Lane Tracking for On-Road Targets

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Abstract—Considerable research has been done on tracking ground targets, including on-road targets. Lane tracking of an on-road target is a new problem in the ground target tracking area. Knowledge of the lane that a target is located in is of particular interest in on-road surveillance and target tracking systems. This paper proposes a method to track the lane of an on-road target based on a Hidden Markov Model (HMM). It is assumed that an image sensor provides raw observation data. The formulation of the problem and our approach are given. Simulation results show that the implemented HMM-based lane estimators can achieve good performance.

Keywords: Ground Target Tracking, Estimation, Hidden Markov Model.

I. INTRODUCTION

Considerable research has been done on tracking ground targets, including on-road targets, e.g., [1]–[6]. Lane estimation (or identification) of on-road target is a new problem that attracted our interest—we have not found any paper published on this problem. Knowing the lane that a target is located in is of particular interest in on-road surveillance and target tracking systems. In addition, if a good estimate of the lane that a target is located in is available it could help improve the estimate of the location and motion of the target.

The Hidden Markov Model has been heavily researched and used over the past several decades [7]–[11], and successfully applied to a wide variety of applications, especially in the speech recognition area [12]–[14].

In this paper, we use the HMM framework to formulate and solve the lane tracking problem. The target motion across lanes is modeled in discrete-time through a Markov chain with some initial and transition probabilities. The multiple lanes of the road are the states of this Markov model. It is assumed that direct observations of the lane of a target with some observation probability matrix (OPM) are available. We derive the OPM based on the assumption that an image sensor (e.g., camera) provides raw observation data in an on-road target motion scenario. The lane tracking is done in terms of both the optimal estimation of the lane sequence and of the current lane of the target, given the observations from the start time to the current time. We call the first estimator the lane sequence estimator (LSE), and the second the lane filter (LF). Performance evaluation of the both estimators was done by a Monte Carlo simulation. The estimators assume knowledge of the transition probability matrix (TPM) of the motion model. Since in practice it is hard to come by with an accurate value of this matrix, we also study the performance of the LSE and LF under mismatch between the TPM used by the estimators and the true TPM (used for ground truth generation).

The paper is organized as follows. Sect. II gives a formulation of the problem as an HMM problem and presents two algorithms for lane tracking of an on-road target. Sect. III derives an observation probability matrix based on the sensor used for this problem. Sect. IV presents simulation and performance evaluation results, including the effect of the TPM on the tracking performance. Conclusions are drawn in Section V.

II. PROBLEM FORMULATION AND LANE ESTIMATION

A. Target Motion Model

It is assumed that a target is moving on a road with N lanes. The target motion across lanes is modeled by a Markov chain \( \{l_t\}_{t=1,2,...} \) as follows.

Let \( l_t \in \{1,2,\ldots,N\} \) denote the lane in which the target is at discrete time \( t = 1,2,\ldots,T \). The initial probability vector and the transition probability matrix (TPM) of the chain are \( \pi = \{\pi_i\}_{i=1}^{N} \) and \( A = \{a_{ij}\}_{i,j=1}^{N} \), respectively. In the context of motion across lanes \( \pi_i = P(l_t = i) \) denotes the a priori probability of the target being in lane \( i \) at the beginning (at time \( t = 1 \)) and \( a_{ij} = P(l_{t+1} = j|l_t = i) \) denotes the probability of the target being in lane \( j \) at time \( t + 1 \) given that it was in lane \( i \) at time \( t \). We assume that the TPM A is time invariant.

B. Observation Model

It is assumed that direct observations of the lane of a target are available. An observation, \( O_t = O_t(i) \), denotes that the target is declared by the sensor to be in lane \( i \) at time \( t \). The observation mechanism is statistically modeled by an observation probability matrix (OPM) \( B = \{b_j(O_t(i))\}_{j,i=1}^{N} \) where \( b_j(O_t(i)) = P(O_t(i)|l_t = j) \) is the probability of observing the target in lane \( i \) given that the target is in lane \( j \) at time \( t \). We assume that the OPM B is time invariant.

C. Lane Sequence Estimator

Given an HMM model \( \lambda = (A,B,\pi) \) and an observation sequence \( O = \{O_1,O_2,\ldots,O_T\} \) we aim at choosing a
lane sequence \( L = l_1, l_2, ..., l_T \) so that the joint probability \( P(O, L|\lambda) \) is maximized.\(^1\)

Since
\[
P(O|L) = b_1 (O_1) b_{i_2} (O_2) ... b_{i_T} (O_T)
\]
and
\[
P(L) = \pi_{i_1} a_{i_1, i_2} ... a_{i_{T-1}, i_T}
\]
we have
\[
P(O, L) = P(O|L)P(L)
\]
After defining the weight
\[
U(l_1, l_2, ..., l_T) = - \left[ \ln (\pi_{i_1} b_{i_1} (O_1)) + \sum_{t=2}^{T} \ln (a_{i_{t-1}, i_t} (O_t)) \right]
\]
it can be obtained that
\[
P(O, L) = \exp [-U(l_1, l_2, ..., l_T)]
\]
The optimal lane sequence estimation is to find
\[
(l^*_1, l^*_2, ..., l^*_T) = \arg \max_{(l_1, l_2, ..., l_T)} [P(O, l_1, l_2, ..., l_T)]
\]
which is equivalent to
\[
(l^*_1, l^*_2, ..., l^*_T) = \arg \min_{(l_1, l_2, ..., l_T)} [U(l_1, l_2, ..., l_T)]
\]
Finding the optimal lane sequence is done through the well known Viterbi algorithm given below. The term \(-\ln (a_{i_{t-1}, i_t} (O_t))\) is the weight associated with the transition \( l_{t-1} \rightarrow l_t \).

**Viterbi LSE Algorithm**

For \( T = 1, 2, \ldots \) recursively compute the optimal lane sequence:

1. **Initialization:**
   
   For \( 1 \leq i \leq N \),
   \[
   \delta_1(i) = -\ln (\pi_{i_1}) - \ln (b_{i_1} (O_1))
   \]
   \[
   \varphi_1(i) = 0
   \]

2. **Recursive Computation:**
   
   For \( 2 \leq t \leq T \), and \( 1 \leq j \leq N \)
   \[
   \delta_t(j) = \min_{1 \leq i \leq N} [\delta_{t-1}(i) - \ln (a_{ij})] - \ln (b_j (O_t))
   \]
   \[
   \varphi_t(i) = \arg \min_{1 \leq i \leq N} [\delta_{t-1}(i) - \ln (a_{ij})]
   \]

3. **Termination:**
   
   \[
   W_{\min} = \min_{1 \leq i \leq N} \delta_T(i)
   \]
   \[
   l^*_T = \arg \min_{1 \leq i \leq N} \delta_T(i)
   \]

4. **Tracking back the optimal lane sequence:**

For \( t = T - 1, T - 2, ..., 1 \),
\[
l^*_t = \varphi_{t+1}(l^*_{t+1}).
\]

It can be easily seen that \( L^*_T = \{ l^*_1, l^*_2, ..., l^*_T \} \) is the optimal lane sequence from time \( t = 1 \) to time \( t = T \).

**D. Lane Filter**

The variable \( \alpha_T(i) \) is defined as
\[
\alpha_T(i) = P(O_1, O_2, ..., O_T, l_T = i)
\]
By Bayes’ rule the probability of the target being in lane \( i \) at time \( t \) given the observation sequence \( O = O_1, O_2, ..., O_T \) is
\[
P(l_T = i|O) = \frac{P(l_T = i, O)}{P(O)} = \frac{\alpha_T(i)}{P(O)}
\]
Further, we have subsequently
\[
\alpha_{T+1}(j) = P(O_1, O_2, ..., O_T, O_{T+1}, l_{T+1} = j)
\]
\[
= \sum_{i=1}^{N} [P(O_{T+1}, l_{T+1} = j|O_1, O_2, ..., O_T, l_T = i) \cdot P(O_1, O_2, ..., O_T, l_T = i)]
\]
\[
= \sum_{i=1}^{N} [P(O_{T+1}|l_{T+1} = j)P(l_{T+1} = j|O_1, O_2, ..., O_T, l_T = i)\alpha_T(i)]
\]
\[
= \sum_{i=1}^{N} [P(O_{T+1}|l_{T+1} = j)a_{ij}\alpha_T(i)]
\]
\[
= b_j (O_{T+1}) \sum_{i=1}^{N} a_{ij}\alpha_T(i)
\]

The probability of the observation sequence \( O = \{O_1, O_2, ..., O_T\} \) is
\[
P(O) = \sum_{i=1}^{N} P(O, l_T = i)
\]
\[
= \sum_{i=1}^{N} \alpha_T(i)
\]

**LF Algorithm**

Recursively compute the optimal lane estimation at time \( T \) for \( T = 1, 2, \ldots \):

1. **Initialization:**
   
   \[
   \alpha_1(i) = \pi_i b_i (O_1), 1 \leq i \leq N
   \]

2. **Recursion:**
   
   For \( t = 1, 2, ..., T - 1 \leq j \leq N \),
\[
\alpha_{t+1}(j) = b_j (O_{t+1}) \sum_{i=1}^{N} [a_{ij}\alpha_t(i)]
\]

\(^1\)The conditioning on \( \lambda \), common in the HMM literature, is dropped in the sequel to simplify the notation.
3. Observation sequence probability:

\[ P(O) = \sum_{i=1}^{N} \alpha_T(i) \quad (21) \]

4. Optimal estimate of the lane at current time \( T \)

\[ l_T^* = \arg \max_{1 \leq i \leq N} P(l_T = i, O) \]

\[ = \arg \max_{1 \leq i \leq N} \alpha_T(i) \]

\[ P(O) \quad (22) \]

The sequence \( L_T^* = \{l_T^*, l_T^2, ..., l_T^n\} \) is the estimation result we obtain at times \( T = 1, 2, \ldots \)

### III. Observation Probability Matrix

Here we propose a method for deriving the observation probability matrix \( B \) for a particular scenario.

It is assumed that a signal processor can provide the displacement \( d \) of the target center from the left edge of the road. For example, if a surveillance system uses raw image data (provided, e.g., by a camera on a satellite), a measurement \( z = d + v \) of this displacement can be extracted by using image processing techniques. We assume that the measurement error \( v \) is zero-mean with Gaussian distribution, truncated outside the road. Then

\[ z \sim f(z) = \begin{cases} \frac{1}{\sigma_v}N(z; d, \sigma_v^2) & \text{if } 0 < z < 2N\Delta \\ 0 & \text{otherwise} \end{cases} \]

where \( N \) is the number of lanes, \( 2\Delta \) is the width of each lane, \( c = \Phi \left( \frac{2N\Delta - d}{\sigma_v} \right) - \Phi \left( \frac{-d}{\sigma_v} \right) \) is a normalization constant, and \( \Phi(\cdot) \) denotes the standard Gaussian cumulative distribution function (cdf).

When a target is in a lane we assume that the central point of the vehicle is on the central line of this lane. Given that a target is in lane \( j \), the probability density function \( f_j(z) \) of a measurement originating from the target is (Fig. 1)

\[ f_j(z) = \begin{cases} \frac{1}{\sigma_v}N(z; (2j - 1)\Delta, \sigma_v^2) & \text{if } 0 < z < 2N\Delta \\ 0 & \text{otherwise} \end{cases} \]

where

\[ c_j = \Phi \left( \frac{2(N - j) + 1)\Delta}{\sigma_v} \right) - \Phi \left( \frac{-2j - 1)\Delta}{\sigma_v} \right) \]

Then, after some straightforward calculation, we have the final result

\[ P(O_i | l_T = j) = P(2(i - 1)\Delta < z < 2i\Delta) \]

\[ = \frac{1}{c_j} \left[ \Phi \left( \frac{2(i - j) + 1)\Delta}{\sigma_v} \right) - \Phi \left( \frac{2(i - j) - 1)\Delta}{\sigma_v} \right) \right] \]

Clearly, the idea of the above derivation is not limited to Gaussian distribution of the error at the output of the image processing.

![Figure 1. Measurement PDF.](image)

For \( N = 3, \Delta = 2 \) m and \( \sigma_v = 2 \) m we obtained that

\[ B = \begin{bmatrix} b_{ij}(O_t(i)) \end{bmatrix}_{j=1}^3_{i=1} \]

\[ = \begin{bmatrix} 0.8114 & 0.1870 & 0.0016 \\ 0.1577 & 0.6845 & 0.1577 \\ 0.0016 & 0.1870 & 0.8114 \end{bmatrix} \]

This OPM was used in the simulations described next.

### IV. Simulation and Performance Evaluation

#### A. Performance measures

In the simulation we calculated the following measures for performance evaluation.

**Probability of being in the correct lane at the current time**

\[ P_T = \frac{1}{N_T} \sum_{r=1}^{N_T} S_T^r \]

where \( S_T^r = 1 \) or 0. \( S_T^r = 1 \) when the lane identified at the current time \( T \) is correct, and \( S_T^r = 0 \) otherwise. \( N_T \) is the number of runs.

**Time-average probability of being in the correct lane**

\[ P_{ac} = \frac{1}{N_T} \sum_{r=1}^{N_T} P_c^r \]

where for each single run \( r \)

\[ P_{ac}^r = \frac{k}{n} \]

\[ (31) \]

is the percentage of times the identified lane is correct and \( n \) is the number of observations in this run.

**Error probability matrix.**

It is calculated via normalization of the confusion matrix, i.e.,

\[ C = \left( \frac{c_{ij}}{\sum_{i=1}^{N} c_{ij}} \right)_{i,j=1}^{N} \]

where \( c_{ij} \) is the number of times that the target is declared in lane \( i \) while it is actually in lane \( j \).
B. Simulation

The parameters of the ground truth are as follows.

\[ N = 3 \]

Initial lane probabilities

\[ \pi = [0.3 \ 0.4 \ 0.3] \]

Lane transition probability matrix

\[ A = \{a_{ij}\} = \begin{bmatrix} 0.92 & 0.07 & 0.01 \\ 0.04 & 0.92 & 0.04 \\ 0.01 & 0.07 & 0.92 \end{bmatrix} \]

The observation probability matrix \( B \) in this case is given by (28), as derived in Sect. III.

The performances of the LSE and LF are illustrated in Figures 2 and 3, respectively. From Figure 2 we can see that the LSE provides good performance. After about 50 time steps the probability of being correct on the lane sequence is around 0.92 and it tends to increase as time goes. From Figure 3, we can also see good performance using the LF algorithm. Note that the performance measures used in Figure 2 and Figure 3 are different: The first one is the average probability of being correct on the lane sequence from time 1 to the current time; The second one is the probability of being correct on the lane at the current time over all the runs.

The error probability matrix of the LSE at \( T = 200 \) is

\[ C = \begin{bmatrix} 0.8600 & 0.1400 & 0 \\ 0.0485 & 0.8738 & 0.0777 \\ 0 & 0.1702 & 0.8298 \end{bmatrix} \]

For the LF at \( T = 200 \) it is

\[ C = \begin{bmatrix} 0.8800 & 0.1200 & 0 \\ 0.0388 & 0.8932 & 0.0680 \\ 0 & 0.1702 & 0.8298 \end{bmatrix} \]

Comparing these error probability matrices we see that, as it should be, the LF has better performance concerning the lane at the current time than LSE.

C. Unknown lane TPM

In order to see the effect of mismatch between the true TPM used in the simulation and the one used by the estimators, we analyze the sensitivity of the elements of the TPM using a 3 by 3 matrix defined as

\[ \Delta = (\Delta a_{ij}) = (a_{ij}^v) - (a_{ij})^v, 1 \leq i,j \leq 3 \]

where \( A = \{a_{ij}\} \) is the ground truth, and \( A_v = \{a_{ij}^v\} \) is the transition matrix used by the HMM estimators. \( \Delta \) is chosen as the matrix \( \Delta \) defined in Sect. IV B. Due to the symmetry of the TPM we analyze the sensitivity of the elements \( a_{11}, a_{12}, a_{22} \).

First, we analyze the effect of \( a_{11} \) on the performance of LSE and LF by comparing the performance in the following four cases.

Case 1: \[ \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] Case 2: \[ \Delta_1 = \begin{bmatrix} .1 & 0 & -.1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Case 3: \[ \Delta_1 = \begin{bmatrix} .2 & 0 & -.2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] Case 4: \[ \Delta_1 = \begin{bmatrix} .4 & 0 & -.4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Secondly, we analyze the effect of \( a_{12} \) in the following three cases.

Case 1: \[ \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] Case 2: \[ \Delta_2 = \begin{bmatrix} 0 & .03 & -.03 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Case 3: \[ \Delta_2 = \begin{bmatrix} 0 & .06 & -.06 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Thirdly, we analyze the effect of \( a_{11} \) with respect to \( a_{12} \) on the performance of LSE and LF by comparing the performance in the following four cases.
Lastly, we analyze the effect of $a_{22}$ in the following four cases.

Case 1:
\[
\Delta_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.1 & 0.2 & -0.1 \end{bmatrix}.
\]

Case 2:
\[
\Delta_3 = \begin{bmatrix} 0.1 & -0.1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_4 = \begin{bmatrix} 0.05 & 0.1 & -0.05 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

Case 3:
\[
\Delta_3 = \begin{bmatrix} 0.2 & -0.2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

Case 4:
\[
\Delta_3 = \begin{bmatrix} 0.4 & -0.4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

Figure 4 shows the performance of the LSE with different TPMs $A_v$. It is seen from Figure 4(a), (b), and (c) that $\Delta_{a_{11}}$ and $\Delta_{a_{12}}$ have some effect on the performance of the LSE, but not very significant. From Figure 4(d) we can see that the performance of LSE is affected by $a_{22}$ very strongly. Figure 5 shows similar results for the LF. It seems that the element $a_{22}$ of TPM has much greater impact on the performance. If the ground truth, especially the true $a_{22}$ of TPM, is unknown, the choice of lane TPM will affect the lane tracking performance heavily. How to identify the lane transition probability matrix appears to be an important problem that we intend to address in the future.

V. CONCLUSIONS

From the simulation and analysis given above we can see that the implemented algorithms, viz., the LSE and LF, provide reasonable performance for the lane tracking problem. After 50 time steps the average probability of correct estimation of the LSE is around 0.92 which is much larger than the probability of correct observation in the observation probability matrix.
Figure 5. Probability of being correct of LF: (a) is for $\Delta_1$, (b) is for $\Delta_2$, (c) is for $\Delta_3$, (d) is for $\Delta_4$. 200 runs.

The probability of being correct at the current time of the LF is also much larger than the probability of correct observation in the observation probability matrix. To estimate the lane sequence, the LSE is better than the LF. To identify the lane at the current time, the LF is better than the LSE. The two algorithms can effectively identify the lane which an on-road target is in. It has been established in the simulation that the choice of lane TPM affects the lane tracking performance heavily. Identifying the lane transition probability matrix is an important problem for further work.

REFERENCES


