Suboptimal JPDA for Tracking in the Presence of Clutter and Missed Detections

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Abstract — A new heuristic for data association on multi-target tracking systems is presented. The algorithm is based on the existing JPDA (Joint Probabilistic Data Association) algorithm, more specifically, on the Suboptimal JPDA approximation. However, compared to Suboptimal JPDA, the presented method exhibits a significant improvement on tracking performance for scenarios in the presence of clutter and missed detections, at negligible increase of computation cost. In fact, it approaches the performance of the classic JPDA, requiring, however, drastically lower computational resources. It is also be shown that the proposed method is more robust with respect to trajectory crossings than the original Suboptimal JPDA.

Keywords: Data association, Target Tracking and Localization, Probability Theory, Bayesian inference.

1 Introduction

JPDA (Joint Probabilistic Data Association) [1] is a well-known approach for observation-to-track association for multi-target tracking systems. At each time step, it enumerates all feasible joint association events $\Theta$ in the current scan, and computes the probability of each of them. A feasible joint association event is a set of nonconflicting validated measurement-to-track pairings in which a measurement can originate from only one source, at most one measurement can originate from a target, and any number of measurements can originate from clutter. Then, it proceeds by calculating the association probabilities $\{\beta_{ij}\}$, where $\beta_{ij}$ is the probability that the $j$-th observation stems from the $i$-th track. Finally, the calculated probabilities are used as update weights for a Kalman or IMM filter.

Two main difficulties aroused from practical experience with JPDA:

- The tendency of track coalescence for closely spaced targets, reported by [2]. This is caused by the cross-correlation between tracks that arise when weighted update is performed [3].

- The high computation cost of JPDA, which reduces its usefulness in practical applications. The exact calculation of association probabilities $\{\beta_{ij}\}$ is a NP-hard problem [4], since the number of joint association events grows exponentially with the number of targets.

Due to being an approximation of the Bayesian filter, JPDA received a lot of attention from the research community and led to various developments usually addressing one (or both) of the problems previously mentioned. This paper focuses on the second problem. The developments made to overcome the high complexity of JPDA involved either improving the generation or processing efficiency of the joint events, avoiding the need to process the entire set of events (as in [5], [6] and [7]), or eliminating the generation of joint events altogether, which usually requires changing the assumptions or correctness conditions of the algorithm (as in [8], [9] and [10]).

In this paper, we focus on improvements over a development of the second group, the Suboptimal JPDA algorithm by Roecker and Phillips [6]. It’s a simple and extremely fast JPDA approximation that gives the same association probabilities as JPDA for some specific simple cases. This is an interesting approach considering that these simple cases may be far more predominant than more complex cases involving large numbers of tracks with a large number of common measurements within their validation gates.

However, as we will show later, the performance of Suboptimal JPDA degrades quickly when such more complex situations arise, particularly, in the presence of measurements which aren’t originated by any target (clutter) and missed detections. To deal with this, we propose a modified algorithm capable of providing an acceptable performance for scenarios with clutter and missed detections, as well as suitable scalability (as heuristic) to scenarios with higher target density. Compared to the basic Suboptimal JPDA computational cost increases only slightly, which justifies its use.
instead of more complex alternatives.

This paper is organized as follows. Section 2 gives a short review of the original JPDA presented in [1], while Section 3 reviews the Suboptimal JPDA algorithm on which this paper is based. Section 4 presents the modified version to track multiple targets observed by radar systems, in the presence of clutter. Section 5 compares the tracking performance and computational performance of the three algorithms, for various air tracking scenarios. Conclusions are given in Section 6.

2 The JPDA

Assuming a Poisson probability mass function for the false measurements, and assuming that each track corresponds to a real target (i.e. all tracks have been initialized), for each scan \( k \), after the generation of the feasible joint events mentioned before, the joint association event probabilities are given by (omitting the subscript \( k \) for convenience)

\[
P\{\theta|Z^k\} = \frac{1}{c} \lambda^k \prod_{j=1}^{m} (\Lambda_{t,j})^{\tau_j} 
\]

\[
\prod_{t=1}^{n} \left[ (P_D^t)^{\delta_t} (1 - P_D^t)^{1 - \delta_t} \right] 
\]

(1)

where \( c \) is the normalization constant, \( Z^k \) denotes the set of measurements until time \( k \), \( m \) is the number of measurements, \( \lambda \) is the spatial density of false measurements, \( \phi \) is the number of false measurements, \( \Lambda_{t,j} \) is the gaussian likelihood of measurement \( j \) being originated by target \( t \), \( n \) is the number of targets, \( P_D^t \) is the probability of target \( t \) being detected, \( \tau_j \) is the measurement association indicator (with a value of one if measurement \( j \) is assigned to a target in \( \theta \)), and \( \delta_t \) is the target detection indicator (with a value of one if a measurement is assigned to target \( t \) in \( \theta \)).

The marginal probability of measurement \( j \) being originated by target \( t \) is then given by:

\[
\beta_{tj} = P_r(\theta_{tj}|Z^k) = \sum_{\theta, \theta_{tj} \in \theta} P_r(\theta|Z^k) 
\]

(2)

and the probability of target \( t \) not having a measurement originated by it is

\[
\beta_{t0} = 1 - \sum_{j=1}^{m} \beta_{tj} 
\]

(3)

Since the number of joint association events grows exponentially with the number of validated measurement/track assignments, the assignments are usually grouped into clusters before the application of the algorithm [10]. Clusters are disjoint sets of measurements and tracks; when a track validates a measurement, both track and measurement are included in the same cluster. Since assignments from different clusters may be considered independent, each cluster may be processed separately.

3 The basic Suboptimal JPDA

3.1 Overview

The Suboptimal JPDA algorithm presented on [6] is a heuristic to obtain the association probabilities \( \beta_{ij} \) without the need of the (costly) generation of joint association events. On each iteration, it consists of the following steps, performed over the list of validated measurement/track pairings:

1. For each track \( t \), form \( A_t \), which is the list of indices of the validated measurements for track \( t \).

2. For each measurement \( j \), form \( C_j \), which is the list of indices of the tracks which validate measurement \( j \).

3. For each track \( t \), form the union of all track index lists from all the measurements that are validated by track \( t \), while excluding the index of track \( t \). The list, denoted by \( L_t \), is given by

\[
L_t = \bigcup_{j \in A_t} C_j - \{t\} 
\]

(4)

4. For each pairing consisting of track \( t \) and its validated measurement \( j \), find

\[
N_{tj} = \max_{k \in A_t, k \neq j} \Lambda_{tk} 
\]

(5)

where \( \Lambda_{tk} \) is the likelihood of measurement \( k \) being originated by target \( t \). If there is a single measurement validated by \( t \), then we should make

\[
N_{tj} = \Lambda_{tj} 
\]

(6)

5. For each pairing consisting of track \( t \) and its validated measurement \( j \), calculate

\[
D_{tj} = \begin{cases} 
\Lambda_{tj}, & L_t = \emptyset \\
\Lambda_{tj} \sum_{u \in L_t} N_{uj}, & L_t \neq \emptyset 
\end{cases} 
\]

(7)

6. Finally, the probability of a track \( t \) associating with measurement \( j \) is given by

\[
\beta_{tj} = \frac{D_{tj}}{B + \sum_{k \in A_t} D_{tk}} 
\]

(8)

and the probability of a track \( t \) not being associated with any measurement is

\[
\beta_{t0} = \frac{B}{B + \sum_{k \in A_t} D_{tk}} 
\]

(9)

where \( B \) is a bias term to account for clutter density, which Roecker and Phillis recommend setting to zero unless the clutter is very dense.
Suboptimal JPDA gives the correct association probabilities (i.e. the same obtained with JPDA) for a given cluster, in the following cases:

- One target and multiple measurements.
- Two target and two measurements, with both measurements validating both tracks.
- N targets and N measurements in complete confusion (all measurements validate all targets, all likelihoods \( \Lambda_{ij} \) equal).

Since the bias term is assumed to be zero, none of the correctness conditions hold if we consider that false measurements may be present\(^1\).

### 3.2 The problem of overfitting

Suboptimal JPDA is a practical engineering solution that takes advantage of the fact that, in many practical applications, the occurrence of more than two target and two measurements in the same cluster is a relatively rare event. Unfortunately, Suboptimal JPDA suffers from overfitting; its performance degrades quickly when the scenario differs considerably from the conditions in which the algorithm gives the correct association probabilities (i.e. with increase of target density, measurement density, number of false measurements, or number of missed detections).

In the presence of clutter, a solution could be attempted using a nonzero bias term \( B \). Helmick [12] suggests the following choice of the bias term:

\[
B = \frac{\lambda(1 - P_D P_G)}{P_D} \tag{10}
\]

where \( P_D \) is the target probability of detection (assumed constant for all targets), and \( P_G \) is the probability of a measurement originated by a target falling into the target’s gate.

The bias term calculated this way, however, leads to largely incorrect association probabilities for even trivial situations. Consider, for instance, a cluster with two tracks and two measurements, with both measurements validating both tracks. Consider the following values for the clutter spatial density, likelihoods, and target detection probability: \( \lambda = 6.3 \cdot 10^{-13}, \Lambda_{11} = 1.1 \cdot 10^{-11}, \Lambda_{12} = 3.4 \cdot 10^{-12}, \Lambda_{21} = 7.2 \cdot 10^{-12}, \Lambda_{22} = 4.0 \cdot 10^{-13} \), and \( P_D = 0.95 \).

Using the JPDA without approximations, the association probabilities are given by \( \beta_{ij} = 0.1609, \beta_{12} = 0.8305, \beta_{21} = 0.8348, \) and \( \beta_{22} = 0.1419 \), whereas using Suboptimal JPDA with \( B \) calculated using (10), with \( P_G = 1 \), they are given by \( \beta_{11} = 1.3 \cdot 10^{-10}, \beta_{12} = 7.4 \cdot 10^{-10}, \beta_{21} = 7.4 \cdot 10^{-10}, \) and \( \beta_{22} = 1.3 \cdot 10^{-10} \). As we will see, the issue is that the bias term cannot be a constant value in order to give the correct probabilities.

### 4 Derivation of an improved suboptimal JPDA approximation

#### 4.1 The case of one target and multiple measurements

For a cluster with a single target and multiple measurements, the joint event association probability, calculated by JPDA, is (omitting the index of the single target for convenience)

\[
P(\theta|Z^k) = \frac{1}{c} \lambda^\phi (P_D)^\delta (1 - P_D)^{1 - \delta_i} \prod_{j=1}^m (\Lambda_j)^{\tau_j} \tag{11}
\]

\[
= \begin{cases}
\frac{1}{2}\lambda m^{-1} P_D \Lambda_j, & \theta = \{\theta_j\} \\
\frac{1}{2}\lambda (1 - P_D), & \theta = \emptyset
\end{cases} \tag{12}
\]

The normalization constant is thus given by

\[
c = \lambda^m (1 - P_D) + \sum_{j=1}^m \lambda^{m-1} P_D \Lambda_j \tag{13}
\]

and the assignment association probabilities become

\[
\beta_j = \frac{\Lambda_j}{c + \sum_{k=1}^m \Lambda_k} \tag{14}
\]

Using Suboptimal JPDA, since \( L = \emptyset \), the association probabilities become

\[
\beta_j = \frac{\Lambda_j}{B + \sum_{k=1}^m \Lambda_k} \tag{16}
\]

As we can see, the association probability calculated by Suboptimal JPDA is indeed correct if we assume \( \lambda = 0 \). For \( \lambda > 0 \), Helmick’s formula (10) for the bias term is similar to the exact bias term given by (15).

#### 4.2 The case of two targets and two measurements

In the case of two targets and two measurements, with both measurements validating both tracks, the joint event association probabilities are given by (assuming \( P_D^1 = P_D^2 = P_D \))

\[
P(\theta|Z^k) = \frac{1}{c} \lambda^\phi \prod_{j=1}^2 (\Lambda_{t,ij})^{\tau_j} \prod_{l=1}^2 [(P_D)^{\delta_l} (1 - P_D)^{1 - \delta_l}] \tag{17}
\]

\[
= \begin{cases}
\frac{1}{2}\lambda \sum_{k=1}^m \Lambda_{1k} P_D^2, & \theta = \{\theta_{11}, \theta_{12}\} \\
\frac{1}{2}\lambda \sum_{k=1}^m \Lambda_{2k} P_D (1 - P_D), & \theta = \{\theta_{12}, \theta_{21}\} \\
\frac{1}{2}\lambda^2 (1 - P_D)^2, & \theta = \emptyset
\end{cases} \tag{18}
\]
The normalization constant is thus given by
\[
c = \lambda^2 (1 - P_D)^2 + 2 \sum_{t=1}^{N} \sum_{j=1}^{2} [\lambda \Lambda_{tj} P_D (1 - P_D)] + \sum_{t=1}^{N} \Lambda_{tj}^2 \frac{P_D^2}{c}
\]
and the assignment association probabilities, for JPDA, become
\[
\beta_{tj} = \frac{\lambda \Lambda_{tj} P_D (1 - P_D) + \lambda \Lambda_{tj} P_D^2}{c}
\]
where \(t \neq u\) and \(j \neq k\), \(b\) is the exact bias term for the one target and multiple measurements case
\[
b = \frac{\lambda (1 - P_D)}{P_D}
\]
and \(c_2\) is given by
\[
c_2 = b^2 + b \sum_{v=1}^{2} \sum_{t=1}^{2} [\Lambda_{tv}] + \Lambda_{11} \Lambda_{22} + \Lambda_{12} \Lambda_{21}
\]
Using Suboptimal JPDA, the values of \(N_{ij}\) are \(N_{11} = \Lambda_{11}, N_{12} = \Lambda_{12}, N_{21} = \Lambda_{22}\) and \(N_{22} = \Lambda_{21}\), and thus, the values of \(D_{ij}\) are \(D_{11} = \Lambda_{11} \Lambda_{22}, D_{12} = \Lambda_{12} \Lambda_{21}, D_{21} = \Lambda_{21} \Lambda_{12},\) and \(D_{22} = \Lambda_{22} \Lambda_{11}\). The association probabilities for Suboptimal JPDA are then given by
\[
\beta_{tj} = \frac{\Lambda_{tj} \Lambda_{uk}}{B + \Lambda_{11} \Lambda_{22} + \Lambda_{12} \Lambda_{21}}
\]
with \(t \neq u\) and \(j \neq k\). Again, Suboptimal JPDA gives the correct association probabilities for the \(\lambda = 0\) case. For \(\lambda > 0\), however, simply using (10) as the bias term would lead to the calculation of severely biased association probabilities.

4.3 The case of multiple targets and one measurement

The case of multiple targets and one measurement within a cluster may happen when a target is not detected, or the measurement originated by it is not validated. A single measurement may also be present if measurements are merged due to limitations on sensor resolution (this particular case is outside the scope of the original JPDA, however). The basic Suboptimal JPDA does not attempt to give the correct association probabilities for the case of multiple targets and one measurement.

On this situation, the joint event association probabilities are given by (omitting the index of the single measurement, and assuming \(P_D^1 = \ldots = P_D^n = P_D\))
\[
P\{\theta|Z^k\} = \frac{1}{c} \lambda^{1 - \tau} (\Lambda_{t})^{\tau} \prod_{t=1}^{n} [(P_D)^{\delta_t} (1 - P_D)^{1 - \delta_t}] = \left\{ \frac{1}{2} \lambda (1 - P_D)^{n-1}, \quad \theta = \{\theta_t\} \right. \]
\[
\left. \frac{1}{2} \lambda (1 - P_D)^{n}, \quad \theta = \emptyset \right. \]

The normalization constant is given by
\[
c = \lambda (1 - P_D)^n + \sum_{t=1}^{n} \Lambda_{t} P_D (1 - P_D)^{n-1}
\]
and the assignment association probabilities, for JPDA, become
\[
\beta_t = \frac{\Lambda_{t} P_D (1 - P_D)^{n-1}}{c}
\]
\[
= \frac{\lambda^{1 - P_D} + \sum_{u=1}^{n} \Lambda_{u}}{P_D}
\]
For Suboptimal JPDA, since every track is validated by a single measurement, then \(N_t = \Lambda_t\), and it follows that \(D_t = \sum_{u=1}^{n} \Lambda_{u}\). Thus, the assignment association probabilities become
\[
\beta_t = \frac{\Lambda_{t} \sum_{u e L_t} \Lambda_{u}}{B + \Lambda_{t} \sum_{u e L_t} \Lambda_{u}}
\]
In this case, as expected, Suboptimal JPDA gives biased association probabilities even for \(\lambda = 0\). In fact, if the bias term is zero, all tracks will have the same probability (100%) of being updated! This shows that Suboptimal JPDA is prone to association errors in the presence of missed detections.

4.4 The “Augmented Suboptimal” JPDA algorithm

Using (15), (21), (29) and the original Suboptimal JPDA as base, we are ready to propose a scalable JPDA approximation that accounts for clutter and missed detections, that we labeled “Augmented Suboptimal” JPDA. For each scan, the algorithm consists in the following steps:

1. For each track \(t\), form \(A_t\), which is the list of indices of the validated measurements for track \(t\).
2. For each measurement \(j\), form \(C_j\), which is the list of indices of the tracks which validate measurement \(j\).
3. For each track \(t\):
   a. Form the union of all track index lists from all the measurements that are validated by track \(t\), while excluding the index of track \(t\). The
list of track indexes, denoted by \( L_t \), is given by

\[
L_t = \bigcup_{j \in A_t} C_j - \{t\} \quad (31)
\]

(b) For the track index list \( L_t \), find the cardinality of the largest measurement index list

\[
c_t = \max_{u \in L_t} |A_u| \quad (32)
\]

(c) If \( A_t = \{j\} \ (|A_t| = 1) \), define

\[
\Omega_t = \Lambda_{tj} \quad (33)
\]

4. For each pairing consisting of track \( t \) and its validated measurement \( j \), if \( |A_t| \geq 2 \), find

\[
\Theta_{tj} = \max_{k \in A_t, k \neq j} \Lambda_{tk} \quad (34)
\]

5. For each pairing consisting of track \( t \) and its validated measurement \( j \), if \( c_t \geq 2 \), \( D_{tj} \) is given by

\[
D_{tj} = \Lambda_{tj} \left( \sum_{u \in L_t} N_{tuj} + b \right) \quad (35)
\]

where \( N_{tuj} \) is

\[
N_{tuj} = \begin{cases} 
\Theta_{uj}, & |A_u| \geq 2, j \in A_u \\
\Omega_u, & |A_u| = 1, j \notin A_u \\
\Lambda_{tj}, & \text{otherwise}
\end{cases} \quad (36)
\]

and \( b \) is

\[
b = \frac{\lambda(1 - P_D)}{P_D} \quad (37)
\]

; else, \( D_{tj} \) is

\[
D_{tj} = \Lambda_{tj} + \sum_{u \in L_t} Q_{uj} \quad (38)
\]

where \( Q_{uj} \) is

\[
Q_{uj} = \begin{cases} 
\Omega_u, & j \notin A_u \\
0, & \text{otherwise}
\end{cases} \quad (39)
\]

6. For every track \( t \), if \( c_t \geq 2 \), \( B_t \) is given by

\[
B_t = b(b + \sum_{u \in A_t} \sum_{j \in A_t} \Lambda_{uj}) \quad (40)
\]

; else, \( B_t \) is given by

\[
B_t = b + \sum_{u \in A_t} \sum_{j \in A_t} \Lambda_{uj} \quad (41)
\]

7. Finally, the probability of a track \( t \) associating with measurement \( j \) is given by

\[
\beta_{tj} = \frac{D_{tj}}{B_t + \sum_{k \in A_t} D_{tk}} \quad (42)
\]

and the probability of a track \( t \) not being associated with any measurement is

\[
\beta_{tu} = \frac{B_t}{B_t + \sum_{k \in A_t} D_{tk}} \quad (43)
\]

### 4.5 Remarks about the Augmented Suboptimal JPDA

The proposed algorithm gives the correct association probabilities (i.e. same as JPDA) for a given cluster, in the following cases:

- One target and multiple measurements, with any number of measurements being false alarms.

- Two target and two measurements, with both measurements validating both tracks. If both targets have the same probability of detection, then any number of measurements may be false alarms.

- Multiple targets and one measurement, with the measurement being possibly a false alarm, if all targets have the same probability of detection.

- \( N \) targets and \( N \) measurements in complete confusion (all measurements validate all targets, all likelihoods \( \Lambda_{ij} \) equal), no false alarms.

Although the Augmented Suboptimal JPDA may seem to be more complex than the original Suboptimal JPDA, its computational performance is quite similar. Table 1 shows the asymptotic complexity of each step of both algorithms, with \( m \) being the the number of measurements and \( t \) being the number of tracks. As it can be seen, the Augmented Suboptimal JPDA has the same complexity (in asymptotic sense) of the original algorithm \( O(mn^2 + m^2n) \).

**Table 1: Asymptotic bounds of suboptimal JPDA approximations**

<table>
<thead>
<tr>
<th>Step</th>
<th>Sub. JPDA</th>
<th>Aug. Sub. JPDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( O(mn) )</td>
<td>( O(mn) )</td>
</tr>
<tr>
<td>2</td>
<td>( O(mn) )</td>
<td>( O(mn) )</td>
</tr>
<tr>
<td>3</td>
<td>( O(mn^2) )</td>
<td>( O(mn^2) )</td>
</tr>
<tr>
<td>4</td>
<td>( O(m^2n) )</td>
<td>( O(m^2n) )</td>
</tr>
<tr>
<td>5</td>
<td>( O(mn^2) )</td>
<td>( O(mn^2) )</td>
</tr>
<tr>
<td>6</td>
<td>( O(m^2n) )</td>
<td>( O(mn^2) )</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>( O(m^2n) )</td>
</tr>
</tbody>
</table>

### 4.6 Considerations for practical implementation

While an in-depth discussion of the problem of track coalescence on JPDA is outside the scope of this paper, nonetheless it needs to be addressed in order to make an algorithm useful for tracking closely spaced targets. Track coalescence is even more prone to occur on both Suboptimal JPDA and Augmented Suboptimal JPDA approximations, because while the sum of track association probabilities is guaranteed to be
unity ($\sum_{j=0}^{m} \beta_{tj} = 1$), the sum of measurement association probabilities is not, leading to possible increase of cross-correlation between tracks.

To deal with track coalescence, we suggest using an approach similar to the Nearest-Neighbor Joint Probabilistic Data Association (NNJPDA) proposed by Fitzgerald [8]. The association probabilities are computed as normal, but instead of performing a weighted update, they become inputs of an assignment algorithm that performs one measurement-to-one track associations. For that purpose, we verify by experiment that the Bertsekas’ auction algorithm [13] adapted to multi-target tracking [14] has better performance than the standard nearest-neighbor technique. Also, instead of using a fixed threshold for pruning low probability assignments, we may instead discard the assignments with $\beta_{tj} < \beta_0$. Track maintenance may be performed by standard techniques that are used with Nearest Neighbor data association, such as M/N and SPRT.

Although the derivation of the Augmented Suboptimal JPDA was based on the assumptions of uniform clutter spatial density $\lambda$ and uniform probability of detection $P_D$, it’s known that for real radar systems, these parameters vary with slant range, topography, and other factors. An obvious way of adapting the Augmented Suboptimal JPDA to these situations is to perform clustering before calculating the association probabilities (as done in JPDA without approximations), and using the average clutter density and probability of detection for each cluster. However, clustering has complexity of about the same order of magnitude of the Augmented Suboptimal JPDA itself. As alternative to clustering one may, for every track, compute the average $P_D$ and $\lambda$ for the measurements that are validated by the track, and use these values to compute $D_{ij}$ and $B_t$ for that track. This will lead to calculation of biased association probabilities, but if the measurements within a cluster are near, the bias will probably be small.

5 Simulation

The goal of the simulation described here is to compare the performance of the original Suboptimal JPDA, the Augmented Suboptimal JPDA proposed on this paper, and the JPDA without approximations (classic JPDA), with respect to tracking performance (track discrimination and rejection of false alarms) and computational performance (amount of CPU time required). The simulation also aims to test the robustness of the Augmented Suboptimal JPDA on scenarios where the conditions for correct calculation of association probabilities are less frequent (check for overfitting).

To generate the measurements, two 2D radars were simulated, with the following properties: period 6 sec., pulse repetition frequency 6000 Hz, range standard deviation 12.5 m, azimuth standard deviation 0.125°, range resolution 137.16 m, azimuth beam width 1.4°, elevation beam width 30°, maximum range 250 nmi, probability of detection 0.95 and false alarm rate $2 \cdot 10^{-6}$. Every target is assumed to have the same probability of detection. The distribution of false alarms is uniform with respect to range, azimuth and elevation, thus it’s non-uniform with respect to cartesian coordinates used on tracking.

Data association is performed using the selected JPDA variant to compute the association probabilities, and the NN-based algorithm described on Section 4.6 to select one measurement-to-one track pairs for update. The original Suboptimal JPDA implementation followed Roecker and Phillis’ recommendation of setting the bias term to zero. The implementation of the classic JPDA used clustering to avoid combinatorial explosion and subsequent depletion of computational resources.

5.1 Simulation scenarios

The first scenario, shown on Figure 1, has two targets which frequent trajectory crossings. The target with lower maneuverability has a speed of 250 m/s, while the target with higher maneuverability has a speed of 315 m/s, in a manner that the targets reach the end of their trajectories simultaneously. The targets move at nearby altitudes (10,058 and 10,088 m, respectively). The position of both radars is also shown.

Figure 1: Scenario 1

Figure 2 shows the measurements generated by radar 1 (for the first Monte Carlo run). Since the radar measurements are 2D, the distinction between clutter and real targets poses a significant challenge.

The second scenario has 200 targets performing randomly-generated trajectories. Target maneuvers include constant turns typical of passenger aircraft. All targets start their trajectories within a 1,620 nm² area;
they may leave this region during the simulation. The radar positions are unchanged from the first scenario.

5.2 Results

To compare the tracking performance of the association algorithms, we used MTT metrics based on [14]. We performed Monte Carlo simulations composed of 50 runs, where on each run, the following metrics are computed:

1. Number of track switches. A track switch is counted whenever a target represented by a track on a certain scan starts to be represented by another track on the next scan, or when it ceases being represented by a track.

2. Cumulative number of duplicate tracks. For each scan, when a target is represented by more than one track, the tracks in excess are counted as duplicate tracks.

3. Cumulative number of false tracks. For each scan, a false track is counted when a track that does not represent any target exists.

For the second scenario only, we also computed the cumulative CPU time used for computing the association probabilities (this is the sum of the CPU times used for computing the association probabilities on each scan).

The results are shown on Figures 3, 4 and 5. Note that the graphical results were normalized with respect to the classic JPDA results, with exception of the cumulative CPU time, which was normalized with respect to the Suboptimal JPDA result.

By analyzing the graphics, we verify that:

- The Augmented Suboptimal JPDA exhibits a superior tracking performance, compared to the original Suboptimal JPDA. Although the Suboptimal JPDA generated a smaller number of false tracks on scenario 2, we verified that this was due to frequent associations of false measurements with true target tracks, which reduced the number of false tracks but provoked a large increase on the number of track switches and duplicate tracks.

- The Augmented Suboptimal JPDA adds a negligible computation time to the basic Suboptimal JPDA. In fact, on this simulation, the basic Suboptimal JPDA had worse computational performance, simply because it wasted more processing time on duplicate tracks and false tracks. The figure also clearly shows that the classic JPDA has much worse computational performance for scenarios with high target density.
6 Conclusions
The paper has developed a suboptimal approximation for the JPDA algorithm for tracking targets in the presence of clutter and missed detections, based on the existing Suboptimal JPDA algorithm developed by Roecker and Phillis. The simulation results show that, for these scenarios, the proposed method has a performance vastly superior to Suboptimal JPDA, approaching the performance of the JPDA without approximations, at no significant increase on computational cost. The presented method is also significantly more robust than Suboptimal JPDA with respect to trajectory crossings, and entirely avoids the combinatorial explosion which may occur in the JPDA without the approximations.

The method developed on this paper has yet to be compared, in terms of tracking performance and computational performance, with other polynomial approximations of JPDA, such as Linear JIPDA [10] and the modified version of JPDA presented on [11]. The methodology used on this paper to obtain a suboptimal algorithm may also be similarly applied to versions of JPDA that avoid track coalescence.

References