Multitarget tracking algorithm - Joint IPDA and Gaussian Mixture PHD filter

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Abstract – Random Finite Set approach is a mathematically rigorous framework for multi-target tracking. It provides a Bayesian recursion of multi-target distribution through Finite Set calculus. But practical implementation of multi-target posterior recursion is difficult because of its combinatorial nature. Probability hypothesis density (PHD) filter is an alternative to this problem where only the first order moment of the complete multi-target posterior is propagated in time. One of the suitable implementations of probability density filter is Gaussian mixture PHD filter. Parallel to this approach, several multi-target tracking algorithms are devised based on corresponding single target tracking algorithms. Joint integrated probabilistic data association is one of the most successful of such algorithms. This article shows that PHD filter recursion reduces to joint IPDA formalism under linear Gaussian assumptions.

Keywords: Multi-target Tracking, Filter, Joint IPDA, PHD, Gaussian Mixture PHD

1 Introduction

Target tracking algorithms, in general, have to handle uncertainties that arise from target dynamic model (process noise), random observation error (observation noise) made by sensors, spurious returns from objects other than targets (false alarms or clutters), missed detection of true targets and etc. Basic Kalman filter ([1, 2]) addresses the problem of process noise and observation noise. Data association based algorithms like ”Nearest Neighbor Filter (NNF)” ([5]), ”Probabilistic Data Association (PDA)” filter ([3]), address the clutter problem. PDA filter outperforms the NNF and therefore has become the standard for the problem where measurement origin uncertainty is present. PDA assigns each observation an weight that measures its association with a given target. Both PDA and Kalman Filter take the existence of target as a certain event. In [6], the PDA is extended to include the ”existence of target” as uncertain event. This new algorithm, called Integrated Probabilistic Data Association (IPDA) filter, calculates the probability of ”target existence event” in order to resolve the uncertainty. The IPDA filter achieves better performance than PDA where the target existence is uncertain.

The algorithms stated above are devised for tracking a single target. To address the multiple target tracking problems, the first approach is to extend the already existing algorithms to track multiple targets simultaneously. As a result, PDA was extended to Joint PDA (JPDA) ([7]) and IPDA was extended to Joint IPDA (JIPDA) filters [9, 10]. JIPDA performs better than JPDA under target existence uncertainty. Fundamentally, JIPDA handles the problem of associating an observation to a target by assigning the observation with a weight that takes into account the existence of other targets as well as clutter returns. Bayes’ theorem provides fundamental mathematical approach in devising these tracking algorithms. While the basic Kalman Filter was derived using Bayesian approach in [11], it was in [12] where all the established algorithms were formulated from Bayes’ theorem. This results in better understanding of the working principle of the established algorithms. During the same period, a multi-target tracker was being formulated in [13, 14]. This approach had Random Finite Set (RFS) mathematics as the basis to model the multi-target density and to update the same recursively using Bayesian formalism. Random Set based formulation models the multiple-target scenario as a random set where the cardinality (total number of targets) of the set can vary along with the state of each of the targets. This is different from the multiple target tracking algorithms, like JPDA or JIPDA where each target is allowed to evolve independently and only the ”observation association” (assigning weight to a certain observation) takes the existence of other targets into account. However, in [24], it was analytically proved that IPDA algorithm can be derived from RFS based filter recursion under linear Gaussian assumptions. But for large number of targets, RFS based Bayesian recursion involves multiple set integrals and is proved to be difficult to implement because of its combinatorial nature. Therefore propagation of entire RFS posterior distribution through Bayes’s theorem was a problem. A suitable alternative was proposed in [15] where only the first order moment of the entire RFS posterior distribution was propagated in
time. This approach is similar to constant gain Kalman Filter for single target tracking. The resulting multitarget tracking filter is called Probability Hypothesis Density (PHD) filter. However, the PHD recursion equations ([15, 17]) do not have a closed form solution in general. Particle system implementation ([19, 20]) and “Gaussian Mixture” assumptions ([21]) result in closed form analytical solutions of PHD filter equations under appropriate conditions. These are easier to implement and therefore are gaining in popularity for multitarget tracking problems. In this paper, an analytical proof will be present to establish the theoretical connection between JIPDA algorithm and Gaussian Mixture PHD (GM-PHD) filter under the following assumptions:

1. Each target follows a linear Gaussian dynamical model
2. Target survival and detection probabilities are state independent,
3. No explicit target birth and spawning event

These conditions reduce GMPHD recursion formula into JIPDA filter algorithms. The presentation of the paper is given below.

In section 2, the basics of PHD filter will be briefly discussed. Section 3 describes the required conditions for GMPHD filter recursion. The resulting GMPHD filter equations are also given in this section. JIPDA modeling for target tracking and its theoretical connection to PHD fundamentals are established in section 4. The section 5 derives the JIPDA filter equations from GMPHD recursion formula. The necessary approximations for JIPDA are also outlined in section 5.3 in order to fully establish the claimed theoretical connection. Conclusion is drawn in 6.

2 The PHD Filter recursion

The PHD filter approximates the optimal multi-target posterior density (in Random finite set approach) by its first order moment [15, 17]. Instead of the complete distribution, the first order moment is propagated in time resulting in multitarget filtering algorithm similar to constant gain Kalman filter for single target tracking. The first order moment is a density which is uniquely defined as following: its integration over a space gives the expected number of targets within that space (as opposed to probability density that integrates to one over the entire space). The recursion of PHD starts with the density at time \(k-1\), \(D_{k-1|k-1}(x_{k-1}|Z_{1:k-1})\) and follows the steps as below

- **Prediction Step**

\[
D_{k|k-1}(x_k|Z_{1:k-1}) = \gamma_k(x_k) + \int \left[ p_{S,k}(x_k|Z_{1:k-1}) f_{k|k-1}(x_k|x_{k-1}) + b_{k|k-1}(x_k|x_{k-1}) \right] \times D_{k-1|k-1}(x_{k-1}|Z_{1:k-1})dx_{k-1} \tag{1}
\]

where
- \(\gamma_k(x_k)\) is the intensity of targets appearing at time \(k\)
- \(p_{S,k}(x_{k-1})\) is the target survival probability
- \(f_{k|k-1}(x_k|x_{k-1})\) is the single target Markov transition density
- \(b_{k|k-1}(x_k|x_{k-1})\) is the intensity of spawning of target from existing ones

- **Update Step**

\[
D_{k|k}(x_k|Z_{1:k}) = [(1 - p_{D,k}(x_k)) + \sum_{z_k \in Z_k} \frac{p_{D,k}(x_k) f(z_k|x_k)}{\lambda_k c_k(z_k) + \psi(z_k|Z_{1:k-1})}] \times D_{k|k-1}(x_k|Z_{1:k-1}) \tag{2}
\]

where
- \(\psi(z_k|Z_{1:k-1}) = \int p_{D,k}(x_k) f(z_k|x_k) D_{k|k-1}(x_k|Z_{1:k-1}) \)
- \(p_{D,k}(x_k)\) is the probability of detection
- \(f(z_k|x_k)\) is the single target likelihood function
- \(\lambda_k\) and \(c_k(z_k)\) are the false alarm (clutter) intensity and false alarm spatial density respectively.

The expected number of targets is given by

\[
N_{k|k} = \int D_{k|k}(x_k|Z_{1:k}) dx_k \tag{3}
\]

3 Gaussian Mixture PHD Algorithm

The recursion of PHD given by (1-2) does not result in closed analytical form in general. However, in [21] it was shown that under certain model conditions, the recursion results in closed form solutions and the algorithm is called “Gaussian Mixture PHD (GMPHD) filter”. These necessary conditions are summarized below.

1. Each target follows a linear Gaussian dynamical model given by

\[
f_{k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; F_{k-1} x_{k-1}, Q_{k-1}) \tag{4}
\]

and each sensor has a linear Gaussian likelihood model given by

\[
f(z_k|x_k) = \mathcal{N}(z_k; H_k x_k, R_k) \tag{5}
\]

where
- \(\mathcal{N}(a; b, c)\) denotes a Gaussian density of random variable \(a\) with mean \(b\) and covariance \(c\).
- \(F_{k-1}\) is the state transition matrix
- \(Q_{k-1}\) and \(R_{k-1}\) are the process and observation noise covariances respectively.
2. Target survival and detection probabilities are state independent,
\[ p_{D,k}(x_k) = p_{D,k} \]
\[ p_{S,k}(x_{k-1}) = p_{S,k} \]

3. The target birth and spawn intensities are in the form of Gaussian mixture, given by
\[ \gamma_k(x_k) = \sum_{i=1}^{J_{\gamma,k}} w_{i,k}^{\gamma,k}N(x_k; m_{i,k}^{\gamma,k}, P_{i,k}^{\gamma,k}) \]
\[ b_{j,k-1}(x_k | x_{k-1}) = \sum_{j=1}^{J_{b,k}} w_{j,k}^{b,k}N(x_k; m_{j,k}^{b,k}, P_{j,k}^{b,k}) + u_{k}^{b,k} \]

where
- \( w_{i,k}^{\gamma,k}, m_{i,k}^{\gamma,k}, P_{i,k}^{\gamma,k} \) are weight, mean and covariance of the \( i \)-th spawn component, \( i = 1, 2, \ldots, J_{\gamma,k} \) and \( J_{\gamma,k} \) is the total number of spawned components
- \( w_{j,k}^{b,k}, m_{j,k}^{b,k}, P_{j,k}^{b,k} \) are weight, mean and covariance of the \( j \)-th birth component, \( j = 1, 2, \ldots, J_{b,k} \) and \( J_{b,k} \) is the total number of birth components

It is also assumed that the density at \( k-1 \) is a Gaussian mixture of the form
\[ D_{k-1|k-1}(x_{k-1}|Z_{1:k-1}) = \sum_{i=1}^{J_{b,k}} w_{i,k}^{b,k}N(x_{k-1}; m_{i,k}^{b,k}, P_{i,k}^{b,k}) \]

Under the stated conditions and assumptions, PHD recursion of (1,2) is reduced to the steps below

- **Prediction**
\[ D_{k|k-1}(x_k|Z_{1:k-1}) = D_{S,k|k-1}(x_k|Z_{1:k-1}) + D_{b,k|k-1}(x_k|Z_{1:k-1}) + \gamma_k(x_k) \]

where
\[ \gamma_k(x_k) \text{ is given by (8)} \]
\[ D_{S,k|k-1}(x_k|Z_{1:k-1}) = \sum_{i=1}^{J_{b,k}} w_{i,k}^{b,k}N(x_k; m_{i,k}^{b,k}, P_{i,k}^{b,k}) \]
\[ m_{S,k}^{i} = F_{k-1}m_{i,k-1}^{b} \]

- **Update**
\[ P_{S,k|k-1}^{i} = F_{k-1}P_{i,k-1}^{b}F_{k-1}^{T} + Q_{k-1} \]
\[ w_{S,k}^{i} = p_{S,k}w_{k}^{i} \]
\[ D_{b,k|k-1}(x_k|Z_{1:k-1}) = \sum_{i=1}^{J_{b,k}} \sum_{j=1}^{J_{b,k}} w_{i,k}^{j}w_{j,k}^{b,k}N(x_k; m_{i,j,k}^{b,k}, P_{i,j,k|k-1}^{b,k}) \]
\[ m_{i,j,k}^{b,k} = F_{b,k-1}m_{i,k-1} + u_{i,k}^{b,k} \]
\[ P_{i,j,k|k-1}^{b,k} = F_{b,k-1}P_{i,k-1}^{b}F_{b,k-1}^{T} + Q_{b,k-1}^{i} \]

It is noted that (13-14) and (17-18) are simple Kalman filter prediction steps. For simplification, these operations can be put in a functional form given by
\[ \begin{bmatrix} m_{S,k}^{i} \n P_{S,k|k-1}^{i} \end{bmatrix} = K_{F}^{pred} \begin{bmatrix} m_{S,k-1}^{i} \n P_{S,k|k-1}^{i} \end{bmatrix} \]

\[ \begin{bmatrix} m_{b,k}^{i} \n P_{b,k|k-1}^{i} \end{bmatrix} = K_{F}^{pred} \begin{bmatrix} m_{b,k-1}^{i} \n u_{b,k}^{i} \n P_{b,k-1}^{i} \n Q_{b,k-1}^{i} \end{bmatrix} \]

**Update**
\[ D_{k|k}(x_k|Z_{1:k}) = (1 - p_{D,k})D_{k|k-1}(x_k|Z_{1:k-1}) + \sum_{z_k \in z_{k}} D_{z,k}(x_k|z_k) \]

where
\[ D_{z,k}(x_k|z_k) = \sum_{i=1}^{J_{b,k}} w_{i,k}^{b,k}N(x_k; m_{i,k}^{b,k}(z_k), P_{i,k}^{b}(z_k)) \]
\[ w_{i,k}^{b,k}(z_k) = \frac{p_{D,k}^{w_{i,k}^{b,k}(z_k)}}{\lambda_{k}c_{k}(z_k) + p_{D,k}^{w_{i,k}^{b,k}(z_k)}} \]
\[ q_{i,k}(z_k) = N(z_k; H_{k}^{m_{k,i,k}^{b,k}(z_k)}, H_{k}^{m_{k,i,k}^{b,k}(z_k)}H_{k}^{T} + R_{k}) \]
\[ m_{k,i,k}^{b,k}(z_k) = m_{k,i,k}^{b,k} + K^{i}_{k}(z_k - H_{k}m_{k,i,k}^{b,k}(z_k)) \]
\[ P_{k,i,k}^{b,k}(z_k) = (I - K^{i}_{k}H_{k})P_{k,i,k}^{b,k}H_{k}^{T} + R_{k} \]

The steps in (25-27) are Kalman filter update equations and can be summarized in the functional form given below
\[ \begin{bmatrix} m_{k,i,k}^{b,k}(z_k) \n P_{k,i,k}^{b,k}(z_k) \end{bmatrix} = K_{F}^{update} \begin{bmatrix} m_{k,i,k}^{b,k}(z_k) \n H_{k} \n P_{k,i,k}^{b,k}(z_k) \n R_{k} \end{bmatrix} \]
4 Joint IPDA and PHD Filter

In IPDA approach, the target state consists of dynamic parameters (position, velocity and etc.) and its existence. IPDA algorithm provides formulation to propagate the joint probability density given by

\[ p(x_{k-1}, \chi_{k-1} = 1 | Z_{1:k-1}) = p(x_{k-1} | Z_{1:k-1}) \]

where \( x_{k-1} \) denotes the dynamic state of the target, \( \chi_{k-1} = 1 \) denotes the existence of target and \( \chi_{k-1} = 0 \) denotes non-existence of the target. In IPDA, the target state density is assumed to be Gaussian. Therefore the expression in (29) becomes

\[ p(x_{k-1}, \chi_{k-1} = 1 | Z_{1:k-1}) = w_{k-1} \mathcal{N}(x_{k-1}; m_{k-1}, P_{k-1}) \]

(30)

where \( w_{k-1} = p(x_{k-1} | Z_{1:k-1}) \). In a multi-target environment, each target is represented by (30). Composite probability density for \( N \) number of such targets, where each of them has a density given by (30), is as follows

\[ \sum_{i=1}^{N} w^i_{k-1} \mathcal{N}(x_{k-1}; m^i_{k-1}, P^i_{k-1}) \]

(31)

The composite density in (31) can be intuitively interpreted as target intensity. For simplicity (but without losing generality), let us assume that there are two targets, \( N = 2 \). The corresponding (assumed) component and composite density are shown in dotted and solid line respectively in Figure 1.

![Target Intensity in state space](image)

Figure 1: Target Intensity in state space

In the figure each dotted line corresponds to the component density function \( w^i_{k-1} \mathcal{N}(x_{k-1}; m^i_{k-1}, P^i_{k-1}) \) and hence does not integrate to one over the entire state space. The interpretation of the composite density is as following: the target intensity within infinitesimally small state-space \( dx \) is

\[ \sum_{i=1}^{N} w^i_{k-1} \mathcal{N}(x_{k-1}; m^i_{k-1}, P^i_{k-1}) |_{x_{k-1} = dx_{k-1}} \]

Therefore the integration of the expression above over the entire space will yield the total number of target within the entire state space.

\[ \int \sum_{i=1}^{N} w^i_{k-1} \mathcal{N}(x_{k-1}; m^i_{k-1}, P^i_{k-1}) dx_{k-1} = \sum_{i=1}^{N} w^i_{k-1} \]

As a result, the composite density in (31), which is derived from the IPDA formalism of target tracking, conforms to the definition of probability hypothesis density described in section 2. The composite density can then be called as probability hypothesis density in PHD terminology and can be expressed as below

\[ D_{k-1|k-1}(x_{k-1} | Z_{1:k-1}) = \sum_{i=1}^{N} w^i_{k-1} \mathcal{N}(x_{k-1}; m^i_{k-1}, P^i_{k-1}) \]

(32)

In IPDA terminology,

1. \( w^i_{k-1} \) is the prior target existence probability of \( i-th \) target
2. \( \mathcal{N}(x_{k-1}; m^i_{k-1}, P^i_{k-1}) \) is the prior state probability density of \( i-th \) target

5 JIPDA Derivation

Based on the equivalence established in section 4 between IPDA based joint target state (including both dynamic parameters and existence) density and the PHD, the recursion of GMPHD (as in section 3) will be shown to be resulting in Joint IPDA (JIPDA) algorithm under some required conditions. These conditions are listed here for clarity.

1. Each target follows a linear Gaussian dynamical model
2. Target survival and detection probabilities are state independent,
3. No explicit target birth and spawning event

These first two approximations are exactly the same as the necessary conditions for GMPHD stated in section 3. The third condition require the birth and spawning densities to be zero.

\[ D_{b,k|k-1}(x_k | Z_{1:k-1}) = 0 \]

\[ \gamma_k(x_k) = 0 \]

Therefore the GMPHD recursion is applicable for JIPDA derivation. Starting from initial density \( D_{k-1|k-1}(x_{k-1} | Z_{1:k-1}) \), these recursion steps will be described next.
5.1 Prediction

\[ D_{k|k-1}(x_k|Z_{1:k-1}) = D_{S,k|k-1}(x_k|Z_{1:k-1}) + D_{b,k|k-1}(x_k|Z_{1:k-1}) + \gamma_k(x_k) \]  
(33)

Target birth and spawning are not explicitly modeled in IPDA approach. Therefore the corresponding intensities, \( D_{b,k|k-1}(x_k|Z_{1:k-1}) = 0 \) and \( \gamma_k(x_k) = 0 \). However, the existing targets survive with probability \( p_{S,k} \) and the corresponding predicted intensity is given as following

\[
D_{S,k|k-1}(x_k|Z_{1:k-1}) = \sum_{i=1}^{N} w^i_{k|k-1}N(x_k; m^i_{S,k}, p^i_{S,k|k-1}) \]  
(34)

\[
[m^i_{S,k}, p^i_{S,k|k-1}] = KFP_{pred}[F_{k-1}, m^i_{k-1}, - , P^i_{k-1}, Q_{k-1}] \]  
(35)

\[
w^i_{k|k-1} = p_{S,k}w^i_{k|k-1} \]  
(36)

According to (33), the predicted density is

\[
D_{k|k-1}(x_k|Z_{1:k-1}) = \sum_{i=1}^{N} w^i_{k|k-1}N(x_k; m^i_{S,k}, p^i_{S,k|k-1}) \]  
(37)

5.2 Update

\[
D_{k|k}(x_k|Z_{1:k}) = (1 - p_{D,k})D_{k|k-1}(x_k|Z_{1:k-1}) + \sum_{z_k \in Z_k} D_{z,k}(x_k; z_k) \]

\[
= \sum_{i=1}^{N} \left[ (1 - p_{D,k})w^i_{k|k-1}N(x_k; m^i_{S,k}, p^i_{S,k|k-1}) \right. \]
\[
+ \sum_{z_k \in Z_k} \left. \sum_{i=1}^{N} \frac{p_{D,k}w^i_{k|k-1}q^i_k(z_k)}{\lambda_k c_k(z_k) + p_{D,k} \sum_{i=1}^{N} w^i_{k|k-1}q^i_k(z_k)} \right] \]
\[
N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \]
\[
= \sum_{i=1}^{N} \left[ w^i_{k|k-1}\beta^i_{k,0}N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \right. \]
\[
+ \sum_{z_k \in Z_k} \left. \sum_{i=1}^{N} \frac{p_{D,k}q^i_k(z_k)}{\lambda_k c_k(z_k) + p_{D,k} \sum_{i=1}^{N} w^i_{k|k-1}q^i_k(z_k)} \right] \]
\[
N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \]
\[
= \sum_{i=1}^{N} \left[ w^i_{k|k-1}\beta^i_{k,0}N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \right. \]
\[
+ \sum_{z_k \in Z_k} \left. \sum_{i=1}^{N} \beta^i_{k,0}N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \right] \]  
(38)

where

\[
\beta^i_{k,0} = (1 - p_{D,k}) \]
\[
\beta^i_{k,z_k} = \frac{p_{D,k}q^i_k(z_k)}{\lambda_k c_k(z_k) + p_{D,k} \sum_{i=1}^{N} w^i_{k|k-1}q^i_k(z_k)} \]
\[
[m^i_{k|k}(z_k), p^i_{k|k}(z_k)] = KF_{update}[m^i_{k|k-1}, z_k, H_k, p^i_{k|k-1}] \]

5.3 JIPDA Approximations

The GMPHD filter results in a Gaussian mixture for density expression as in (38). The next step for GMPHD is to employ either “track to peak” assignment ([25]) or “data-association” ([26]) scheme. However, IPDA approximates the sum of Gaussians for each \( i \) (each term within the outermost summation sign in (38)) by a single Gaussian. For \( i-th \) target, the updated state density is the Gaussian mixture approximation of component densities. This approximation can be obtained from (38) and is shown below.

\[
[w^i_{k|k-1}\beta^i_{k,0}N(x_k; m^i_{S,k}, p^i_{S,k|k-1}) \]
\[
+ \sum_{z_k \in Z_k} w^i_{k|k-1}\beta^i_{k,z_k}N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \]
\[
= \sum_{z_k \in Z_k} \left[ \beta^i_{k,0} + \sum_{z_k \in Z_k} \beta^i_{k,z_k} \right] w^i_{k|k-1} \]
\[
\times \left[ \beta^i_{k,0}N(x_k; m^i_{S,k}, p^i_{S,k|k-1}) \right. \]
\[
+ \sum_{z_k \in Z_k} \left. \sum_{i=1}^{N} \beta^i_{k,z_k}N(x_k; m^i_{S,k}(z_k), p^i_{S,k|k}(z_k)) \right] \]
\[
= w^i_{k|k}N(x_k; m^i_{k|k}, p^i_{k|k}) \]  
(39)

where

\[
w^i_{k|k} = \left[ \beta^i_{k,0} + \sum_{z_k \in Z_k} \beta^i_{k,z_k} \right] w^i_{k|k-1} \]  
(40)

\[
\beta^i_{k,0} = \frac{\beta^i_{k,0}}{\beta^i_{k,0} + \sum_{z_k \in Z_k} \beta^i_{k,z_k}} \]  
(41)

\[
\beta^i_{k,z_k} = \frac{\beta^i_{k,z_k}}{\beta^i_{k,0} + \sum_{z_k \in Z_k} \beta^i_{k,z_k}} \]  
(42)

\[
m^i_{k|k} = \beta^i_{k,0}m^i_{S,k} + \sum_{z_k \in Z_k} \beta^i_{k,z_k}m^i_{S,k}(z_k) \]  
(43)
\[ P_{k|k} = \beta_{k,0}^j \left( P_{S,k|k-1} + (m_{S,k} - m_{k|k})(m_{S,k} - m_{k|k})' \right) \]
\[ + \sum_{z_k \in Z_k} \beta_{k,z_k}^j \left( P_{k|k} - m_{k|k}^i(z_k) - m_{k|k}^i(z_k) - m_{k|k}^i)' \right) \]  

(44)

The updated density in (38) can now be expressed by substituting the approximate Gaussian obtained in (39).

\[ D_{k|k}(x_k|Z_{1:k}) = \sum_{i=1}^{N} w_{k|k}^i \mathcal{N}(x_k; m_{k|k}^i, P_{k|k}^i)(45) \]

In Joint IPDA terminology,

1. \( w_{k|k}^i \) is the posterior existence probability of \( i-th \) track,
\[ w_{k|k}^i = \pi(x_k^i = 1|Z_{1:k}) \]

2. \( \beta_{k,0}^j, \beta_{k,z_k}^j \) are the joint data association probabilities of \( i-th \) target in (multiple target tracking context)

5.4 Track Management in JIPDA

The states, as well as the existence probabilities, of existing \( N \) targets are updated according to (45). In order to provide a measure of track quality, JIPDA divides the track list based on the existence probabilities \( w_{k|k}^i \). The general scheme for such division are summarized below.

1. "Confirmed Track" : \( i-th \) track is confirmed if \( w_{k|k}^i \geq \gamma_{conf} \)
2. "Terminated Track" : \( i-th \) track is terminated if \( w_{k|k}^i < \gamma_{term} \)
3. "Tentative Track" : \( i-th \) track is tentative if \( \gamma_1 \leq w_{k|k}^i \leq \gamma_2 \)

where the thresholds, \( \gamma(\cdot) \), are chosen suitably. Moreover, several tracks may also be merged if they share a common measurement history over past few (suitably chosen number of) scans and/or if they are close (suitably defined). Once the track management scheme is applied, all or some of the \( N \) targets may survive to continue into the next scan. Accordingly the target density of (45) is readjusted to

\[ D_{k|k}(x_k|Z_{1:k}) = \sum_{i=1}^{M} w_{k|k}^i \mathcal{N}(x_k; m_{k|k}^i, P_{k|k}^i) \]  

(46)

where \( M \leq N \). Moreover, Joint IPDA does not explicitly model target birth or spawning. However, it initiates new target at \( k \) based on two-point initiation ([27]) or other suitable method. Each new target is assigned with a certain existence probability and initial state density (of Gaussian form). These initiated targets then are added to the overall target density and the final expression becomes

\[ D_{k|k}(x_k|Z_{1:k}) = \sum_{i=1}^{M} w_{k|k}^i \mathcal{N}(x_k; m_{k|k}^i, P_{k|k}^i) \]  

(47)

where

- \( M_{new} \) is the number of new tracks initiated at \( k \)
- \( w_{k|k}^i, m_{k|k}^i \) and \( P_{k|k}^i \) are initial target existence probability, mean and covariance for \( j-th \) initiated target respectively.

Terminating existing targets and adding new targets in the density do not pose any problem with the definition of density because the integration of the adjusted density in (47) over the entire space still provides the total number of targets within the space. This expression of density in (47) becomes the input density for time \( k+1 \) and thus completes the proof of Joint IPDA recursion from GMPHD formulation.

6 Conclusion

Single target tracking based algorithms are commonly extended to multi-target tracking context. Integrated probabilistic data association (IPDA) was originally devised for single target tracking in clutter and was later extended to multiple target tracking problem in the form of Joint IPDA (JIPDA). Joint IPDA allows the targets to evolve independently (same as single target tracking case) but its data association methodology takes care of the presence of clutter and other existing targets in the same space. A mathematically sound framework based on Random Finite Set calculus is also proposed in literature for multiple target tracking and several algorithms have been proposed based on this framework. PHD filter is such a development and is gaining popularity in literature. Gaussian mixture PHD (GM-PHD) filter is a form of PHD filter where closed form solution of PHD recursion is obtained under linear Gaussian assumption of target dynamic transition and sensor observation model. This paper has presented an analytical proof that JIPDA recursion can be derived from GMPHD filter equations under the same conditions as required by GMPHD filter and with the restriction of "no target birth and spawning". However, JIPDA proposes track initiation to cater for these events.

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