Adaptive UKF for Target Tracking with Unknown Process Noise Statistics

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Abstract - With an application to target tracking with unknown process noise, adaptive UKF is presented. In this new algorithm, modified Sage-Husa noise statistics estimator is introduced to estimate the system process noise variance adaptively. By estimating the noise covariance online, the proposed method is able to compensate the errors resulting from the change of the noise statistics. Such a mechanism can improve the state estimation accuracy and enlarges its application scope. The simulations show that adaptive UKF can provide better performance in tracking accuracy than the standard UKF, especially in the case of unknown prior system noise statistics.

Keywords: Tracking, UKF, modified Sage-Husa estimator, adaptive method.

1 Introduction

Nonlinear filtering problems abound in many diverse fields including economics, and statistics and numerous statistical and array processing engineering problems such as time series analysis, communications, target tracking, and satellite navigation [1, 2, 3]. In radar tracking applications, target dynamics are usually modeled in Cartesian coordinates, while the measurements are directly available in the original sensor coordinates. Hence, tracking in Cartesian coordinates using sensor coordinates measurements is actually a nonlinear state estimation problem.

As is well known, the most widely used filter in radar tracking application is extended Kalman filter (EKF), which is based upon the principles of linearizing the nonlinear models using Taylor series expansions. Due to the assumptions of local linearization and the computation of the Jacobin matrix of the state vector, EKF may provide poor performance or diverges [4, 5].

In recent years, a large number of suboptimal approaches based on linearization techniques have been developed. Julier and Uhlm ann presented a novel filter named the unscented filter (UKF) [6-8]. Unlike the EKF, UKF uses a set of chosen samples to represent the state distribution. UKF shows higher approximation accuracy without the calculation of the Jacobian and Hessina matrices. The UKF method has proven to be superior to EKF in target tracking and other applications and attracted great attention [6-9].

In the application of target tracking, however, the exact knowledge of the process noise matrix which is required in the framework of Kalman filter are usually unknown and time-varying in practice. The use of wrong prior statistics in the Kalman filter can lead to large estimation errors or even to a divergence of errors. Because of the uncertain process noise, the standard UKF provides poor performance in robustness and tracking accuracy.

In this paper, an adaptive UKF is proposed to address these problems. In the proposed algorithm, we adopt modified Sage-Husa noise statistics estimator to estimate and adjust the process noise covariance. Based on the Sage’s Estimator for constant noise statistics [2], an exponentially weighted fading-memory estimator called modified Sage-Husa estimator is proposed in [10], which can recursively estimate the unknown time-varying mean and covariance matrices simultaneously. Embedding the recursive modified Sage-Husa virtual noise statistics estimator into the UKF, the adaptive UKF can handle the recursive state filtering in the presence of unknown process noise covariance matrices.

This work is organized as follows. The problem definition is described in Section 2. Section 3 introduced general unscented Kalman filter. The proposed adaptive UKF is presented in Section 4. Simulation results and analysis are given in Section 5. Finally, Conclusions are discussed in Section 6.

2 Problem definition

In target tracking the motion of a target can be described generally in Cartesian coordinates by the following state-space model.

\[ x_{k+1} = Fx_k + v_k \]  

(1)
where $\mathbf{x}_k$ is the target motion state vector of Cartesian coordinates at time $k$. $\mathbf{F}$ is the state transition matrix, $\mathbf{G}$ is the noise gain matrix. $\mathbf{v}_k$ is the system noise process which is modeled as a zero-mean white Gaussian random process with known covariance matrix $\mathbf{Q}_k$.

The radar measurement polar coordinate of the target position at discrete time includes range and bearing is related to the Cartesian coordinate target state as follows.

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{w}_k$$  \hspace{1cm} (2)

where $\mathbf{z}_k$ is the vector of polar coordinates measurements, $\mathbf{w}_k$ is the observation noise process additive zero-mean Gaussian noise vector with variance $\mathbf{R}_k$. Then Target tracking becomes the nonlinear problem of estimating the target states from the noisy polar measurements.

### 3 Standard UKF

In this section the principle of classical UKF is introduced. Consider the general discrete nonlinear system:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{v}_{k-1})$$  \hspace{1cm} (3)

$$\mathbf{y}_k = h(\mathbf{x}_k, \mathbf{w}_k)$$  \hspace{1cm} (4)

where, $\mathbf{x}_k$ and $\mathbf{y}_k$ denote the state variable and observations at time $k$, $f$ and $g$ are known possibly nonlinear functions. $\mathbf{v}_k$ and $\mathbf{w}_k$ are independently distributed (i.i.d.) system noise and observation noise sequences respectively, which are assumed to Gaussian white noise with mean.

$$\mathbf{v}_k \sim \mathcal{N}(\mathbf{v}_k, \mathbf{Q}), \mathbf{w}_k \sim \mathcal{N}(\mathbf{w}_k, \mathbf{R}).$$

In UKF, the state distribution is represented by the deterministically chosen sample points which can capture specific mean and covariance of the distribution [6, 7]. The nonlinear function is applied to each of these points to yield a transformed sample, and then the predicted mean and covariance are calculated from the transformed sample.

The UKF estimation can be described briefly as follows

**Initialization:**

$$\bar{\mathbf{x}}_0 = E(\mathbf{x}_0)$$  \hspace{1cm} (5)

$$\mathbf{P}_0 = E((\mathbf{x}_0 - \bar{\mathbf{x}}_0)(\mathbf{x}_0 - \bar{\mathbf{x}}_0)^T)$$  \hspace{1cm} (6)

**Sigma point’s calculation:**

A set of $2n+1$ weighted samples are deterministically chosen as follows

$$\chi_{k-1} = \left[ \hat{\mathbf{x}}_{k-1|k-1} \quad \hat{\mathbf{x}}_{k-1|k-1} \pm ((n + \lambda)\mathbf{P}_{k-1|k-1})^{1/2} \right]$$  \hspace{1cm} (7)

**Time update:**

$$\chi_{k|k-1} = f(\chi_{k-1})$$  \hspace{1cm} (8)

$$\bar{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n} w_i^p \chi_{i,k|k-1}$$  \hspace{1cm} (9)

$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2n} w_i^p \left[ \chi_{i,k|k-1} - \bar{\mathbf{x}}_{k|k-1} \right]$$

$$\left[ \chi_{i,k|k-1} - \bar{\mathbf{x}}_{k|k-1} \right]^T + Q$$  \hspace{1cm} (10)

**Measurement update**

$$\gamma_{k|k-1} = h(\chi_{k|k-1})$$  \hspace{1cm} (11)

$$\bar{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1}$$  \hspace{1cm} (12)

where

$$w_0 = \lambda/(n + \lambda)$$  \hspace{1cm} (13)

$$w_i^m = w_i^f = 1/2(n + \lambda) \hspace{0.5cm} i = 1, 2, \cdots, 2n$$  \hspace{1cm} (14)

$$w_i^f = \lambda/(n + \lambda) + (1 - \alpha^2 + \beta)$$  \hspace{1cm} (15)

$$\lambda = \alpha^2(n + k) - n$$  \hspace{1cm} (16)

$$\alpha = \frac{1}{\sqrt{n + \lambda}}$$  \hspace{1cm} (17)

$$\beta = \frac{\lambda}{n + \lambda}$$  \hspace{1cm} (18)

$$\sigma_i^2 = \frac{\lambda}{n + \lambda} \hspace{1cm} i = 1, 2, \cdots, 2n$$  \hspace{1cm} (19)

**Measurement update**

$$\gamma_{k|k-1} = h(\chi_{k|k-1})$$  \hspace{1cm} (17)

$$\bar{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1}$$  \hspace{1cm} (18)

$$\mathbf{P}_{y_k|x_k} = \sum_{i=0}^{2n} w_c \left[ \gamma_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right] \left[ \gamma_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right]^T + R$$  \hspace{1cm} (19)

$$\mathbf{P}_{y_k|x_k} = \sum_{i=0}^{2n} w_c \left[ \gamma_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right] \left[ \gamma_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right]^T + R$$  \hspace{1cm} (20)
\[ K_k = P_{x_k|y_k} P_{y_k|y_k}^{-1} \]  
(21)

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \tilde{y}_{k|k-1})
\]  
(22)

where the variable definitions are given as follows: \( \{w_i\} \) is a set of scalar weights, \( n \) is the state dimension, \( \lambda \) is a scaling parameter. Three parameters \( \alpha \), \( \beta \) and \( k \) are introduced to tune the filter [7, 8].

4 Adaptive UKF

4.1 Modified Sage-Husa estimator

When the process noises are unknown and time-varying, the filtering algorithm cannot be recursively carried out in a common way. One solution to this problem is to estimate the statistical parameters of the virtual noises online along with the recursive estimate of the state. With noise statistics estimator to estimate the covariance of virtual process noise, an adaptive filter (AUKF) can be established. It should be noted that adaptive filtering algorithms cannot estimate process and measurement noise simultaneously. Usually, the measurement noise statistic is relatively well known compared to the system model noise.

Assume \( Q_k \) are unknown, then the corresponding Sage-Husa process noise statistics estimator is given by [11, 12]

\[
\hat{q}(k) = \frac{1}{k} \sum_{j=1}^{k} \left[ \hat{x}(j|k) - f(\hat{x}(j-1|k)) \right]
\]  
(23)

\[
\hat{Q}(k) = \frac{1}{k} \sum_{j=1}^{k} \left[ \hat{x}(j|k) - f(\hat{x}(j-1|k)) - q \right]
\]

\[
\left[ \hat{x}(j|k) - f(\hat{x}(j-1|k)) - q \right]^T
\]  
(24)

But in maneuvering target tracking, the effect of the latest residual should be emphasized much more. This can be achieved by different weighting coefficient for each item in the above formula. By using the exponentially weighted fading memory method [10, 12], weighting coefficient \( \{\beta_j\} \) can be chosen as follows

\[
\beta_j = \beta_{j-1} b^j, \quad 0 < b < 1, \quad \sum_{j=0}^{k-1} \beta_j = 1.
\]

According to the above characteristics of the weighting coefficient \( \{\beta_j\} \), we can deduce that

\[
\beta_j = d_{k-j} b^j, \quad d_{k-j} = (1-b)/(1-b^k) \quad j = 0, 1, \ldots, k-1
\]

A recursive modified Sage-Husa estimator can be obtained as follows

\[
\hat{q}_k = (1-d_{k-j}) \hat{q}_{k-1} + d_{k-j} \left[ \hat{x}_{k|k} - f(\hat{x}_{k-1|k-1}) \right]
\]  
(25)

\[
\hat{Q}_k = (1-d_{k-j}) \hat{Q}_{k-1} + d_{k-j} \left[ K_k V_k V_k^T K_k^T + P_{k|k} \right]
\]

\[
- F_{k-1} P_{k-1|k-1} F_{k-1}^T
\]  
(26)

where, \( b \) is a forgetting factor, and \( 0.95 < b < 0.995 \). \( V_k \) is residual

\[
V_k = y_k - H_k \bar{x}_{k|k-1}
\]

4.2 Adaptive UKF

Embedding the modified Sage-Husa virtual noise statistics estimator into the UKF, we obtain the adaptive UKF (AUKF), which combines the advantages of the UKF and the modified Sage estimator. In the follows, we give the detailed algorithm of the AUKF.

Initialization:

\[
\hat{x}_0 = E(x_0)
\]  
(27)

\[
P_0 = E[(x - E(x_0))(x - E(x_0))^T]
\]  
(28)

\[
\hat{Q}_0 = Q_0, \quad \hat{q}_0 = q_0
\]  
(29)

Prediction:

\[
\chi_{k-1} = \left[ \hat{x}_{k-1|k-1} - \hat{x}_{k-1|k-1} + ((n+\lambda)P_{k-1|k-1})^{1/2} \right]
\]

\[
\hat{x}_{k-1|k-1} - ((n+\lambda)P_{k-1|k-1})^{1/2}
\]  
(30)

\[
\chi_{k|k-1} = f(\chi_{k-1}) + \hat{q}_k
\]  
(31)

\[
\bar{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \chi_{i,k|k-1}
\]  
(32)

\[
P_{k|k-1} = \sum_{i=0}^{2n} w_i^m \left[ \chi_{i,k|k-1} - \bar{x}_{k|k-1} \right] \left[ \chi_{i,k|k-1} - \bar{x}_{k|k-1} \right]^T
\]

\[
+ \hat{Q}_{k-1}
\]  
(33)

Measurement update

\[
\gamma_{k|k-1} = h(\chi_{i,k|k-1})
\]  
(34)
\[ P_{y, y_k} = \sum_{i=0}^{2n} w^i_c (\gamma_{i,k|k-1} - \bar{y}_{k|k-1}) \]

\[ \gamma_{i,k|k-1} - \bar{y}_{k|k-1} = \gamma_{i,k|k-1} \]

\[ P_{x, y_k} = \sum_{i=0}^{2n} w^i_c (\gamma_{i,k|k-1} - \bar{x}_{k|k-1}) \]

\[ \gamma_{i,k|k-1} - \bar{x}_{k|k-1} = \gamma_{i,k|k-1} \]

\[ R_k = P_{x, y_k} \gamma_{y, y_k}^{-1} \]

\[ K = P_{x, y_k} \gamma_{y, y_k}^{-1} \]

\[ P_{k|k} = P_{k|k-1} - K_k P_{y, y_k} K^T \]

\[ \dot{q}_k = (1 - d_{k-1} \dot{q}_k - d_{k-1}) [K_k V_k V_k^T K_k + P_{k|k}] \]

\[ -F_{k-1} P_{k|k-1} F_{k-1}^T \]

5 Numerical simulations

5.1 Simulation scenarios

Consider the scenario of a tracking problem as follows. For simplicity, assume the radar is located at the origin. The nonlinear system dynamics at discrete time can be described as

\[ x_k = F x_{k-1} + v_k \]

where, \( v_k \) is a zero-mean Gaussian noise vector with unknown covariance \( Q_k \).

\( Q_k \) is designed to change according to

\[ Q_k = \begin{cases} \text{diag} \{0.05^2, 0.05^2\} & 0 \leq k \leq 125 \\ \text{diag} \{0.4^2, 0.4^2\} & 126 \leq k \leq 215 \\ \text{diag} \{0.2^2, 0.2^2\} & 216 \leq k \leq 340 \\ \text{diag} \{0.8^2, 0.8^2\} & 341 \leq k \leq 370 \\ \text{diag} \{0.2^2, 0.2^2\} & 371 \leq k \leq 450 \end{cases} \]

The intervals between the samples are \( T = 5s \). An aircraft target has a nearly CV motion form (50000m, 50000m) with an initial velocity of 200m/s for 125s before executing a \( 1^\circ/s \) coordinated turn for 90s. Then it flies westward for another 125s, followed by a \( 3^\circ/s \) turn for 30s. After the turn, it continues to fly at constant velocity.

Measurements in a polar format taken by radar at discrete time include range and bearing, given by

\[ z_k = \begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = h(x_k) + w_k \]

\[ = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \tan^{-1} \left( \frac{y_k}{x_k} \right) \end{bmatrix} + \begin{bmatrix} v_{r_k} \\ v_{\phi_k} \end{bmatrix} \]

where \( w_k \) is an additive zero-mean Gaussian noise vector with variance \( R_k = \text{diag} \{ \sigma_r^2, \sigma_\phi^2 \} \), \( \sigma_r = 20m \) and \( \sigma_\phi = 1^\circ \) are standard deviations for range and bearing, respectively.

The following initialization of the state estimates and covariance matrix is used

\[ \hat{x}_0 = [50010 -118 50010 2]^T \]

\[ \hat{P}_0 = \text{diag} \begin{bmatrix} 100 & 5 & 100 & 5 \end{bmatrix} \]

5.2 Simulation results and analysis

The proposed adaptive UKF (AUKF) and standard UKF are applied to the scenario. Fifty Monte Carlo simulations are performed and all the filters use the same trajectories. The root mean square error (RMSE) is utilized to evaluate the performances of the filters

\[ RMSE = \left( \frac{1}{N} \sum_{k=1}^{N} (\hat{x}_k - x_k)^2 \right)^{1/2} \]

For reference, the true track of the target and state estimations are shown in Figure 1. The Figures 2-3 Compare the RMSEs of the position and velocity estimates. In addition, The Average target state estimation RMSE (based on 100 Monte Carlo runs) is also shown in Table 1.
It can be seen from the Figures.1-3 that the AUKF performs significantly than the standard UKF. Since the unknown process noise can lead to large estimation errors, the state estimate of standard UKF sometimes may deviate from the true state very large, especially when target executes a coordinated turn. Figures.2-3 compare the RMSEs of different filters across 450 time steps, show that AUKF can approximate the true state better than standard UKF and have relatively consistent stability. Note that as expected in Table1, the average position and velocity estimations RMSE of AUKF are obviously less than that of standard UKF.

The reason lies in that the modified Sage-Husa estimator can estimate the unknown process noises online whereas the standard UKF depends on the fixed prior knowledge about the process noises. In new algorithm, the estimated noise statistics are further used by the master UKF to adaptively compensate the influence caused by inaccurate a priori knowledge and changing statistics of system noise.

The average computational times (one Monte Carlo run) on dual 2.2GHz Intel processors for a tracking period of 450 time steps are 3.1304s, and 4.0257s for UKF and AUKF algorithms respectively. In comparison to UKF, AUKF presents better accuracy with adding less computation cost and complexity. Some increase in computational burden and complexity is considered acceptable.

In summary, according to the contents mentioned above, it can be concluded that the performance of the AUKF is superior to standard UKF.

### 6 Conclusions

In this paper, a new adaptive unscented Kalman filter, which is based on modified Sage estimator, has been proposed for target tracking with unknown system noise statistics. The simulation results show that the proposed filter provides better performance in tracking accuracy than the Unscented Kalman filter.

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