Use of Prior Information in Active Sonar Tracking

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Abstract – A Bayesian tracking model is proposed that uses measurement likelihood functions based on predicted signal-to-noise ratios (SNR) in active sonar data. The predicted SNR modeling can incorporate prior information, such as the presence of known discrete and persistent clutter objects. The likelihood model assumes an exponential distribution of returns with a mean based on the predictive model that incorporates assumed SNR of the targets, known clutter, and background clutter, and the beam response and waveform ambiguity functions. Two variations of an example based on simulated frequency modulated (FM) and continuous wave (CW) signals is used to assess target detection and localization performance. Significant enhancements are observed when prior knowledge of clutter is incorporated into the measurement model in these idealized examples.

Keywords: Bayesian tracking, active sonar, clutter.

1 Introduction

The undersea active sonar problem presents a rich diversity of contextual information which can be vital for situational awareness but too often is ignored by automated tracking and classification systems. Such information may include, for example, surface ship traffic monitored by passive sonar, radar, or automatic identification system (AIS) receivers. It may also include ship wrecks, rock outcroppings, and other fixed acoustic scatterers that have been identified through prior surveillance. Clearly the incorporation of prior knowledge, when done properly, will lead to increased performance. The technical challenges are in determining what useful information is available and how to incorporate it.

In a standard active sonar tracking problem, clutter is treated as random, temporally uncorrelated fluctuations in the background intensity [1, 2]. High amplitude clutter may lead to false track initializations, but the lack of motion consistency will tend to cause the track to be dropped. While the majority of clutter may be modeled in this manner, a realistic environment will contain numerous sources of persistent clutter as well. Without additional association logic, such clutter will inevitably lead to false tracks.

The problem is similar to that of passive sonar interferers, where loud surface vessels can mask quiet targets of interest. The effects of these loud interferers can be mitigated by use of adaptive beamforming methods [3] or in some cases the presence of interferers can be removed from the data before it is processed [4]. An approach without altering the data is to incorporate knowledge of interferers directly in the measurement likelihood function of a Bayesian tracker [5].

A Bayesian formulation of the tracking and classification problem provides a natural framework to incorporate prior knowledge [6]. While the prior distribution would seem an obvious place for this, it can only reference the state of an unknown target(s) of interest. Thus, information such as a coastline or group of islands might limit the range of possible target states and thereby be incorporated into the prior. Information regarding known clutter, however, will affect the measurement but not necessarily the target state. The measurement likelihood function is therefore a natural place to incorporate such information. One may remove the clutter-originated returns and thereby redefine what constitutes a measurement, but this requires one to determine exactly which returns originate from the clutter object. The Bayesian approach allows for a statistical modeling of a known clutter object’s contribution to the overall measurement.

In this paper, we consider the use of prior information for active sonar tracking of an unknown target in the presence of clutter. We assume that there are both randomly scattered clutter returns that vary from scan to scan and some persistent objects, such as wrecks, that lead to returns with consistent positions and known mean SNR. The tracker compares two assumptions: that either a single target is present or no target is present. The outputs are the probability that a target is present and a distribution over the state space of where the target is likely to be, assuming it is indeed present.

We introduce a method for accounting for known clutter objects by modifying the measurement likelihood function in the Bayesian tracker described in Section 2. We use predictive SNR modeling of the targets and clutter objects, as described in Section 3. The implementation and examples are described in Section 4, and the results are given in Section 5. Future work is discussed in 6 and the conclusions are summarized in 7. An appendix provides additional derivations.

2 Tracking Overview

Consider a Bayesian tracking scheme for which the state space consists of the number of targets present (either zero
or one) in the modeled state space and the target’s kinematic state, \( s \), given that it is present. Let \( p_n \) be the probability that a single target is present in a particular region of interest, and let \( \rho_n(s) \) be the probability density function (PDF) for the target kinematic state after the \( n \)th measurement \( y \). (Note that \( n = 0 \) corresponds to the prior distribution.) This is represented by the clutter and target likelihood functions, \( L_n(y|\emptyset) \) and \( L_n(y|s) \), respectively. Measurement updates on the kinematic PDF are performed using Bayes’ theorem,

\[
\rho_n(s) = \frac{L_n(y|s)\rho_{n-1}^+(s)}{E_n},
\]

where \( \rho_{n-1}^+(s) \) is the motion updated PDF and

\[
E_n := \int L_n(y|s)\rho_{n-1}^+(s)\,ds.
\]

Similarly, the target probability is updated by

\[
p_n = \frac{E_n\rho_{n-1}^+(s)\rho_{n-1}(s)}{\rho_{n-1}^+(s)\rho_{n-1}(s) + E_n},
\]

where \( \rho_{n-1}^+ \) is, again, the motion updated probability. Additional description of the motion model and birth/death process used in the Bayesian tracker is provided in Ref. [7].

The form of the likelihood function is very important. For example, one common approach in bearing-only Bayesian tracking is to assume that the measurement error is Gaussian in bearing [8, 9]. In practice, the errors can be very non-Gaussian, and often individual contacts are not associated with the target of interest. We use a model that is based more closely on the actual signal propagation and processing.

We define the likelihood model in terms of measured signal-to-noise (SNR) values from the normalized matched filter output of a standard active signal processing string. Both frequency modulated (FM) and continuous wave (CW) transmit waveforms are considered. Each SNR value (squared amplitude) \( z_k \) is associated with a particular echo time of arrival (TOA) \( \tau_k \), angle of arrival (AOA) \( \phi_k \), and for (CW) Doppler frequency shift \( \nu_k \). Each measurement \( y \) consists of \( K \) returns \( y_k \) such that \( y = (y_1, \ldots, y_K) \), where for a CW waveform \( y_k = (z_k, \tau_k, \phi_k, \nu_k) \), and for an FM waveform \( y_k = (z_k, \tau_k, \phi_k) \). The SNR values are assumed to be independent and exponentially distributed. The likelihood function is

\[
L_n(y|s) = \prod_{k=1}^{K} \frac{1}{\mu_k(s)} \exp[-z_k/\mu_k(s)],
\]

where \( \mu_k(s) \) represents the mean SNR that one would expect to receive from measurement element \( y_k \), e.g. at \( \tau_k \) and \( \phi_k \) for an FM source signal, given that the target state is \( s \). The use of this prediction is the core of the approach to modeling the measurement likelihoods and incorporating information about known clutter objects. The general idea is to compare the observed signal to the predicted based on an assumed model, as described in more detail in Section 3.

We choose to use thresholded measurements in order to reduce the false alarm rate and computational cost. Accordingly, only individual elements with SNRs that exceed a set threshold \( \eta \) are incorporated in to the measurement. Let \( k = (k_1, \ldots, k_I) \) denote an ordered sequence of indices corresponding to these threshold crossings. (If there are no threshold crossings, then \( k = \emptyset \).) For such measurements the likelihood is (see Appendix for derivation)

\[
L_n(k, y|s) = L_n(\emptyset, y|s) \prod_{i=1}^{I} \frac{1 - F_{\mu_k(s)}(\eta|s)}{F_{\mu_k(s)}(\eta|s)} f_k(z_{k_i}|s, \eta),
\]

where

\[
L_n(\emptyset, y|s) = \prod_{k=1}^{K} F_{\mu_k(s)}(\eta|s),
\]

\[
f_k(z_{k_i}|s, \eta) = \frac{1}{\mu_k(s)} \exp[-(z_{k_i} - \eta)/\mu_k(s)],
\]

and

\[
F_{\mu_k(s)}(\eta|s) = 1 - \exp[-\eta/\mu_k(s)].
\]

### 3 SNR Predictive Modeling

Under the hypothesis that no target is present and there are no persistent clutter objects, we assume a uniform clutter background for all points \( k \) in measurement space, i.e.,

\[
\mu_k(\emptyset) = \sigma_0^2.
\]

If a single target is assumed to be present, then

\[
\mu_k(s) = \mu_k(\emptyset) + \sum_{m=1}^{M} \sigma_m^2 h_k(s_m),
\]

where \( \sigma_0^2 \) is the target SNR and \( h_k(s_m) \in [0, 1] \) is the product of the array beam response \( b(\phi) \) and waveform ambiguity function \( \chi_{CW}(\tau, \nu) \) for a target in state \( s_m \). The exact functions are described in the data generation and tracker model sections (Sections 4.1 and 4.2 respectively).

If there are \( M \) known clutter objects, each one can be modeled using the same method as the target object. Let \( s_m \) denote the current estimate of the kinematic state of the \( m \)th such object, let \( \sigma_m^2 \) denote the mean SNR of the \( m \)th clutter object, and let \( h_k(s_m) \in [0, 1] \) denote the product of the array beam response and waveform ambiguity function for a clutter object \( m \) with state \( s_m \). The total clutter is contribution is

\[
\mu_k(\emptyset) = \sigma_0^2 + \sum_{m=1}^{M} \sigma_m^2 h_k(s_m).
\]

If a target is assumed to be present, then the total mean received SNR at point \( k \) in measurement space should be

\[
\mu_k(s) = \sigma_0^2 + \sum_{m=1}^{M} \sigma_m^2 h_k(s_m) + \sigma_k^2 h_k(s).
\]

Eq. 12 gives the predicted mean SNR at point \( k \) in measurement space (e.g. a time and bearing pair) conditioned on the assumption that the target is in state \( s \). If the target
and a clutter object are near each other in state space, then they will have a similar range and bearing. If one considers a measurement near those, then reverberation from both objects will contribute to the mean SNR at that point. If the objects are far apart, there will be only the contribution of each single object in its own neighborhood. A target that passes near a known clutter object will be detectable because the predicted and actual mean SNR values for the corresponding points in measurement space will be larger than predicted assuming the target is not near the clutter. If Doppler data is available, then the predicted contributions from the stationary object and that for the moving object will map from the state space into different regions of the measurement space (separated in velocity). We model point targets and clutter, leaving distributed features (e.g., large bathymetric features) for future work.

4 Implementation and Examples

4.1 Data generation

The data are generated according to the model in Sections 2 and 3, with the background clutter set to a mean SNR of $\sigma^2 = 1$ (units of amplitude squared). We assume that scans are available every 120 seconds, and that the CW and FM scans are simultaneous in different bands, with windowing in the signal processing separating the two. As common in many signal processing schemes, the possible measurements are binned into equally spaced discrete values over the measurement space. Specifically, for the FM measurements, there are 18000 time bins and 72 beams. For CW, there are 108 time bins, 72 beams, and 71 Doppler bins.

The SNR at each combination of time, bearing, and (for CW) Doppler shift could be one of the $K$ individual elements $y_k$ of the overall measurement. For each of these points in the measurement space, a received SNR is simulated for the given target state and clutter states by sampling an exponential distribution with mean $\mu_k(s)$, calculated using Eq. 12. These returns are then thresholded at 10 dB for both FM and CW signals. Assuming only uniform clutter, this gives an average of 58 threshold crossings per scan for FM and 25 for CW.

The ambiguity functions $h_k(s)$ and $h_k(s_m)$ for the target and the clutter objects, respectively, are identical in structure. The beam response, as modeled here, is given by

$$b(\phi) = \sin^2(\phi/\Delta \phi),$$

where $\phi = \phi(s) - \phi_k$, $\phi(s)$ is the bearing to the hypothesized target (or clutter object) state, and $\Delta \phi$ is the nominal beam width, which is set to 0.0873 radians (i.e., 5 degrees).

For the CW waveform, the ambiguity function is

$$\chi_{\text{CW}}(\tau, \nu) = \left(1 - \frac{|\tau|}{T}\right) \text{sinc} \left[\nu T \left(1 - \frac{|\tau|}{T}\right)\right],$$

where $T$ is the pulse length. For a linear FM, it is given by

$$\chi_{\text{FM}}(\tau, \nu) = \chi_{\text{CW}}(\tau, \nu - \tau B/T),$$

where $B$ is the bandwidth. Combining these results and letting $\tau(s)$, $\phi(s)$, and $\nu(s)$ be the TOA, AOA, and Doppler shift (respectively) corresponding to the hypothesized state $s$, we have

$$h_k(s) = b(\phi(s) - \phi_k)|\chi(\tau(s) - \tau_k, \nu(s) - \nu_k)|^2.$$

4.2 Tracker assumptions

We use a grid-based implementation of the Bayesian tracker due to its stability, which allows a more rigorous study of varying parameters of the problem than with an alternative such as a particle filter. While particle filters are very useful in some applications, the inherent randomness and necessary tuning to a particle problem make them less useful for comparing different implementations of other aspects of the Bayesian tracking such as the likelihood functions. For example, likelihoods that have large flat regions or discontinuities may require a different particle filter implementation than is optimal for a smooth, well-behaved likelihood function.

We define a position grid of $N_x \times N_y$ uniform cells and a velocity grid of $N_{vx} \times N_{vy}$ uniform cells, where $N_x = N_y = 91$ and $N_{vx} = N_{vy} = 11$. A prior distribution that is uniform in position and a Gaussian with mean at (0, 0) in velocity is assumed. The initial probability of a target being present $p_0$ is set to 0.5. The Gaussian prior in velocity is chosen because of its stability properties over long motion updates (see Ref. [7] for more details). One negative of using this prior is that initially the most likely speed of the target is zero, which is more consistent with clutter than with a target of interest. Future work will consider additional prior distributions, as this may allow for more discrimination between clutter and targets when there are few measurements.

The likelihood function (Eq. 5) is evaluated by calculating predicted means $\hat{\mu}_k(s_i)$ for each target state grid cell. The ambiguity functions used for these calculations are broader (e.g., falling off more slowly and smoothly from one to zero) than in the data generation stage in order to mitigate the sparse sampling on the state space. If the grid is too coarse and the ambiguity function too narrow, then a measurement could fall into a grid cell and still lead to a calculation of zero likelihood for that grid cell. Ideally one would integrate across the volume of the cell. A parallel research effort is developing efficient methods for this integration.

Specifically, the waveform ambiguity function assumes a Gaussian waveform with linear frequency modulation as stated in [10] and defined such that

$$|\chi(\tau, \nu)|^2 = \exp\left[-\pi(B_{\text{eff}}^2 \tau^2 + t_{\text{eff}}^2 \nu^2)\right],$$

where $\tau = \tau(s) - \tau_k$, $\nu = \nu(s) - \nu_k$, $B_{\text{eff}}$ is the effective bandwidth, and $t_{\text{eff}}$ is the effective pulse length. For this example, $\nu = 0$ for the FM waveform. The effective bandwidth is set to the true bandwidth divided by 300 for FM and the true bandwidth divided by 2.5 for CW. The effective pulse length is set to the true pulse length for FM and to the true pulse length divided by 2.5 for CW. The different form and additional scaling of the parameters are used in order to broaden the functions out so that they adequately cover the discrete state space and the discrete measurement space. As another
means of mitigating the discretization problem, the state \( s_i \) is sampled uniformly randomly from cell \( i \) in the state space grid in order to add diversity to the sample space and to reduce the impact of artifacts that can arise from a fixed pattern of sampling, such as using the center of each cell.

For example, using a 5x5 position grid and a 3x3 velocity grid then if only the center of each grid cell were used, there would only be one value used for each position grid cell. For a grid cell with center at position (1 km, 1 km) and sides length 2 km, there are actually 9 such grid cells, one for each of the possible combinations of velocities. This particular position cell will actually be sampled 9 times. With a random sampling, nine different positions that fall in that original cell will be sampled instead of just one.

In the FM case (in which the bearing and time delay measurements are independent of velocity), this leads directly to a better sampling of the overall state space. In the CW case (with correlated time delay and Doppler estimates), this adds diversity to the sample and on average can be expected to improve the accuracy of the likelihood function evaluation.

Perfect knowledge of the position and mean SNR of the fixed clutter objects is assumed. However, the exact measurements are not known because they are random functions of the actual state of the object. Similarly, the mean SNR of the target is assumed to be known, but the position, velocity, and actual SNRs of the measurements are not known. The long range goal of this method is to estimate the target and clutter SNRs from the data, but that remains future work (see Section 6). The SNR threshold of the signal processor is known to be 10 dB for FM and CW signals.

### 4.3 Example scenario

For the example summarized in Fig. 1, we consider a state space with dimensions ranging from -35 km to 30 km East in position, -30 km to 30 km North in position, and ranging from -12 m/s to 12 m/s in both North velocity and East velocity. A single source is placed at the origin, and a single receiver is placed at (0 km, 5 km). The assumed pulse length is 1 second and the assumed maximum detection range (due to windowing in the signal processing) is a time delay of 35 seconds. This creates a hard outer detection limit in time delay. There is also a blanking region near the source-receiver pair.

Three fixed clutter objects are present, as follows:

1. **Fixed at (15 km, 15 km) with mean SNR of 15 dB**
2. **Fixed at (-17 km, 16 km) with mean SNR of 15 dB**
3. **Fixed at (0 km, -4 km) with mean SNR of 15 dB**

The target originates at coordinates of (-25 km, 28 km) and moves with a constant velocity of (3.5 m/s, -5.0 m/s). The mean target SNR is assumed to be fixed at 15 dB independent of position. As shown in Fig. 1, the target trajectory enters the detection region near the second clutter object. It then continues through a region with no fixed clutter objects before approaching the third clutter object. In this example, the SNRs of the target and all fixed clutter objects are the same.

This tests a basic case in which they are very similar and hypothetically more difficult to distinguish.

The crossovers with known clutter are important test cases. The first clutter crossing is just as the target enters the detection region. It is important that a method that removes the clutter measurements does not also remove returns from the actual target. For example, one method to remove clutter objects from the tracker is to search the measurement space near a clutter object and just remove any returns that fall inside this neighborhood. However, this also would remove any returns that actually came from a target that fell in that region.

### 5 Analysis of Tracking Performance

The output of the tracker has two main components: the probability that a target is present (or the **target probability**), and the distribution of the state of a target conditioned on it being present, i.e. inside the modeled state space. We examine these results in two scenarios using the example problem and data described above. In the first scenario, the known clutter objects are ignored in the tracking stage. This means that the tracker uses the predicted mean SNR from Eq. 10 when evaluating the likelihood function. In the second scenario, the known clutter is modeled and thus the likelihoods are evaluating using the predicted mean SNR from Eq. 12.

#### 5.1 Probability of target being present

In the example, the target is present in the modeled state space for the entire simulation. It begins outside of the detection region, enters the detection region, and then exits. Therefore, the target probabilities output by the tracker should reflect this behavior. The actual posterior (i.e. after measurement updates) target probabilities are show in Fig. 2.

In the base case (with no known clutter modeling), the target probability is one for the entire run. This is expected because the clutter objects yield relatively large returns that are consistent with a target being present during the entire simulation, even when the target is outside the detection region.
In a practical sense, this leads to a large false alarm rate; the tracker is concluding that there is a target present when the target is invisible to the sensors. In truth, it is not a false alarm because the target is actually present in the state space during the entire simulation. However, the same (but now incorrect) conclusion would be drawn if the target were not present. While the target is outside the detection region, it has no effect on the tracker and we expect no target detection.

In the case with known clutter modeling, the target probability starts at the prior value of 0.5 and falls to near zero. This initial decline reflects that the received measurements are not very consistent with the existence of a target in the detection region. As these measurements accrue, the evidence for the absence of a target dominates the prior probability of 0.5. At time 30 minutes, the target has entered the detection region and yields several loud returns that are consistent with the presence of a target, so the target probability jumps to one.

The target probability remains at one until very near the end of the simulation. Although not obvious in the figure, the target probability begins to decrease ever so slightly starting at time 168 minutes, although this drop is insignificant until time 184 minutes. Interestingly, the target leaves the detection region at time 166 minutes. The delay in the target probability can be explained by two factors.

First, just before the target leaves the detection region, it is very well localized (as will be discussed in the next subsection). The motion update stage of the Bayesian tracker projects the estimate of the target state forward in time using the position and velocity estimates as well as the diffusion and damping models. Consequently, the tracker predicts that the target will move outside of the detection region.

The second factor is the form of the likelihood model outside of the detection region. The likelihood of any feasible (i.e. inside the detection region) measurement conditioned on the target being in a state outside the detection is constant, and related to the background noise and the clutter only (as in Eq. 11). Since the (motion-updated) prior puts nearly all of the probability mass outside the detection region, the likelihood function that results from the random clutter and the modeled fixed clutter is very inconsistent with a target inside the detection region, which effectively raises the likelihood probability that the target, if present, is outside the detection region. Since this is consistent with the prior distribution, there is little reason to lower the target probability because everything is still in agreement with the assumption of a target being present outside the detection region.

5.2 Localization performance

The localization error is a very important measure of tracker performance. The error in the posterior MAP (maximum a posteriori) estimate of the target position is shown in Fig. 3. For the base (no clutter) model, a trend appears. It seems as if the tracker has a baseline performance level that follows a “v” shape and then suffers deviations from this baseline. However, this is really just an artifact of the scenario. When the clutter is not modeled in the tracker’s likelihood, the tracker tends to track the clutter objects, and the MAP estimate will usually fall on one of the clutter objects. Since the target path takes it near two clutter objects, it is possible for small errors in localization to occur just because the target is near a clutter object that is being tracked erroneously.

For example, when the target enters the detection region, it is near one clutter object. Erroneously tracking the clutter object yields a small localization error. Additionally, when a clutter object and the target near each other, there will be more threshold crossings in that region. Combined, the unknown clutter object and target appear more like a target than any clutter object alone. As the target moves away from the clutter object, the localization errors spike to higher levels.

There are times where the base tracker successfully localizes the target when it is far from clutter objects, such as the period from 74 minutes to 82 minutes. However, the MAP estimate then moves back toward the clutter object at (0 km, -4km). Because the target is approaching this point, the error in the localization decreases to near zero as it passes by the clutter object. As the target moves away, the MAP estimate
jumps to the clutter object at (15 km, 15 km) before returning to the object at (0 km, -4 km). For the remainder of the simulation, the MAP estimate stays primarily on this object, while occasionally moving to the target or another clutter object.

The localization performance with the known clutter model is markedly improved. The error while the target is outside the detection region is understandably large. As shown in Fig. 4, the MAP estimate falls outside of the detection region. As an artifact of the current implementation, it actually tends to fall in one of the corners of the state space when the PDF in the non-detection region is uniform and greater than the PDF inside the detection region. The important point is that it is outside the detection region. This shows that the tracker is “carving out” the detection region as shown in Fig. 4; essentially, the model knows that if there were anything in the detection region, it would be hearing it. Since it hears nothing, it concludes there is not likely to be anything in that region.

As noted in Section 5.1, the target probability is dropping during this time period. If there were no birth-death model, this target probability would reach a fixed value related to the ratio of the sizes of the detection region and state space. However, as the target probability drops, the probability of a target being “born” somewhere in the state space increases. As currently modeled, the birth distribution is uniform in position, so the birth model will increase the probability that a target is inside the detection region. This is logical in that as time has passed, something that was not present before could be present now. The presence of this mass inside the detection region will then be contradicted by the next measurement. This mismatch in the prior and posterior further reduces the target probability, since a target is not being heard in a region where it would be expected (with prior probability) to be and where it would be heard if it were there. A topic for future research is the use of non-uniform birth distributions that better reflect real world target dynamics.

Once the target enters the detection region, the position error drops to near zero. The strong, immediate “detection” in terms of high target probability and excellent localization is due primarily to the relatively large SNR of the target. The MAP state estimate remains very good for the remainder of the run, indicating that the clutter is being discounted properly and that the target is not being ignored in clutter regions. This is particularly noteworthy just when the target enters the detection region near the clutter object because there is no specific prior information suggesting that the target is in that region. Thus the measurements from the actual target are quickly recognized as “target like.”

The target leaves the detection region at time 166 minutes, but the MAP position estimates remain very good. The motion updated prior marginal velocity distribution at time 168 minutes is shown in Fig. 5. While the MAP estimate for the velocity is a bit off, the general direction of motion is consistent with the target moving out of the detection region. Hence, the motion-updated prior estimate will have a MAP position estimate near the true target location. Additionally, the posterior estimate after the next measurement will be similar to the prior in that region. There will be no returns associated with the target since it is outside the detection region. Once the clutter objects are modeled, there is nothing in the detection region that is consistent with a target. Consequently, the likelihood inside the detected region will be “carved out” (as in Fig. 4), leaving significant likelihood outside the detection region. This uniform likelihood outside the detection region reinforces the high prior probability in the regions near the target as was predicted by the motion update.

5.3 Additional example

In this subsection, a variation of the original example (defined in Section 4) is considered. Here, only the clutter object at (15 km, 15 km) is in the data, and its mean SNR is reduced to 13 dB (from 15 dB). Additionally, the target SNR now varies (linearly in dB) from 7.4 dB at the edge of the detection region to a maximum of 17 dB for bistatic ranges less than or equal to 30 km. This means that the target and clutter have identical mean SNRs at a bistatic distance of 39 km, which first occurs...
at time 46 minutes in the simulation.

The tracker assumes a mean 13 dB clutter object at (15 km, 15 km) and a constant mean target SNR of 17 dB. This tests a case in which the target is initially quieter than a clutter object and then becomes louder than a clutter object. We also considered examples in which the assumed mean SNRs of the clutter objects (for tracking) were 3-5 dB lower or higher than the true mean SNRs and found very similar results.

The localization results are shown in Fig. 6. For the base model, the tracker initially tracks the clutter object. However, around 60 minutes, it suddenly locks on to the target. At this point in the simulation, the mean target SNR is much louder than the clutter object, and also more consistent with the expected target SNR. Consequently, the model recognizes that object as more “target-like”. Note that there is some delay (compared to when the target becomes louder than the clutter) in the switch due to the prior probability that has accumulated around the clutter object. The target localization is very good until just before the target leaves the detection region.

As the target approaches the detection limit, its SNR is decreasing and it actually becomes quieter than the clutter object. However, the motion updated prior at each measurement maintains probability mass around the target for several updates. Eventually, the lack of strong returns near the target and the presence of returns at the clutter object dominate the updates and the tracker locks on to the clutter object again.

With known clutter modeled, the tracker never locks on to the clutter object. While the target is outside the detection region, the MAP estimate moves around the corners of the state space (as mentioned earlier). Shortly after the target enters the detection region at 30 minutes, the tracker begins to track it, though it jumps between believing there is a target present and believing there is no target present, as shown in Fig. 7.

At time 46 minutes, the tracker locks on to the target until it exits the detection region. At this point, the target has reached its maximum mean SNR of 17 dB, which is the value assumed in the tracker. The localization and target probabilities also accurately reflect the target state for some time after it leaves the detection region due to the motion update.

6 Future work

The first avenue for future work is to apply the predicted-SNR based tracking model to real-world data sets. Preliminary work on this has been promising, but additional modifications to the signal processing of this raw data and to the tracker assumptions appear to be necessary. The second avenue of future work involves imprecisely characterized clutter. For example, if the clutter objects are moving, such as merchant traffic, the real-time position may not be known precisely. Even if AIS data is available, there are often lags or other errors in the message transmissions and processing. Similarly, the SNR of the known objects is uncertain, even for fixed wrecks due to changing environmental conditions or sensor geometries. Additionally, bathymetric features are often large in spatial extent, and therefore multiple highlights can occur from the same object. This might be captured by a set of distributed, imprecisely localized discrete objects. Consequently, future work will focus on assigning prior probabilities to the clutter locations, as well as to the SNR.

These estimates might be improved on-line via some bootstrapping procedure. Ideally, $\sigma_T^2$ and $\sigma_m^2$ in Eq. 12 would be estimated jointly from prior information and the observed measurements. One would like the tracker to respond robustly to targets that are either louder or quieter than expected. Similarly, as more measurements are collected, information about presumed clutter objects should improve. The improved modeling of the clutter should in turn improve the modeling of the target, resulting in better tracking performance.

7 Conclusion

This paper introduced a Bayesian tracking model with measurement likelihoods that are based on predicted SNR values. This model enables the incorporation of prior information about clutter objects. The likelihood model essentially compares the observed SNR with the predicted SNR under two hypotheses: one with a target present and one without the target. The relative likelihoods combine with the prior information to estimate a target probability. The measurement
likelihood function in the case assuming a target is present combines with the prior distribution to yield a posterior distribution of the target state.

By incorporating predicted SNR returns from clutter objects, the model essentially expects to hear something loud in that region, even when the target is not there. This effectively discounts returns from just the clutter object. If the target were in that region, too, then the number of threshold exceedances would be higher. Hence, returns just from the clutter object do not suggest the presence of a target.

The example problems show that the model is very effective at removing the influence of clutter objects whose positions and mean SNRs are perfectly known, even when these SNRs are the same as the target. It also demonstrates that the method still allows for effective target tracking, even when the target trajectory passes near the clutter objects. However, these conclusions are based on somewhat idealized scenarios with artificial data. The work outlined in Section 6 is needed before these conclusions can be generalized.

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Appendix

Consider a sequence of independent, univariate random variables $Y_1, \ldots, Y_K$ with corresponding PDFs $f_1, \ldots, f_K$ and CDFs $F_1, \ldots, F_K$. In the context of this paper, each $Y_k$ corresponds to, say, the measured SNR at a given point (i.e., time delay, bearing, and Doppler shift) in measurement space. The distribution of each $Y_k$ will also depend upon whether we assume a target is present or not.

Now suppose we are interested in the subset of values which exceed a given threshold $\eta > 0$. Let $K_1 < \cdots < K_I \in \{1, \ldots, K\}$ denote the (random) indices of those exceedances. If these threshold exceedances constitute our definition of a measurement, then the corresponding likelihood function is the probability density of obtaining the particular set of indices (i.e., points in measurement space) $k = (k_1, \ldots, k_I)$ with a corresponding set of values $y = (y_1, \ldots, y_K)$. We shall denote this likelihood function $L(k, y)$. For $I = 0$, this takes the simple form

$$L(\emptyset, y) = \prod_{k=1}^{K} F_k(\eta), \quad \tag{18}$$

while, for $I = K$, we have

$$L(1, \ldots, K, y) = \prod_{k=1}^{K} \left[ 1 - F_k(\eta) \right] f_k(y_k|\eta), \quad \tag{19}$$

where the conditional PDF is

$$f_k(y_k|\eta) := \frac{f_k(y_k)}{1 - F_k(\eta)} 1_{[\eta, \infty)}(y_k). \quad \tag{20}$$

If, however, $I = 1$, then

$$L(k_1, y) = P \left[ Y_{k_1} = y_{k_1} | I = 1, K_1 = k_1 \right] P \left[ I = 1, K_1 = k_1 \right]$$

$$= f_{k_1}(y_{k_1}|\eta) \prod_{k=1}^{K_1-1} F_k(\eta) \prod_{k'=k_1+1}^{K} F_{k'}(\eta)$$

$$= L(\emptyset, y) \frac{1 - F_{k_1}(\eta)}{F_{k_1}(\eta)} f_{k_1}(y_{k_1}|\eta),$$

$$\tag{21}$$

More generally, for $I \in \{1, \ldots, K\}$ and $1 \leq k_1 < \cdots < k_I \leq K$, we have

$$L(k, y) = L(\emptyset, y) \prod_{i=1}^{I} \frac{1 - F_{k_i}(\eta)}{F_{k_i}(\eta)} f_{k_i}(y_{k_i}|\eta) \quad \tag{22}$$

References


