Track Initiation for Blind Mobile Terminal Position Tracking Using Multipath Propagation

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Abstract—Blind localization and tracking of mobile terminals in urban scenarios is an important requirement for offering new location based services, handling emergency cases of non-subscribed users, public safety etc. In this context, we propose a track-before-detect scheme, taking explicit advantage of multipath propagation in an urban terrain by using a priori information about the known locations of the main scattering objects such as buildings. A ray-tracing technique allows the prediction of the directional and temporal structure of the received multipath components for an arbitrary transmitter position. We consider a single observing station where the direction and the relative time of arrival of the received multipath components can be estimated by an antenna array. By a likelihood function, which is algorithmically defined for a randomly distributed set of potential transmitter positions, these measurements are compared with those being expected by ray tracing. This likelihood function is the key component of a track-before-detect scheme providing initial state estimates for mobile transmitter tracking using a particle filtering technique.

Keywords: localization techniques, multipath propagation, particle filtering, track before detect, target tracking.

I. INTRODUCTION

There is a rapid growth of wireless applications that require the knowledge of the mobile terminal’s location [1]. In most cases, the cooperative position estimation methods [2], [3] can be used. In this contribution, however, we concentrate on the blind localization that presumes no cooperation of the mobile terminal with the location reference station. This problem is typical of non-subscribed user localization, e.g. in emergency, security, and safety applications [4].

In a preceding paper [5], we formulated the boundary conditions and proposed a possible solution. According to [5], the blind localization task is carried out via a single observing station (OS) equipped with an antenna array by taking explicit advantage of multipath propagation. The measured spatial and temporal characteristics of the radio channel are assumed to contain information on the spatial location of the mobile station (MS) relative to the OS position. It is typical for the blind case that the time of arrival (ToA) estimation submits only excess (or relative to the first incoming component) delay of the multiple propagation paths. In other words, even under benign line of sight (LoS) conditions the measured spatial/temporal characteristics alone are insufficient for the MS position estimation due to the lack of range information. Hence we propose to compensate the missing geometrical information by exploiting the a-priori knowledge about the geometric structure of the environment (buildings etc.). The key idea implies some “comparison” of measured spatial/temporal radio parameters with pre-calculated reference data. Thus the pre-calculation of the reference data is applied by a ray tracing (RT) (see [6]) algorithm based upon the above mentioned geometric database of the propagation environment.

The comparison of measured and predicted parameters occurs within the likelihood function defined in [5], where it was also shown that the positioning accuracy suffers from the false and undetected multipaths caused by the measurement process. In order to mitigate this effect, the tracking of path parameters was proposed. On the one hand this preprocessing feature decreases the probability of false alarms since they appear randomly during the measurement time. On the other hand, it increases the detection probability of the true multipaths whose parameters vary deterministically depending on the dynamics of the MS and OS. Both result in a robust and accurate localization of the MS under certain assumptions and generalizations. It was assumed, e.g., that the geometrical model reproduces the real environment perfectly and a simplified synthetic scenario was used for the experiments.

In this contribution the localization algorithm operates under more realistic conditions. A 3D model of a real world scenario was used to generate the measurements, which is an important intermediate step before the localization algorithm can be applied to the real measurement data.

The remainder of the paper is organized as follows. In Section 2 we recall the definition of the likelihood function from [5] and propose its approximation, which is numerically less
intensive. In Section 3 we introduce the 3D model of the real world environment and discuss the multipath parameter tracking issue. Section 4 presents the experimental results.

II. MEASUREMENT MODEL AND LIKELIHOOD FUNCTION

A. Measurement model

In this subsection, we will introduce the underlying measurement model. In the following discussions, we will suppress the time index whenever there is no danger of ambiguity. Let us denote the set of measured path parameters for a particular time index by:

\[ z = \{ z^k \}_{k=1}^{n_k} \]  \hspace{1cm} (1)

\( n_k \) is the number of the measured propagation paths which can vary with time. \( z^k \) collects the parameters characterizing the \( k \)-th measured multipath component and has the following structure:

\[ z^k = [\tau^k \, \varphi^k]^T. \]  \hspace{1cm} (2)

Each multipath component is specified by its relative delay:

\[ \tau^k \in [0, \tau_{\text{max}}], \]  \hspace{1cm} (3)

with \( \tau_{\text{max}} \) denoting the measured delay spread, and by its azimuth direction of arrival:

\[ \varphi^k \in [-\pi, \pi]. \]  \hspace{1cm} (4)

We denote the known OS position by

\[ r = [x_r \, y_r]^T \]  \hspace{1cm} (5)

and the MS position is

\[ d = [x_d \, y_d]^T. \]  \hspace{1cm} (6)

Both are allowed to vary with time. In our simulations, we obtained the “measured path parameters” by modeling the radio wave propagation between \( r \) and \( d \) by means of a RT algorithm. The set of modeled path parameters is denoted by

\[ h(r, d) = h_d = \{ h_d^i \}_{i=1}^{n_T}. \]  \hspace{1cm} (7)

and represents the parameters that could be measured if there were no disturbing factors due to the measurement process. Note that \( h(r, d) \) is a nonlinear function since the number of the propagation paths and the values of their parameters depend in a nonlinear way on the position of the MS and OS. Thereby, \( n_T \) is the true number of paths at the MS position \( d \). Parameters characterizing the \( i \)-th true multipath component are contained in a vector

\[ h_d^i = [\tau_d^i \, \varphi_d^i]^T, \]  \hspace{1cm} (8)

where \( \tau_d^i \) is an excess/relative delay obtained from the originally calculated length of the corresponding ray \( l_d^i \) within the RT analysis by subtracting the length of the shortest ray in the set and dividing it by the speed of light:

\[ \tau_d^i = \left( l_d^i - \min \left( \{ l_d^j \}_{j=1}^{n_T} \right) \right) / c_{\text{light}}. \]  \hspace{1cm} (9)

During the measurement process the true parameters are affected by different types of errors. Therefore, the measured path parameters are not identical to the true ones \( h(r, d) \). The low SNR value causes missing detections of the true propagation paths or conversely produces false paths. Furthermore, we have to consider the measurement uncertainties which distort the true parameter values. The modeling mismatch issue, however, is not considered in this work, thus, we assume that the real measurement environment is perfectly reproduced by the RT.

We assume that missing detections occur randomly with the probability \( 1 - P_D \), where \( P_D \) is a detection probability. Vector \( m \) comprises the indices of the detected propagation paths and the set of detected path parameters is a subset of \( h_d \) and is denoted by \( h_d^m \).

The generation of the false propagation paths is a random process as well. We model the number of false paths (also referred to as spurious paths, false alarms or clutter) \( m \) as a Poisson-distributed random variable with the mean number of false alarms \( n_F \). Since the false paths originate from the noise space within the eigenvalue decomposition, their parameters are uniformly distributed in the delay and DoA domain. \( h_d^m \) consists of the incomplete set of true paths and the set of false paths. Finally, we extend the measurement model to additive measurement noise and obtain the following measurement equation:

\[ z = \{ h_d^k + w^k \}_{k=1}^{n_k} \]  \hspace{1cm} (10)

\( w^k \) denotes the measurement noise with entries

\[ w^k = [w_{\tau}^k \, w_{\varphi}^k]^T, \]  \hspace{1cm} (11)

where \( w_{\tau}^k \sim \mathcal{N}(0, \sigma_{\tau}^2) \) and \( w_{\varphi}^k \sim \mathcal{N}(0, \sigma_{\varphi}^2) \) are the realizations from Gaussian distributions. Let \( \sigma_{\tau}^2, \sigma_{\varphi}^2 \) denote the noise variances and

\[ C_k = \text{diag}(\sigma_{\tau}^2, \sigma_{\varphi}^2) \]  \hspace{1cm} (12)

the noise covariance matrix of the \( k \)-th measured path. The values of the noise variances depend on the array configuration, system bandwidth, SNR and are typically different for every measured propagation path. For simplicity’s sake, we assume equal variances for all paths. The measurement model is now complete. In the next section we define the underlying likelihood function.

B. Data Association and Likelihood function

The definition of the likelihood function is one of the central points of the proposed localization procedure. The likelihood function provides a measure of proximity between the multipath parameters predicted by the RT analysis for an arbitrary MS position and the measured multipath parameters obtained by the antenna array at the OS. In calculating the match between the modeled and measured path parameters, we consider the types of error that distort the path parameters and those that either cause missing detections of multipath
components or produce the false ones. This leads to a combinatorial association problem [7], [8] since there are many ways to interpret the measured data. Since we have no a priori information about the location of the MS, the straightforward strategy is to sample the region of interest. Let us assume a sampled, hypothetical MS position specified by two Cartesian coordinates:

\[
\mathbf{s}_p = [x_p, y_p]^T.
\]  

(13)

In total let there be \( P \) hypothetical MS positions with \( p = 1 \ldots P \) that can be randomly chosen or arranged in a grid. \( P \) is thus a design parameter of the localization algorithm to be chosen appropriately depending on the size and the density of the environmental scenario. We model the radiowave propagation between the known OS position \( \mathbf{r} \) and \( \mathbf{s}_p \) by means of RT analysis in the same manner as in (7). The set of predicted path parameters is denoted by

\[
\mathbf{h} (\mathbf{r}, \mathbf{s}_p) = \mathbf{h}_p = \{ \mathbf{h}^i_p \}_{i=1}^{n_p},
\]  

(14)

representing the pendant to the measured parameters defined in (10). Here, \( n_p \) is the number of the predicted propagation paths at the hypothetical MS position \( \mathbf{s}_p \). Parameters characterizing the \( i \)-th predicted multipath component are contained in a vector

\[
\mathbf{h}^i_p = [\tau^i_p, \varphi^i_p]^T.
\]  

(15)

Note that only those paths are considered whose excess delays lie within the measured delay spread \( \tau_{\text{max}} \) defined in (3), that is \( \{ \tau^i_p \}_{i=1}^{n_p} \leq \tau_{\text{max}} \).

We denote the likelihood function by \( p (\mathbf{z}|\mathbf{s}_p) \), which is a conditional probability density and thus can be written as a sum over all possible data interpretations according to the Total Probability theorem:

\[
p (\mathbf{z}|\mathbf{s}_p) = \sum_{E_{i_1...i_{n_p}}} p (\mathbf{z}, E_{i_1...i_{n_p}}|\mathbf{s}_p)
\]

\[
= \sum_{i_1=0}^{n_K} \cdots \sum_{i_{n_p}=0}^{n_K} p (\mathbf{z}|E_{i_1...i_{n_p}}, \mathbf{s}_p) p (E_{i_1...i_{n_p}}|\mathbf{s}_p).
\]  

(16)

We denote a possible data interpretation by \( E_{i_1...i_{n_p}} \) where \( i_1 \ldots i_n \) is an association vector of modeled to measured propagation paths, with

\[
i_j = \begin{cases} 0, & \text{no association, path is not detected or is due to clutter} \\ k \in \{1, \ldots, n_K\}, & \text{\( j \)-th predicted path is associated with the \( k \)-th measured path} \end{cases}
\]  

(17)

Note that one measured path can be associated only with one predicted path. Since the number of measured and predicted paths can differ, there can be several not associated paths. E.g. \( E_{0210} \) represents a possible data interpretation that means that \( n_p = 4 \), i.e. there are 4 predicted propagation paths. Furthermore, the first and the fourth predicted path were not associated; the second predicted path was associated with the second and the third predicted path with the first measured path. Let us elaborate on the terms from (16). Under the assumption that the measured propagation paths are independent of each other, we obtain a factorized likelihood model conditioned on an association hypothesis \( E_{i_1...i_{n_p}} \) (see [8]):

\[
p (\mathbf{z}|E_{i_1...i_{n_p}}, \mathbf{s}_p) = \prod_{k=1}^{n_K} p (\mathbf{z}^k|E_{i_1...i_{n_p}}, \mathbf{s}_p)
\]

\[
= \prod_{j \in I_0} p_{C} (\mathbf{z}^j) \cdot \prod_{j \in I} p_{A} (\mathbf{z}^j|\mathbf{h}^j_p),
\]  

(18)

where \( I = \{ j \in \{1, \ldots, n_p \} \wedge i_j \neq 0 \} \) is the subset of \( n \) indices corresponding to the predicted paths which are associated with the measured paths and \( I_0 = \{ j \in \{1, \ldots, n_p \} \wedge i_j = 0 \} \) is a subset of \( n_K - n \) not associated paths. In the above, \( p_{C} (\mathbf{z}^j) \) denotes the clutter likelihood model for the \( j \)-th measured path, which is assumed to be uniform over the Field of View of the sensor referred to as \( \text{FoV} = 2\pi \tau_{\text{max}} \). \( p_{A} (\mathbf{z}^j|\mathbf{h}^j_p) \) denotes the association likelihood for an \( i_j \)-th measured path associated with the \( j \)-th predicted path. Since the measurement noise is assumed to be independent and Gaussian (see (12)), the likelihood for the \( i_j \)-th measured multipath component, under the hypothesis that it is associated with the \( j \)-th predicted path, is given by

\[
p_{A} (\mathbf{z}^j|\mathbf{h}^j_p) = \mathcal{N} (\mathbf{h}^j_p; \mathbf{z}^j, \mathbf{C}^j).
\]  

(19)

Following the assumptions made above the expression (18) simplifies to

\[
p (\mathbf{z}|E_{i_1...i_{n_p}}, \mathbf{s}_p) = |\text{FoV}|^{-1} n_K |n_K - n| \prod_{j \notin I} p_{C} (\mathbf{h}^j_p; \mathbf{z}^j, \mathbf{C}^j).
\]  

(20)

The second factor in (16) \( p (E_{i_1...i_{n_p}}|\mathbf{s}_p) \) is referred to as association prior (see [8]). We assume the prior of the association hypothesis to be independent of the state and past values of the association hypothesis and thus can be expressed as:

\[
p (E_{i_1...i_{n_p}}|\mathbf{s}_p) = p (i_1 \ldots i_{n_p}|n, n_K, n_p) \prod_{l \in I} p_{P} (n_K - n) p (n|n_p) p (n_p).
\]  

(21)

Here, the first term describes the probability of a single hypothesis under the assumption that all hypotheses are equivalent and is given as

\[
p (i_1 \ldots i_{n_p}|n, n_K) = \frac{1}{N_H} = \left( \frac{n_p}{n} \right)^{n_p} \frac{n_K^1}{(n_K - n)!}.
\]  

(22)

\( N_H \) is the number of valid hypotheses that follows from the number of ways of choosing a subset of \( n \) elements from the available predicted propagation paths \( n_p \) multiplied by the number of possible associations between associated \( n \) and measured \( n_K \) paths. Note that \( n_p \) is a hypothetical value of the true number of measured paths \( n_T \), which is normally unknown. Since we have no a priori information about \( n_T \), we assume a uniform prior for all values of \( n_p \):

\[
p (n_p) = \frac{1}{\max \{ n_p \}_{p=1}^P + 1}.
\]  

(23)
The second term in (21) expresses the probability of \( n_K - n \) false alarms:

\[
p_F(n_K - n) = \frac{(n_F)^{(n_K-n)}}{(n_K-n)!} \cdot e^{-n_F},
\]

which is assumed to follow a Poisson distribution with rate parameter \( n_F \). Finally, the third factor in (21) denotes the probability of \( n \) associated paths, which is assumed to follow the binomial distribution:

\[
p(n|n_p) = \binom{n_p}{n} P_D^n (1 - P_D)^{(n_p-n)}
\]

(25)

incorporating all possible ways to group \( n \) paths among \( n_p \) assumed true measurements. All measured propagation paths share the same known detection probability \( P_D \) according to the measurement model introduced in subsection II-A. Under the assumptions discussed above, the likelihood function can be expressed as

\[
p(z|s_p) \propto \sum_{E_{1 \ldots n_p}} \binom{n_p}{n} P_D^n \prod_{j \in E} N(h_j^i; z^i, C^i) (1 - P_D)^{(n_p-n)}.
\]

(26)

The number of possible associations \( N_H \) within the introduced likelihood function can be enormous. It increases exponentially with the number of measured and predicted paths. Therefore, suitable techniques for the complexity reduction are crucial, e.g. gating [9].

C. Likelihood function approximation

In the previous subsection, we derived the likelihood function formula that was successfully validated in synthetic scenario (see [5]). However, a further analysis showed that under certain conditions, e.g. in scenarios with reach multipath propagation (number of multipaths is greater than 10) or in dense multipath scenarios (multipaths are not well separated in the parameter space) the calculation of the likelihood function becomes computationally intractable. These conditions are quickly achieved in complex real world scenarios where multiple bounce scattering has to be considered. In order to cope with the numerical burden we apply an approximation to the likelihood function. Instead of calculating the abundance of hypotheses weights in (26), we propose to consider only one, most significant hypothesis, expecting that it will reflect the behavior of the complete likelihood function. This approximating association hypothesis can be found using the following procedure. Firstly, we calculate the association likelihoods for each measured path being associated with each of \( n_p \) predicted paths according to (19) and obtain \( n_K \times n_p \) values. In the next step, we apply gating in order to collect the significant association values. Then the validated association values are arranged in a list (see e.g. table I), containing in the first column the association weight, in the second column the corresponding measured path index, and in the third the predicted path index. Now, we search for the highest association weight in the first column of table I and store it in \( \nu_1 \). Let the corresponding measured and predicted path indices be \( k \) and \( j \) respectively. We then remove all associations with participating \( k \)-th measured path and \( j \)-th predicted path from the list in order to obtain a unique pairing in the resulting hypothesis. The procedure is continued with the search and remove steps in the reduced association list until it is empty. The \( m \) stored association weights are then used to calculate the sought approximated likelihood function:

\[
\bar{p}(z|s_p) \propto \frac{\binom{n_p}{n} P_D^n \prod_{i=1}^{\nu_1} \nu_i}{(1 - P_D)^{(n-p)}}.
\]

(27)

III. REAL WORLD SCENARIO AND PATH TRACKING

A. Real world RT scenario

The numerical experiments presented in this paper were carried out in a 3D model of the real world scenario created for the town center of Ilmenau, Germany. Figure 1 presents

![Fig. 1. 3D model of the real world scenario.](image)

the view of the scenario, which is much more complicated than the previously used synthetic scenario (see [5]). The great number of buildings in the scenario represents not only a challenging task for the RT algorithm. It also results in a much higher number of multipaths, inevitably leading to numerical
problems during the calculation of the likelihood function (see subsection II-C). In the receiver-transmitter constellation (see Figure 1), each multipath was allowed to interact with the obstacles twice at most; furthermore, the dynamic power range was set to 30 dB. The total number of detected multipaths was 47.

B. Path parameter tracking

In order to mitigate the effect of positioning accuracy degradation caused by false and not detected multipaths, the tracking of measured path parameters was proposed in [5]. The applied Kalman filter bank works well in a simple synthetic scenario from [5], however it is not able to cope with association ambiguities as well as the problem of track formation or deletion occurring in the dense multipath environment. For the handling of the path parameter tracking task in the real world scenarios, we use a technique proposed in [10], the integrated probabilistic data association filter (IPDAF). IPDAF is an extension of the well known PDAF (see [9] for the details) and assumes that track existence is an event with a corresponding probability. It provides recursive expressions for both the probability of track existence and data association. An extension to multitarget case, so called joint IPDAF (JIPDAF), is presented in [11].

During tracking, path tracks were confirmed if the probability of existence exceeded the confirmation threshold and terminated if the probability fell below the termination threshold. Here, the confirmation threshold was set to 0.4 and termination threshold to 0.05. The initial probability of track existence was 0.2.

The flow chart of the whole initialization algorithm is presented in Figure 5. Here, \( \hat{g}_t \) denotes the path tracks estimated via JIPDAF existing at the time point \( t \) and \( \hat{P}_t \) are the corresponding uncertainties. \( T_D \) denotes the delay time needed for the confirmation of the path tracks and \( T_p \) is the measurement period. Note that for the calculation of (27), which occur within a Sampling Importance Resampling (SIR) filter \( \hat{g}_t \) and \( \hat{P}_t \) are used instead of the raw data \( z_t \) and \( C_t \). In [5], it is shown that the path tracking procedure decreases the probability of false alarms since they appear randomly during the measurement period. Furthermore, it increases the detection probability of the true multipaths whose parameters vary deterministically depending on the dynamics of the MS and OS.

IV. EXPERIMENTS AND RESULTS

In this section, we present the simulation results of the proposed space state initialization technique for the blind MS tracking. We use the real world scenario presented in section III-A. We initiate the track at different MS locations depicted in Figure 2 in order to evaluate the performance of the algorithm under different environmental conditions. Here we choose LoS as well as NLoS situations. The measurement noise covariance was set to \( C_t = \text{\text{diag}} \left( \left( \frac{5m}{c_{\text{Light}}} \right)^2, \left( 5° \right)^2 \right) \). Furthermore, we assumed \( P_D = 0.8 \) and \( n_F = 10 \). We have applied a SIR filter, a well-known particle filtering technique (see [12], [13]), for the MS position initialization. Within the SIR procedure, we used the proposed approximation of the likelihood function (27) with preceding path parameter tracking described in section III-B. The result of the sequential state estimation is saved in \( \hat{s} \) and its uncertainty \( \hat{P}_s \) (see Figure 5).

The following rough assumptions concerning the initial MS state were considered within the SIR algorithm. The MS is located somewhere outdoors in the region of interest depicted in Figure 3, i.e. the location possibility is equal for all outdoor positions. Hence we model the initial position uncertainty as a uniform distribution in the region of interest except for the indoor areas. We assume measurement rates of at least 1 observation per second, i.e. the state variation of the MS moving with a maximum velocity of up to 5km/h (valid for pedestrian area) is negligible. Therefore we limit the MS state
to the x and y coordinates only and define the following state dynamics function:

$$s_{p,t} = I \cdot s_{p,t-1} + \omega_{p,t-1},$$

(28)

where \(s_{p,t}\) denotes the sampled MS position \(p\) at time \(t\), specified in (13) also referred to as a particle. \(I\) is a \(2 \times 2\) identity matrix and \(\omega_{p,t-1}\) is a process noise with the entries \(\omega_{p,t-1} = [\omega_x \ \omega_y]^T\), which are modeled as realizations from Gaussian distributions \(\omega_x \sim \mathcal{N}(0, \sigma_x^2)\) and \(\omega_y \sim \mathcal{N}(0, \sigma_y^2)\). In our example \(\sigma_x\) and \(\sigma_y\) were set to 5m. The measurement equation is given by (10) and the approximated likelihood function available for pointwise evaluation is defined in (27). Therefore, the assumptions required to use the SIR are satisfied (see [12]). Figure 3 shows 1000 samples randomly chosen from the region of interest that approximate the initial position uncertainty. During the first cycle of the SIR algorithm, the likelihood weight is evaluated for every particle. The subsequent resampling step eliminates particles with low likelihood weights and multiplies particles with high likelihood weights. In our simulations we used a source moving with the velocity of 1m/s, i.e. in the second SIR cycle the parameters were measured at the position 1m away from the start position.

Figure 4 presents the particle distributions and the estimated MS positions depicted by black dots for all 5 cases after the fifth SIR cycle. The ellipses indicate the 2σ regions of the estimated state covariance matrices. Note that true MS positions depicted by squares lie within the 2σ regions.

![Figure 4. Estimated MS locations (black dots); true MS locations (gray squares).](image)

**V. CONCLUSIONS**

We presented a track-before-detect method for initialization of blind mobile terminal tracking in urban scenarios. The key role, thereby, plays the proposed likelihood function which determines the proximity of the measured and predicted multipath components with respect to all possible association hypotheses between them. The measurements of the multipath components are provided by an OS equipped with an antenna array. The predicted temporal and spatial structure of the multipath components is generated by means of the RT analysis using a priori information about the location of the scattering objects. In order to mitigate the impact of missing and false propagation paths on the positioning result, we proposed the preprocessing of the measured path parameters by means of a multitarget tracking algorithm - JIPDAF.

The likelihood function, which is algorithmically defined for a randomly distributed set of potential MS positions, was applied within the particle filtering technique and was tested on data generated from a 3D model of the real-world scenario. In order to overcome a combinatorial complexity arising in dense multipath scenarios, an approximation of the likelihood function that considers only one hypothesis was introduced. The presented simulation results show that this approximation yields sufficient accuracy of state initiation and finally leads to a reasonably good position estimation.

Topics of our future research will be consideration of errors caused by the imperfect geometrical model and the track maintenance part of the MS tracking algorithm.

**REFERENCES**


