to the parameters of the system.

- An optimization algorithm for antenna allocation:
  First, it is shown how additional constraints are considered in order to guarantee the uniqueness of the solutions. Then, considering different constraints over the geometry of the problem, the final antenna allocation problem is written in a standard Semi-definite Programming (SDP) form.

The rest of this paper is organized as follows. Section II presents a brief overview of MIMO radars with collocated antennas. CRLB is derived for the MIMO system in Section III. Section IV deals with the antenna allocation problem where the convex optimization framework is described. Simulation results that are the main part of the paper are given in Section V. Finally, the paper is concluded in Section V.

**A. Notations**

The list of notations used in this paper are as follows:

- $A = D(a)$: a diagonal matrix with $A_{ii} = a_i$ and $A_{ij} = 0, i \neq j$.
- $\Re(a)$: the real part of the complex variable $a$.
- $\Im(a)$: the imaginary part of the complex variable $a$.
- $\mathcal{N}(\mu, \Sigma)$: a Gaussian function with mean $\mu$ and the covariance matrix $\Sigma$.
- $E_x(f)$: expected value of function $f$ over the random variable $x$.
- $tr(A)$: the trace of matrix $A$.
- $A^H$: the Hermitian transpose.
- $A(:,j)$: the $j$-th column of matrix $A$.
- $A(j,:)$: the $j$-th row of matrix $A$.

**II. MIMO RADARS WITH COLLOCATED ANTENNAS**

Consider an array of antennas with $M$ transmitters and $N$ receivers.

**Definition 1**: Define $s_{ti} = [x_{ti} \ y_{ti}]'$ and $s_{rj} = [x_{rj} \ y_{rj}]'$ as the location of the $i$-th transmitter and the $j$-th receiver in a $2$-dimensional surveillance region, respectively.

**Assumption 1**: There is a single target available in the region where $x = [x \ y]'$ denotes the location of the target. Also, the complex reflection of the target is modeled by a complex random variable $\alpha = \Re(\alpha) + j\Im(\alpha)$.

**Assumption 2**: It is assumed that the target reflection obeys a Swerling type I model [15] where $\Re(\alpha) \sim \mathcal{N}(\Re(\bar{\alpha}), \sigma_\alpha^2)$ and $\Im(\alpha) \sim \mathcal{N}(\Im(\bar{\alpha}), \sigma_\alpha^2)$.

**Assumption 3**: It is assumed that the distance between any two antennas is much smaller than the distance of the array to the target. It is also assumed that the arrays of transmitters and receivers are collocated.

**Definition 2**: Define $h[k] = [h_1^*[k] \cdots \ h_M^*[k]]^H$ as the transmitted waveform in the $k$-th snapshot with $K$ being the number of total snapshots.

$\bar{r}_{bin}$ denote the index of the resolution cell and the resolution width, respectively.

**Assumption 4**: Transmitters send orthogonal signals with a diagonal cross-correlation matrix being defined as

$$R = \frac{1}{K} \sum_{k=1}^{K} h[k]h^H[k] = D ([P_1 \cdots P_M])^T$$

where $P_m$ denotes the total transmitted power by the $m$-th antenna.

**Definition 3**: Assuming that the target is located in the $c$-th cell, define $r^{c*} = \| x \|^2$ as the distance of the target to the origin. Then, the ratio parameter $\beta^c$ is defined as follows:

$$\beta^c = \frac{r^{c*} + (1 - c^*)r_{bin}}{r_{bin}}$$

Now, given all assumptions, the received output of matched filter in the $c$-th resolution cell can be written as follows [9]:

$$\eta_c = \begin{cases} (1 - c^*)\phi & c = c^* - 1 + w \\ \beta^c \phi & c = c^* + w \end{cases}$$

where $w$ denotes a complex Gaussian noise with independent real and imaginary parts being distributed as $\{ \Re(w), \Im(w) \} \sim \mathcal{N}(0, \sigma_w^2)$, and $\phi$ is the contribution of the target on the received signal that is written as $\phi = \alpha \psi$ with the following form for the unknown term in the right-hand side of the equality [9]:

$$\psi = \sqrt{K} \text{VEC}(AR^H)$$

Here, VEC($A$) stands for the matrix vectorization operator, and $A$ stands for the steering matrix defined as follows [12]:

$$A = b(a)^H$$

$$a = \exp \left( -\frac{2\pi}{\lambda} \left[ \sin(\theta) \cos(\theta) \right] S_t \right)$$

$$b = \exp \left( -\frac{2\pi}{\lambda} \left[ \sin(\theta) \cos(\theta) \right] S_r \right)$$

Now, the mean received output of matched filter is defined as $\eta^* = [\Re(\eta^c) \Im(\eta^c)]^T$ with $\Re(\eta^c) = \Re(\eta_{c-1}) \Re(\eta_c)$ and $\Im(\eta_c)$ being the imaginary part with replacing the $\Im$ operator. The terms $\Re(\eta_c)$ and $\Im(\eta_c)$ (with $c \in \{c^* - 1, c^*\}$) can be found by calculating $\Re(\psi)$ and $\Im(\psi)$ as follows and then replacing in (3), respectively:

$$\Re(\psi) = \bar{\alpha} \left( \Re(\psi) - \Im(\psi) \right)$$

$$\Im(\psi) = \bar{\alpha} \left( \Re(\psi) + \Im(\psi) \right)$$

**Definition 4**: Given the vector of the output of matched filter as $\eta = [\eta_{c-1}^T \eta_c^T]^H$, define $\eta^* = [\Re(\eta^c) \Im(\eta^c)]^H$.
with \( \Omega \) being defined as follows:
\[
\Omega(S_t, S_r, R) = (I_{1 \times M} \otimes S_t - S_t \otimes I_{1 \times N}) \left( R^\dagger \otimes I_{1 \times N} \right)
\]
(12)
where \( \otimes \) is the Kroncker product, and \( 1_{a \times b} \) is a \( a \times b \) matrix with all entries being one.

Given the signal model in (3) and the mean output of matched-filter in (10), the following proposition provides the distribution of the output of matched-filter [9]:

**Proposition 1:** In a scenario with a single target being located in the \( c^* \)-th resolution cell, the output of the matched-filter received by a collocated MIMO radar with \( M \) transmitters and \( N \) receivers (e.g. \( \eta^* \)) is Gaussian distributed with mean \( \eta^* \) and covariance \( \Sigma \) defined as follows:
\[
\Sigma = \left( \begin{array}{cc}
\Sigma_{cc}(c^*-1) & \Sigma_{c(c-1)} \\
\Sigma_{c(c-1)} & \Sigma_{c(c-1)}
\end{array} \right)
\]
with the following definitions for \( \Sigma_{cc} \) and \( \Sigma_{c(c-1)} \) terms:
\[
\Sigma_{cc} = \begin{cases} 
K \sigma^2 (1 - \beta^c)^2 + \sigma^2 & c = c^* - 1 \\
K \sigma^2 (\beta^c)^2 + \sigma^2 & c = c^* 
\end{cases}
\]
(14)
\[
\Sigma_{c(c-1)} = K \sigma^2 (1 - \beta^c) \beta^c I_{2MN}
\]
(15)

### III. CRAMER-RAO LOWER BOUND

CRLB gives the best Minimum Mean Squared Error (MMSE) bound for any unbiased estimator [1]. Here, the goal is to estimate the unknown parameters of a single target.

**Definition 5:** For the target located in the \( c^* \)-th resolution cell, define the state and parameter vector \( X \) and \( \Theta^* \), respectively, as follows:
\[
X = \left[ x_t \ y_t \ \Re(\alpha) \ \Im(\alpha) \right]'
\]
(16)
\[
\Theta^* = \left[ \theta \ \beta^* \ \Re(\alpha) \ \Im(\alpha) \right]'
\]
(17)

The CRLB is the inverse of the Fisher-Information-Matrix (FIM) defined as follows:

**Definition 6:** Assuming \( y \) as the received noisy measurements and \( \theta \) as the parameters of the system, define the following matrix operator:
\[
J_{\theta \theta'} = E_y \left[ \frac{\partial \log p(y|\theta)}{\partial \theta} \left( \frac{\partial \log p(y|\theta)}{\partial \theta} \right)' \right]
\]
(18)

Referring to the definition of \( \eta^* \) and its distribution provided by Proposition 1, define \( J_{XX'} \) as the desired FIM, which can be written in the following form [9]:
\[
J_{XX'} = \Gamma J_{\Theta^* \Theta^*} \Gamma'
\]
(19)

Here, \( \Gamma \) is defined as
\[
\Gamma = \begin{bmatrix}
\frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} & 0 & 0 \\
\frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
(20)

Now, the FIM derivation is equivalent to finding the unknown term \( J_{\Theta^* \Theta^*} \) in (19), which can be broken into the following sub-terms:
\[
J_{\Theta^* \Theta^*} = \begin{bmatrix}
J_{\theta^2} & J_{\theta \beta^2} & J_{\theta \Re(\alpha)} & J_{\theta \Im(\alpha)} \\
J_{\beta \theta^2} & J_{\beta \beta^2} & J_{\theta \Re(\alpha)} & J_{\beta \Re(\alpha)} \\
J_{\theta \Re(\alpha)} & J_{\beta \Re(\alpha)} & J_{\Re(\alpha)\Re(\alpha)} & J_{\Re(\alpha)\Im(\alpha)} \\
J_{\theta \Im(\alpha)} & J_{\beta \Im(\alpha)} & J_{\Im(\alpha)\Re(\alpha)} & J_{\Im(\alpha)\Im(\alpha)}
\end{bmatrix}
\]
(21)

**Definition 7:** Define the following structure for the inverse of \( \Sigma \):
\[
\Sigma^{-1} = \begin{bmatrix}
k_1 I_{2MN} & k_2 I_{2MN} & k_3 I_{2MN}
\end{bmatrix}
\]
(22)

where \( \{k_1, k_2, k_3\} \) terms can be found by referring to the definition of \( \Sigma \) in (13).

Referring to the signal model, an analytical form for each entry of the FIM is provided as follows:

**Proposition 2:** Assume a single target scenario where the target is located in the \( c^* \)-th resolution cell. Each entry of the FIM an be calculated as follows:
\[
J_{\theta \theta'} = K \left( \sum_{j=1}^{MN} p'(\Omega(\cdot, j)|\Omega(\cdot, j)) \right) C_{\theta \theta'}
\]
(23)

**Definition 8:** Assuming \( y \) as the received noisy measurements and \( \theta \) as the parameters of the system, define the following matrix operator:
\[
J_{\theta \theta'} = E_y \left[ \frac{\partial \log p(y|\theta)}{\partial \theta} \left( \frac{\partial \log p(y|\theta)}{\partial \theta} \right)' \right]
\]
(24)

Referring to the definition of \( \eta^* \) and its distribution provided by Proposition 1, define \( J_{XX'} \) as the desired FIM, which can be written in the following form [9]:
\[
J_{XX'} = \Gamma J_{\Theta^* \Theta^*} \Gamma'
\]
(25)

with \( p = [\cos(\theta) - \sin(\theta)]' \), and the following definitions for the constant terms:
\[
C_{\theta^2} = k_1 (1 - \beta^c)^2 + k_2 (\beta^c)^2 + 2 k_3 \beta^c (1 - \beta^c)
\]
(26)
\[
C_{\beta \beta^2} = k_1 + k_2 - 2 k_3
\]
(27)
\[
C_{\theta \Re(\alpha)} = C_{\theta \Im(\alpha)} = C_{\theta \Re(\alpha)} = C_{\theta \Im(\alpha)} = C_{\theta \Re(\alpha)} = C_{\theta \Im(\alpha)}
\]
(28)

**IV. OPTIMAL ANTENNA ALLOCATION**

It is observed that the localization performance is affected by the distribution of antennas. Consider a scenario with two antennas \( (N = 2, M = 2) \), where each antenna plays the role of both the transmitter and the receiver. There is also a single target with parameters \( [30^\circ, 33.1^\circ] \), which is located in \( \{r, \theta\} = \{825m, 30^\circ\} \). The CRLB of DOA estimates is now shown in Figure 1 in terms of different inter-sensor distances for the designed scenario. It can be observed that the geometry
of sensors (inter-sensor distances) affects the performance bound of DOA estimation, where the best geometry achieves the performance 33% better than the worst geometry. While the relationship between the performance and the geometry of antennas was shown graphically for the case with two antennas, in a large-scale scenario with more antennas, such a graphical tool is not available. Therefore, this section deals with a convex formulation of the antenna allocation problem for collocated MIMO radars where the CRLB is used as the performance metric.

A. The Optimization Algorithm

Referring to (23), it can be inferred that only terms $J_{\Theta}$ are a function of the antenna locations where $\zeta \in \{\theta, \beta^c, \Re(\alpha), \Im(\alpha)\}$. On the other hand, it is easy to show that:

$$\sum_{j=1}^{MN} \Omega(:, j) = 0$$  \hspace{1cm} (34)

It is also obvious that $J_{\theta^2}$ is a convex function of $\Omega$ terms [4]. The following corollary now summarizes some important notes about the optimization framework:

**Corollary 1:** In a collocated MIMO radar with $M$ transmitters and $N$ receivers, where a single target located in the $c^*$-th resolution cell, the CRLB is a function of inter-antenna differences. In addition, all entries of FIM are independent of the inter-sensor differences except $J_{\theta^2}$, which is also a convex function of the unknown differences.

**Definition 8:** Define the difference between the $m$-th transmitter and the $n$-th receiver as follows:

$$\Delta s_{mn} = s_{tm} - s_{rn}$$  \hspace{1cm} (35)

The antenna allocation problem can be now dealt with by minimizing the trace of CRLB, maximizing the determinant of FIM, or minimizing the maximum eigenvalue of CRLB [12]. The following lemma proposes the convex optimization algorithm for antenna allocation in a collocated MIMO radar system:

**Lemma 1:** Consider a collocated MIMO radar with $M$ transmitters and $N$ receivers. In addition, assume that there is a single target located in the $c^*$-th resolution cell. Then, a convex optimization algorithm that finds an optimal placement of antennas is given as follows:

$$\max \{\Delta s_{11}, \cdots, \Delta s_{MN}\} \quad J_{\theta^2}$$  \hspace{1cm} (36)

**Proof:** The optimization problem can be formulated as minimizing the determinant of the CRLB, which is equivalent to maximizing $|J_{\Theta}^*|$. In addition, the system matrix $\Gamma$ defined in (20) is independent of the location of the antennas. Therefore, the final goal is to maximize $|J_{\Theta^*}^{\Theta^*}|$. Referring to the FIM derived in (21), the following new form is first written:

$$J_{\Theta^*}^{\Theta^*} = \begin{bmatrix} J_{\theta^2} & b' \\ b & B \end{bmatrix}$$  \hspace{1cm} (37)

where $b$ and $B$ are the blocked vector and matrix formed by remaining entries of $J_{\Theta^*}^{\Theta^*}$ in (21). Now, the determinant term can be written as:

$$|J_{\Theta^*}^{\Theta^*}| = |B||J_{\theta^2} - b'Bb|$$  \hspace{1cm} (38)

It is known that both $B$ and $b$ are independent of the antenna placement. Therefore, determinant maximization can be achieved by maximizing $J_{\theta^2}$ with respect to $\Omega$. However, referring to (12), it is observed that $\Omega$ is a linear function of $\Delta s_{mn}$ terms. The optimization problem can be finally simplified to maximizing $J_{\theta^2}$ with respect to $\Delta s_{mn}$ terms. \hfill \blacksquare

The final optimization problem can be now constructed by imposing constraints on the inter-antenna distances. The new optimization problem can be now written as follows:

$$\max \{\Delta s_{11}, \cdots, \Delta s_{MN}\} \quad \sum_{m=1}^{M} \sum_{n=1}^{N} p^* \Delta s_{mn} \Delta s_{mn}^t \quad S.T \quad ||\Delta s_{mn}||_2 \geq d_{mn}, \forall \{m, n\}$$  \hspace{1cm} (39)

where $\{d_{mn}\}$ are design parameters. Note that, in writing the above equation, it is assumed that the transmitted powers are all the same and unitary ($P_1 = P_2 = \cdots = P_M = 1$).

B. More Constraints and SDP Formulation

The optimization problem in (39) provides a set of $\{\Delta s_{mn}\}$s for which the determinant of FIM is maximized. However, there might be multiple solutions for the location of sensors that lead to the same set of difference vectors.

**Definition 9:** Given $\{\Delta s_{11}, \cdots, \Delta s_{MN}\}$ as a set of difference vectors that form a specific geometry of antennas, define $s_c = [s_c^x, s_c^y]^t$ as the center of mass of the configuration. It is now known that there are an infinite number of antenna locations with the same set of difference vectors but different centers of the mass. For example, recall the two-antenna scenario in the last section. The optimization algorithm finds the optimal $\Delta s_{12}$. The optimal solution can be regarded as a set of lines connecting the location of two antennas. However, there are an infinite number of such lines with the same slope where each new line can be constructed from the other one by a simple reflection. To remedy this problem, a new constraint is added to the optimization problem as follows:

$$\sum_{m=1}^{M} s_{tm} + \sum_{n=1}^{N} s_{rn} = 0$$  \hspace{1cm} (40)
Table I
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{\text{max}}$</td>
<td>Maximum coverage range of transmitters</td>
<td>50 (km)</td>
</tr>
<tr>
<td>$t_{\text{min}}$</td>
<td>Range width</td>
<td>30 (m)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wave-length</td>
<td>30 (cm)</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of snapshots</td>
<td>128</td>
</tr>
<tr>
<td>$\sigma_t^2$</td>
<td>Variance of the scatterers</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>$\sigma_n^2$</td>
<td>Variance of the additive noise</td>
<td>1</td>
</tr>
<tr>
<td>$P^*$</td>
<td>Transmitted power</td>
<td>1 (w)</td>
</tr>
</tbody>
</table>

The above constraint guarantees that the center of the mass of the array is in the origin. Merging the above constraint to the main optimization algorithm in (39), the final problem is not in a standard form due to the appearance of $s_{\text{min}}$ and $s_{r_n}$ terms. The following theorem shows how the final optimization algorithm can be written into a standard form:

**Theorem 1:** Consider a single-target scenario with a collocated MIMO radar being used as the measurement tool. Defining $T^* = \{T_1^{11}, \ldots , T^{MN}\}$, $S^* = \{s_1, \ldots , s_R\}$, and $t = [t_1^{11} \ldots t_{MN}]$, the optimal placement of transmitters and receivers that maximizes the determinant of FIM is found by solving the following Semi-definite Programming (SDP) optimization problem:

$$\max_{T^*, S^*, t} \quad \sum_{m=1}^{M} \sum_{n=1}^{N} t_{mn}$$

S.T. $s_{\text{min}} \leq \{s_{tm}, s_{rn}\} \leq s_{\text{max}}, \forall \{m, n\}$

$$\sum_{m=1}^{M} s_{tm} + \sum_{n=1}^{N} s_{rn} = 0$$

$$\text{tr} (T^* n^* P) \geq t_{mn}$$

$$\begin{bmatrix} 
-2 & s_{tm} - s_{rn} \\
-s_{tm}^2 - s_{rn}^2 & -d_{mn}^2 \\
\end{bmatrix} \preceq 0$$

$$\begin{bmatrix} 
1 & s_{tm} - s_{rn} \\
\frac{1}{T_{mn}} & s_{tm}^2 - s_{rn}^2 \\
\end{bmatrix} \preceq 0$$

with $P = P^* n$, $\preceq$ as the generalized inequality operator, and $s_{\text{min}}$ and $s_{\text{max}}$ as the minimum and maximum coverage areas, respectively.

The above optimization problem can be now efficiently solved using standard packages [10].

V. NUMERICAL RESULTS

In this section, it is studied how the optimal allocation of antennas in the surveillance region affects the localization performance of the MIMO radar system. The parameters of the designed MIMO radar are shown in Table I. A single target is located at [410 - 710](tm). The parameters of the target are also chosen to be as follows:

$$\Theta = [\frac{-\pi}{3} .33 3 3]$$

A. Same Transmitters and Receivers

Assume that there are $M$ antennas available where each antenna can both transmit and receive signals. Two antenna configurations are considered in this part. First, the Uniform-Linear-Array (ULA) structure is taken where the distance between each two antennas is $\frac{\lambda}{2}$. The second configuration is the optimal geometry of antennas found by the optimization algorithm proposed in this paper. For simulations, it is assumed that $d_{mn} = \lambda, \forall \{m, n\}$, and $s_{\text{min}}$ and $s_{\text{max}}$ are chosen to be $[-\lambda - \lambda]$ and $[\lambda \lambda]$, respectively. The optimal allocation of antennas is now shown in Figure 2 for different number of antennas. In addition, Figure 3 presents the CRLB of localization for the optimal and the ULA structure separately. It can be observed that the CRLB of the optimal configuration is much lower than that of the ULA structure. The improvement becomes more significant when the number of antennas is smaller. For example, for the case with $M = 2$ antennas, the CRLB of the optimal structure is 10 times lower than that of the ULA configuration while the improvement decays to 2 times lower at $M = 5$ antennas. When the number of antennas increases, the gap between the optimal and ULA CRLB becomes tighter because the Signal-to-Noise Ratio (SNR) is large enough to make up the poor geometry of antennas.

It is also beneficial to study the performance of the localization algorithm with the designed optimal configuration. To do this, assume $M = 3$ is fixed as the number of antennas. Besides the optimal and ULA configurations, a random antenna allocation is also used for the test where the antennas are randomly distributed in the underlying surveillance region. The localization RMSE is now calculated at different target SNRs where all results are obtained after 100 Monte Carlo simulations. Figure 4 presents the resulting RMSE for each of the above-represented configurations. It is observed that the optimal configuration achieves the lowest RMSE while the ULA provides the worst results. The random allocation also gives an RMSE between the optimal and ULA configurations although other random distributions of antennas may provide higher RMSE results.

B. Different Transmitters and Receivers

Now, we assume that each antenna can either transmit or receive signals. For simplicity, it is also assumed that $M = N$. The optimization algorithm is now applied to the MIMO radar system with $M$ transmitters and $N$ receivers. The optimal configuration is then shown in Figure 5 for different numbers of transmitters and receivers. It is observed that for the case with $M = N = 2$ antennas, the optimal configuration is similar to the structure found for $M = 4$ in Figure 2. The same conclusion can be also made for the case with $M = N = 3$ in Figure 5 and $M = 9$ in Figure 2. It is also observed that transmitters and receivers are clustered based on minimizing the mutual distance between each two antennas. Referring to Figure 6, the observation is confirmed for the case with $(M+N) = 8$ and different values of $M$ and $N$. It can be seen that the antennas with the least distances to each other fall in the same category (transmitters or receivers).
Figure 2. The obtained optimal configuration for different number of antennas.

Figure 3. Localization CRLB for the ULA and optimal configurations.

Figure 4. The location RMSE for different optimal, random, and ULA configurations ($M = 3$).

Figure 5. Optimal configuration of antennas for the case with separated transmitters and receivers.

Figure 6. Optimal configuration of antennas with the same values of ($M+N$) but different values of $M$ and $N$.

C. Target Geometry

The DOA of the target also affects the optimal configuration of antennas. Let us consider the case where each antenna is used as both the transmitter and the receiver. When $M = 4$ antennas are available, the optimization algorithm is implemented to find the optimal configuration of antennas. Figure 7 shows the results for different target DOAs. The results shown in Figure 7 imply that the optimal configuration at each DOA can be obtained from other configurations by a rotation around the center of mass. This characteristic could be also noticed by referring to (39) where the target DOA information appears in the matrix $P$. Now, the localization CRLB of the optimal configuration is shown at each target DOA in Figure 8. Since the CRLB is a function of the location of the target, it is also observed that the performance of the optimal configuration is affected by the DOA of the target. For example, for the DOA around $0$ rad, the worst performance is achieved while values around $\frac{\pi}{6}$ rad provide the lowest location CRLB.

The algorithm proposed in this paper assumes an exact
knowledge of the location of the target. If there is an uncertainty in the target location, the obtained configuration is not necessarily optimal. To explore how the uncertainty affects the localization performance, first, an optimal configuration is designed knowing that \( \theta = \frac{\pi}{6} \). Then, the obtained configuration is used for other hypothesized target DOAs. Figure 9 presents the resulted CRLB when the optimal configuration at \( \theta = \frac{\pi}{6} \) is used. For comparison, the CRLB is calculated for the optimal configuration at each DOA separately. It can be observed that for small uncertainties, both shown CRLBs are very close. Nevertheless, when the uncertainty increases the gap between CRLBs becomes higher with 100% difference at \( \theta = \frac{2\pi}{5} \) as the worst case. By further simulations, it can be observed that the uncertainty gap becomes tighter when more antennas is used. In other words, the more the antennas, the less sensitive the CRLB to the uncertainty in the target DOA.

Remark 1: The expectation in (18) is taken over both received signals and the target location. It is normally assumed that the location of the target is known and, therefore, the expectation is only taken over the received signals. The uncertainty gap discussed in this section occurs due to neglecting the uncertainty in the target location. It is possible to consider an uncertainty region for the target DOA and, then, reformulate the CRLB calculation. In this case, the designed optimal configuration becomes more robust to the uncertainty in the location of the target.

VI. CONCLUSION

This paper considered the antenna allocation problem in MIMO radar systems with collocated antennas. Deriving the CRLB, it was first shown that the distribution of antennas affects the localization performance. Then, it was justified that the FIM can be written as a convex function of the inter-antenna difference vectors when a single target is available in the surveillance region. Considering distance and geometric constraints, the final optimization problem was formulated as a standard SDP. Simulation results showed the superior localization performance under the new optimal configuration of antennas. As a future work, the optimization algorithm is being formulated for the case with multiple targets falling inside the same resolution cell.

REFERENCES


