Multitarget Tracking in the Presence of Wakes

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Abstract—In this paper we focus on targets which, in addition to reflecting signals themselves, also have a trailing path behind them, called a wake. When the detections are fed to a tracking system like the Probabilistic Data Association Filter, the estimated track can be misled and sometimes lose the real target because of the wake. This problem becomes even more severe in multitarget environments where targets are operating close to each other in the presence of wakes. To prevent this, we have developed a probabilistic model of the wakes in a multitarget environment. This model is used to augment the Joint Probabilistic Data Association Filter (JPDAF). Simulations of two crossing targets with wakes show that this modification gives good results and the number of lost tracks is significantly reduced.

Keywords: Multitarget, tracking, data association, wake.

I. INTRODUCTION

Targets in real tracking scenarios may be detected by their reflection of signals emitted from a radar [6], a sonar [14], or by the use of optical sensors [8], [13]. In addition to target-originated measurements there will also be a number of detections due to noise and clutter, called false alarms. A well-known tracking method to handle targets in clutter is the Probabilistic Data Association Filter (PDAF) [2], [5]. The PDAF accounts for the measurement origin uncertainty by calculating for each validated measurement at the current time the association probabilities to the target of interest.

In a multitarget environment [4] the association of measurements is more problematic because the individual targets no longer can be considered separately as in the PDAF. For this purpose the Joint Probabilistic Data Association Filter (JPDAF) [2], [9], [10] was developed to consider a known number of targets in the data association simultaneously. This method evaluates the measurement-to-target association probabilities for the latest set of measurements and then combines them into the state estimates.

A more powerful source of false measurements is the wake phenomenon that appears behind certain targets. This could be air bubbles from a diver, the wake behind a ship, or the wake from ballistic vehicles in the reentry stage. One possible approach to this problem is to handle both the target and the wake behind it as an extended target. A problem with this approach is the varying and unknown size of the wake which may reach far behind the target yielding a large bias. In this paper it is emphasized that the wake-dominated measurements should not be considered as part of the target, but rather as a special kind of clutter. When these measurements are fed to the tracking system, it becomes important to associate them correctly to prevent a lost track. In [1] a probabilistic editing method is used to handle the wake-dominated measurements in the tracking algorithm. This probabilistic editing method is based on a single measurement extracted for each time-step, and that this measurement originates from either the target or the wake. In [12] a modified PDAF is developed to handle false measurements originating from the bubbles behind a diver (the wake). This modified single target tracking method does not restrict the number of false measurements for each time-step, but assumes a set of measurements where each false measurement originates from either random clutter or the wake. In this paper we extend the modified PDAF to handle multiple targets in the presence of wakes. A probabilistic wake model is used for each target in the multitarget environment that has a wake behind it. These single wake models are combined to form a joint wake model, and the modified JPDAF is developed to incorporate this additional joint wake model.

In Section II the tracking model and data association is presented for a single target. In Section III the modified JPDAF is developed for a multitarget environment. The data association methods are then compared in Section IV by simulations of two crossing targets, before conclusions are given in Section V.

II. BACKGROUND

A. Model of Tracking

The standard discrete linear model in tracking is

\[ x_{k+1} = F x_k + v_k \quad \text{and} \quad z_k = H x_k + w_k \]  

where

- \( x \): target state
- \( F \): transition matrix
- \( z \): measurement
- \( H \): measurement matrix
- \( v \): process noise
- \( w \): measurement noise
- \( k \): time index

The process and measurement noises are assumed independent, white and Gaussian with covariance matrices

\[ E\{v_k v_k^T\} = Q \quad \text{and} \quad E\{w_k w_k^T\} = R \]  

In this system, assuming equal process and filter model, a Kalman filter would be optimal as long as there is only one single measurement \( z_k \) at each time \( k \). In real data this is unfortunately not true due to false measurements originating

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from noise and clutter. Instead, a set of \( m_k \) measurements \( Z_k = \{ z_k(1), z_k(2), \ldots, z_k(m_k) \} \) is available at time \( k \) and a form of data association is needed.

### B. Standard PDAF

The approach of the PDAF is to calculate the association probabilities for each validated measurement at the current time to the target of interest. The posterior track probability density is therefore a mixture of Gaussian probability density functions (pdf), but is then forced back to Gaussianity by moment-matching for the succeeding scan. For a derivation of the PDAF see [2], and in the following a brief overview of the PDAF will be given.

Assume that the target state at time \( k-1 \) is estimated as \( \hat{x}_{k-1|k-1} \) with associated covariance \( P_{k-1|k-1} \). This means that the estimate is conditioned on the entire past up to time \( k-1 \). Then the following assumptions are made:

(a) The track is already initialized.

(b) The past information about the target is summarized approximately by the Gaussian distribution

\[
p(x_k|Z^{k-1}) \approx \mathcal{N}(x_k; \hat{x}_{k|k-1}, P_{k|k-1})
\]

where \( Z^{k-1} = \{ Z_0, Z_1, \ldots, Z_{k-1} \} \)

(c) A validation region or gate is set up for each time step to select the candidate measurements for association.

(d) At time \( k \) there are \( m_k \) validated measurements but at most one of them can be target-originated. The rest are assumed due to i.i.d. uniformly spatially distributed false alarms, independent across time.

(e) Detections of the real target occur independently over time with known detection probability \( P_D \).

At each time \( k \), the algorithm goes through the following steps:

1) Predict the target state, associated covariance and measurement at time \( k \) based on the estimates at \( k-1 \):

\[
\begin{align*}
\hat{x}_{k|k-1} &= F \hat{x}_{k-1|k-1} \\
P_{k|k-1} &= FP_{k-1|k-1}F^T + Q \\
\hat{x}_{k|k} &= H \hat{x}_{k|k-1} \\
\end{align*}
\]

2) Compute the innovation covariance for the true (target-originated) measurement

\[
S_k = HP_{k|k-1}H^T + R
\]

and use \( S_k \) to form the measurement validation gate where the validated measurements \( Z_k \) result in \( m_k \) innovations:

\[
\nu_k(i) = z_k(i) - \hat{x}_{k|k-1} \quad i = 1, \ldots, m_k
\]

3) Calculate the association probabilities \( \beta_k(i) \),

\[
\beta_k(i) = \frac{c e^{-\frac{1}{2} \nu_k(i)^T S_k^{-1} \nu_k(i)}}{2 \pi^{|S_k|^{1/2}} m_k^{1/2} P_D \nu_k(i)}
\]

Here \( c \) is a normalizing constant to ensure that \( \sum_{i=0}^{m_k} \beta_k(i) = 1 \). \( V_k \) is the volume of the gate and \( P_G \) is the probability that the true measurement falls inside the gate. In (8) a diffuse prior [2] is used for the point mass function (pmf) of the number of false measurements in the validation region.

4) Calculate the Kalman gain and the combined innovation

\[
W_k = P_{k|k-1} H^T S_k^{-1} \quad \text{and} \quad \nu_k = \sum_{i=1}^{m_k} \beta_k(i) \nu_k(i)
\]

5) The state estimation covariance is updated by

\[
P_{k|k} = \beta_k(0) P_{k|k-1} + \left[ 1 - \beta_k(0) \right] \left( P_{k|k-1} - W_k S_k^{-1} W_k^T \right) + W_k \sum_{i=0}^{m_k} \beta_k(i) \nu_k(i) \nu_k(i)^T - \nu_k \nu_k^T
\]

where the last term in (11) is the “spread of the innovations.”

### C. Modified PDAF

Targets with a wake behind them may cause detections from the wake that mislead the tracking algorithm and are likely to result in a lost track. To prevent this, an extension of the regular PDAF incorporating a special probabilistic model of the wake was developed in [12]. The PDAF with the wake model is illustrated in Fig. 1, and takes into account that false measurements can originate from either the wake with pdf \( p_W(\cdot) \) and a priori probability \( P_W \), or from i.i.d. uniformly distributed noise/clutter with a priori probability \( 1 - P_W \), independently across time. This modification affects the PDAF.
in the calculation of the $\beta_k(i)$ in (8) and yields

$$
\beta_k(i) = \left\{ \begin{array}{ll}
   c \frac{e^{-\frac{1}{2}v_k(i)^T S_k^{-1} v_k(i)}}{1 - P_{k,t} + P_{GW} P_W(z_k(i))} & i = 1, \ldots, m_k \\
   c \frac{1}{2\pi S_k} m_k^{1 - \nu_k} & i = 0 
\end{array} \right.
$$

(12)

The denominator in $\beta_k(i)$ for $i = 1, \ldots, m_k$ is the pdf of a false measurement

$$
p(z_k(i)|\text{measurement } i \text{ is false}) = \frac{1 - P_{GW}}{V_k} + \frac{P_W}{P_{GW}} p_W(z_k(i))
$$

(13)

where $P_{GW}$ is used to account for restricting the density of the wake model $p_W(z_k(i))$ to the validation gate. The calculation of $P_{GW}$ for a linear $p_W(\cdot)$ is presented in detail in [12]. As expected, in the limit as $P_W$ goes to zero, (12) becomes (8).

III. PROBABILISTIC DATA ASSOCIATION FOR MULTIPLE TARGETS IN THE PRESENCE OF WAKES

In a multitarget environment the data association algorithm needs to handle situations where a measurement could originate from different targets. For this purpose, the JPDAF was developed, and a derivation of this standard algorithm is given in [2]. Another problem arises when these targets have wakes behind them that result in misleading wake detections. In this section we will modify the JPDAF to handle this problem.

A. Assumptions

Assume there are a known number $N_T$ of established targets at time $k-1$. For each target $t$, where $t = 1, \ldots, N_T$, the target state is estimated as $\hat{x}_{k-1}^{t}$ with associated covariance $P_{k-1|k-1}^{t}$. Then the following assumptions are made:

(a) Measurements from one target can fall in the validation gate of a neighboring target.

(b) The past information about target $t$ is summarized approximately by the Gaussian distribution

$$
p(x_{k}^{t}|Z^{k-1}) \approx N(\hat{x}_{k}^{t}; \hat{x}_{k-1}^{t}, P_{k-1}^{t})
$$

(14)

(c) At time $k$ there are $m_k$ validated measurements in the union of their validation gates, but for each target $t$ at most one measurement can be target-originated. The rest are assumed due to the wakes with pdf $p_W(\cdot)$ and a priori probability $P_W$, or from i.i.d. uniformly distributed noise/clutter with a priori probability $1 - P_W$, independent across time.

In Fig. 2 an example of the pdf’s for two targets that are starting to cross each other is shown. Here both targets have a wake behind them, and the joint wake model (the sum of each target’s single wake model) increases linearly behind the targets inside the joint validation region. The joint validation region contains all the candidate measurements, and restricts the spatially uniform distribution representing the noise/clutter. Further details about the joint wake model and the validation region are given in Appendix A.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Probability density functions for two targets with crossing trajectories. The distribution of the targets are Gaussian and overlap each other. The wakes behind the targets are modeled as linear increasing pdf’s, and the noise/clutter is uniformly spatially distributed inside the joint validation region.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Two targets with a measurement in the intersection of their validation gates are shown with corresponding validation matrix $\Omega$.}
\end{figure}

B. Joint Association Events

Define the validation matrix $\Omega$ to represent all feasible association events at time $k$ (the time index $k$ is omitted for simplicity where it does not cause confusion)

$$
\Omega = [\omega(j,t)] \\
\text{ where } j = 1, \ldots, m \text{ and } t = 0, \ldots, N_T
$$

(15)

Here, $\omega(j,t)$ is a binary element indicating if measurement $j$ lies in the validation gate of target $t$. The index $t = 0$ means that the measurement is from none of the targets and therefore it is a false measurement. An example where a measurement may originate from two targets, i.e., it lies in both targets’ validation gates, is shown with the corresponding validation matrix $\Omega$ in Fig. 3. For all these possible joint association events, conditional probabilities have to be derived.

A joint association event $\theta$ describes an unambiguous association between the measurements and the targets at time $k$

$$
\theta = \bigcap_{j=1}^{m} \theta(j,t_j)
$$

(16)

where

- $\theta(j,t)$ is the event that measurement $j$ originates from target $t$, where $j = 1, \ldots, m$ and $t = 0,1,\ldots,N_T$.
\* \( t_j \) is the index of the target to which measurement \( j \) is associated in the event under consideration. \( \theta \) can also be represented by the event matrix

\[
\Omega_\theta = [\omega_\theta(j, t)]
\]

consisting of the units in \( \Omega \) corresponding to the associations in \( \theta \)

\[
\omega_\theta(j, t) = \begin{cases} 1 & \text{if the event } \theta(j, t) \text{ is part of } \theta \\ 0 & \text{otherwise} \end{cases}
\]

Using this, a feasible association event needs to fulfill the following requirements:

1) a measurement can have only one source, i.e.

\[
\sum_{t=0}^{N_T} \omega_\theta(j, t) = 1 \quad \forall \ j
\]

2) at most one measurement can originate from a target

\[
\delta_\theta^t \triangleq \sum_{j=1}^{m} \omega_\theta(j, t) \leq 1 \quad t = 1, \ldots, N_T
\]

The binary variable \( \delta_\theta^t \) is called the target detection indicator since it indicates whether a measurement is associated to a target \( t \) or not in event \( \theta \). It is also convenient to define two more binary variables

\[
\tau_\theta(j) \triangleq \sum_{t=1}^{N_T} \omega_\theta(j, t)
\]

\[
\phi_\theta \triangleq \sum_{j=1}^{m} [1 - \tau_\theta(j)]
\]

where \( \tau_\theta(j) \) is the measurement association indicator to indicate if measurement \( j \) is associated to a target or not, and \( \phi_\theta \) is the number of false (unassociated) measurements in event \( \theta \).

C. Modified JPDAF Including a Wake Model

The joint association event probabilities are derived using Bayes’ formula

\[
P\{\theta_k|Z^k\} = P\{\theta_k|Z_k, m_k, Z^{k-1}\} = \frac{1}{c} p[Z_k|\theta_k(m_k, Z^{k-1}|P[\theta_k|Z^{k-1}, m_k} = \frac{1}{c} p[Z_k|\theta_k(m_k, Z^{k-1}|P[\theta_k|m_k} (23)
\]

where \( c \) is a normalizing constant. In the last line of the above equation the irrelevant conditioning term \( Z^{k-1} \) has been omitted. The likelihood function of the measurements in (23) is derived by assuming that the states of the targets, conditioned on the past observations, are mutually independent

\[
p[Z_k|\theta_k(m_k, Z^{k-1}] = \prod_{j=1}^{m_k} p[z_k(j)|\theta_k(j, t_j), Z^{k-1}] (24)
\]

Measurements not associated with a target are assumed either from the wakes with pdf \( p_W(z_k(j)) \) and a priori probability \( P_W \), or from uniformly distributed noise/clutter with a priori probability \( (1 - P_W) \). Defining \( V_k \) as the volume of the joint validation gate, the pdf of a measurement given its origin is

\[
p[z_k(j)|\theta_k(j, t_j), Z^{k-1}] = \begin{cases} \mathcal{N}[z_k(j); \hat{z}_k^{t_j|k-1}, S_k^{t_j}] & \text{if } \tau_{\theta_k}(j) = 1 \\ P_W \frac{p_W(z_k(j))}{P_W} + (1 - P_W) \frac{1}{V_k} & \text{if } \tau_{\theta_k}(j) = 0 \end{cases}
\]

where \( \hat{z}_k^{t_j|k-1} \) is the predicted measurement for target \( t_j \) with associated innovation covariance \( S_k^{t_j} \). The constant \( P_W \) is used for restricting \( p_W(z_k(j)) \) to the joint validation region, and has an analytical expression derived in Appendix A. Using the above equation, (24) can be written as

\[
p[Z_k|\theta_k(m_k, Z^{k-1}] = \prod_{j=1}^{m_k} \left\{ \mathcal{N}[z_k(j); \hat{z}_k^{t_j|k-1}, S_k^{t_j}] \right\}^{\tau_{\theta_k}(j)} \times \left\{ P_W \frac{p_W(z_k(j))}{P_W} + (1 - P_W) \frac{1}{V_k} \right\}^{1-\tau_{\theta_k}(j)} (26)
\]

Next, the last term in (23) will be derived. Let \( \delta_\theta \) be the vector of detection indicators corresponding to event \( \theta_k \)

\[
\delta_\theta = [\delta_\theta^1, \ldots, \delta_\theta^{N_T}] (27)
\]

The vector \( \delta_\theta \) and the number of false measurements \( \phi_\theta \) are both completely defined when \( \theta \) is given. This yields using the definition of conditional probabilities [11]

\[
P\{\theta_k|m_k\} = P\{\theta_k, \delta_\theta, \phi_\theta|m_k\} = P\{\theta_k|\delta_\theta, \phi_\theta, m_k\} P\{\delta_\theta, \phi_\theta|m_k\} (28)
\]

The first term in (28) is obtained using combinatorics:

1) In event \( \theta_k \) there are assumed \( m_k - \phi_\theta \) targets detected.

2) The number of events \( \theta_k \), where the same targets are detected, is given by the number of ways of associating \( m_k - \phi_\theta \) measurements to the detected targets from a set of \( m_k \) measurements.

By assuming each such event a priori equally likely, one has

\[
P\{\delta_\theta, \phi_\theta|m_k\} = \frac{1}{m_k} \frac{P_{\delta_\theta}}{m_k - \phi_\theta} = \frac{\phi_\theta!}{m_k!} (29)
\]

The last term in (28) is, assuming \( \delta \) and \( \phi \) independent,

\[
P\{\theta_k|m_k\} = \prod_{t=1}^{N_T} P_D^{\delta_t^k}(1 - P_D^{\delta_t^k})^{1-\delta_t^k} \mu_F(\phi) (30)
\]

where \( P_D^{\delta_t^k} \) is the detection probability of target \( t \) and \( \mu_F(\phi) \) is the prior pmf of the number of false measurements. The indicators \( \delta_t^\phi \) have been used to select the probabilities of detection and no detection events according to the event \( \theta_k \) under consideration. Combining (29) and (30) into (28) yields the prior probability of a joint association event

\[
P\{\theta_k|m_k\} = \frac{\phi_\theta!}{m_k!} \prod_{t=1}^{N_T} (P_D^{\delta_t^k}(1 - P_D^{\delta_t^k})^{1-\delta_t^k} \mu_F(\phi) (31)
\]
The pmf of the number of false measurements \( \mu_F(\phi) \) can, as in the case of the PDA, have two versions, parametric or non-parametric.

1) Parametric JPDA uses a Poisson pmf

\[
\mu_F(\phi) = e^{-\lambda V} \frac{(\lambda V)^\phi}{\phi!}
\]

which requires the spatial density \( \lambda \) of the false measurements.

2) Nonparametric JPDA uses a diffuse prior

\[
\mu_F(\phi) = \epsilon \quad \forall \phi
\]

which does not require the parameter \( \lambda \).

Using the nonparametric model and combining (31) and (26) into (23) yields the joint association event probabilities

\[
P\{\theta_k|Z^k\} = \frac{\phi_0^1}{c} \prod_{t=1}^{N_T} (P_{D}^t)^{\delta_t^k} (1 - P_{D}^t)^{1-\delta_t^k}
\times \prod_{j=1}^{m_k} \left\{ \mathcal{N} \left[ z_k(j); \hat{\nu}_k, \sigma_v^k \right] \right\}^{\tau(j)}
\times \left\{ P_W \frac{P_Z(z_k(j))}{P_{GW}} + (1 - P_W) \frac{1}{V_k} \right\}^{1-\tau(j)}
\]

where the constants \( \epsilon \) and \( m_k \) are brought into the normalization constant \( c \). For comparison, the joint association event probabilities derived in [2] for the standard JPDAF is

\[
P\{\theta_k|Z^k\} = \frac{\phi_0^1 \cdot V_k^{-\phi_0}}{c} \prod_{t=1}^{N_T} (P_{D}^t)^{\delta_t^k} (1 - P_{D}^t)^{1-\delta_t^k}
\times \prod_{j=1}^{m_k} \left\{ \mathcal{N} \left[ z_k(j); \hat{\nu}_k, \sigma_v^k \right] \right\}^{\tau(j)}
\]

where the third line in (34) is substituted with \( V_k^{-\phi_0} \). As for the modified PDAF, (34) reduces to (35) in the limit as \( P_W \) goes to zero. Finally, marginal association probabilities are obtained by summing over all the joint association events in which the marginal event of interest occurs

\[
\beta_k^j(j) \triangleq P\{\theta_k(j,t)|Z^k\} = \sum_{\theta_k} P\{\theta_k|Z^k\} \omega(j,t)
\]

\[
\beta_k^j(0) \triangleq 1 - \sum_{j=1}^{m_k} \beta_k^j(j)
\]

By using these association probabilities in (8), the state estimation equations are exactly the same as in the PDAF, (5) - (11).

IV. SIMULATION RESULTS

In this section we compare the data association methods described previously (PDAF, Modified PDAF, JPDAF and Modified JPDAF). To do this, a multitarget tracking problem of two crossing targets in the presence of wakes are simulated for varying trajectory crossing angles \( \gamma \), see Fig. 4. For both targets a two-dimensional direct discrete time nearly constant velocity model [3] is used in (1) and (2):

\[
F = \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix}
\quad
H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
Q = \begin{bmatrix}
\frac{T^4}{4} & \frac{T^3}{2} & 0 & 0 \\
\frac{T^3}{2} & \frac{T^2}{4} & 0 & 0 \\
0 & 0 & \frac{T^4}{4} & \frac{T^3}{2} \\
0 & 0 & \frac{T^3}{2} & \frac{T^2}{4}
\end{bmatrix}
\quad
R = \begin{bmatrix}
\sigma_p^2 & 0 \\
0 & \sigma_r^2
\end{bmatrix}
\]

The parameters in (38) and other simulation design parameters are given in Table I, and a standard Kalman filter is used as the tracking algorithm. Both targets are initialized with speed 0.5 m/s and a course according to the trajectory crossing angle \( \gamma \). To ensure a controlled crossing for the true trajectories, the added process noise \( Q \) is set low, but not to zero. The small amount of process noise is used to exploit situations where the targets’ positions are not totally overlapping at the crossing point, but where one target will cross in the wake from the other target. The measurements in real sonar and radar applications are obtained in polar coordinates yielding range dependent cross-range errors. By assuming a uniform measurement error inside a resolution cell, the variance is given by the squared size of the cell divided by 12. This error can be approximated as Gaussian with the same variance. In this paper, the measurement noise is assumed constant over the surveillance area, with an approximately size of a
resolution cell as 1.8 m$^2$. This size is comparable to e.g a sonar system. Target-originated measurements are generated by adding measurement noise to the true trajectories, and then clutter and wake measurements (false alarms) are added. Denote the surveillance area covering the full trajectories of the targets as $S$, the wake area as $W$, and their respective volumes $V_S$ and $V_W$. Then, the number of clutter and wake measurements are both Poisson distributed with parameters

$$\lambda_{\text{clutter}} = P_{FA}V_S$$

$$\lambda_{\text{wake}} = \frac{P_W}{1 - P_W}P_{FA}V_W$$

where $P_{FA}$ is the probability of false alarm in a unit volume. The clutter measurements are located uniformly in $S$, and wake measurements are distributed exponentially decreasing behind the target and Gaussian distributed sideways with parameters $\lambda_w$ and $\sigma_w^2$, respectively. One simulation run consists of 300 time-steps, and the true target-originated measurements are detected with probability $P_D$, independently over this period.

To reduce the computational load, the different versions of the multitarget tracking algorithms are substituted with their analogous single target tracking algorithms as long as targets are not “sharing” measurements. In other words, the standard PDAF is used instead of the JPDAF, and the modified PDAF is used instead of the modified JPDAF when the targets are apart.

Tracks are declared as lost if the position error exceeds 3 meters in front and to both sides of the true target, but are extended to 10 meters in the direction behind the target. This extension is to allow tracks with a small bias behind the true target due to the wake-originated measurements. The relatively low threshold at 3 meters is needed for small trajectory crossing angles $\gamma$ so that tracks following the wrong target are declared as lost. At the same time 3 meters are sufficient not to declare tracks as lost in cases where they are able to get back on track again. The results of lost tracks after 5000 Monte Carlo runs for trajectory crossing angles $\gamma$ between $5^\circ$ and $30^\circ$, with $1^\circ$ spacing, are shown in Fig. 5. The standard filters (PDAF and JPDAF) have serious problems, and the JPDAF shows no improvement compared to the PDAF during the crossing period. For low trajectory crossing angles, $\gamma$ below $25^\circ$, the modified JPDAF performs best. This is comparable to situations where two targets are moving almost together, and the targets are “sharing” measurements for a longer period of time. For $\gamma$ above $25^\circ$ the modified single target tracking algorithm (PDAF) is preferable due to its equally good performance and lower computational load.

In Fig. 6 the average position errors are shown for the different filters for Target 1 at trajectory crossing angle $\gamma = 15^\circ$ (similar results for Target 2). These results are based only on tracks that are not declared as lost. The standard filters are outperformed by the modified filters, which are close to the filter with perfect data association (using only the true target-originating measurements). The different modified filters perform similarly, and only during the crossing period does the multitarget tracking algorithm perform slightly better than the single target tracking filter. It is also interesting to see the RMS error in the directions parallel and perpendicular to the target’s true velocity. This is shown in Fig. 7 for $\gamma = 15^\circ$. As expected, the error is much higher in the direction parallel to the velocity because this is the direction of the wakes. False detections from the wake, if not sufficiently accounted for, will draw the estimated position behind the target and into the wake, and create a bias in the estimate. The error in the direction perpendicular to the velocity is much smaller compared to the parallel direction, but during the crossing period the estimate for one target will be drawn towards the other target, also known as track coalescence [7]. This is most problematic for the modified single target tracking algorithm because it accounts for the wake behind its own target, but has no information about the nearby target which
also has a wake behind it. Also notice that the perpendicular error before crossing is actually lower for the “non-perfect” data association filters, especially the standard filters, than the perfect data association filter. The reason for this is the high density of the wake measurements normally distributed (zero mean) in the direction perpendicular to the target’s velocity. When these measurements are taken into the probabilistic data association, the weighted sum of all candidate measurements will give a smaller error in the perpendicular direction than using only the true target-originated measurement.

V. Conclusions

An important factor in a multitarget tracking system is to correctly associate each measurement received from a detector to its origin. The JPDAF has been a solution to this problem in many implemented systems due to its effectiveness and low computational demand. In the JPDAF all false measurements are assumed due to i.i.d. uniformly spatially distributed noise or clutter. This assumption is not adequate for targets in the presence of wakes, because detections originating from the wake may result in a lost track if they are not properly accounted for. The solution presented extends this to incorporate a model of the wakes behind the targets in a multitarget environment. The purpose of this wake model is to weight wake-originated measurements lower than in a regular JPDAF to avoid the tracks following these measurements and therefore be forced to turn into the wake. To achieve this, we presented a model formed by the sum of single models each linearly increasing behind their associated targets. Simulations of two crossing targets shows that the wake model presented is a useful modification of the JPDAF, especially when the trajectory crossing angle between the targets is small. The wake model is also necessary for higher trajectory crossing angles, but in these situations it seems to be enough with a modified single target tracking filter (PDAF).

APPENDIX A

SPECIFICATION OF THE JOINT WAKE MODEL

The probability $P_{GW}$ in (25), used to restrict the density of the joint wake model $p_{W}(z_k)$ to the joint validation region, has to be calculated for each scan by integration of $p_{W}(z_k)$ inside the region. The joint wake model is the sum of all single wake models $p_{W}^t(z_k)$ behind each target $t$ under consideration. Hence, the $P_{GW}^t$ has to be calculated for each target and then summed up

$$P_{GW} = \sum_{t=1}^{N_T} P_{GW}^t \quad \text{and} \quad p_{W}(z_k) = \frac{1}{N_T} \sum_{t=1}^{N_T} p_{W}^t(z_k)$$  \quad (41)

In this section an analytical expression for the integration of a single wake model inside the joint validation region will be derived. Let $\vec{z}$ be the position of the predicted measurement of target $t$ with velocity $v$. The wake model is linear increasing with length $L$ behind the predicted position of the target, i.e., the direction opposite to $v$, and uniform with width $W$ in the direction perpendicular to the target’s velocity $v$.

$$p_{W}^t(z_k) = p_l(l)p_w(w) = \frac{2l}{L^2W}$$  \quad (42)

where

$$p_l(l) = \frac{2l}{L^2} \quad 0 \leq l \leq L \quad p_w(w) = \frac{1}{W} \quad 0 \leq w \leq \frac{W}{2}$$  \quad (43)

and $l$ and $w$ are the respective distances behind and sideways (relative to $v$) to the target. The joint validation region containing all candidate measurements in the multitarget environment is a circle with radius $r$ and center $c$. Assume a Cartesian coordinate system with origin at position $c$ and $y$-axis parallel to $v$ but in the opposite direction, see Fig. 8. Define the two front corners of the wake model with elements $\alpha$ and $\beta$ for
the $x$-axis, and $\rho$ for the $y$-axis

$$\rho = (c - z)^T v / |v|$$

$$\alpha = \sqrt{c - z^2 - \rho^2 - w/2}$$

$$\beta = \sqrt{c - z^2 - \rho^2 + w/2}$$ (44)

The integration depends on if the front corners $[\alpha \rho]^T$ and $[\beta \rho]^T$ are inside or outside the joint validation region (circle), and will be broken into one, two or three parts. To do this, define three binary variables $\delta_{\rho}$, $\delta_{\alpha}$ and $\delta_{\beta}$ as follows:

$$\delta_{\rho} = \begin{cases} 1 & \text{if } \rho < 0 \\ 0 & \text{otherwise} \end{cases}$$ (45)

$$\delta_{\alpha} = \begin{cases} 1 & \text{if } \alpha^2 + \rho^2 > r \\ 0 & \text{otherwise} \end{cases}$$ (46)

$$\delta_{\beta} = \begin{cases} 1 & \text{if } \beta^2 + \rho^2 > r \\ 0 & \text{otherwise} \end{cases}$$ (47)

Then the integral can be written as

$$P_{GW}' = \frac{2}{L^2 W} \left\{ \delta_{\rho} \delta_{\alpha} \int_{\max(\alpha, -r)}^{\sqrt{r^2 - \rho^2}} \int_{-\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} (y - \rho) dy dx ight.$$

$$+ \int_{\alpha(1 - \delta_{\alpha}) - \rho \sqrt{r^2 - \rho^2}}^{\alpha(1 - \delta_{\alpha}) + \rho \sqrt{r^2 - \rho^2}} \int_{-\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} (y - \rho) dy dx$$

$$+ \left. \delta_{\rho} \delta_{\beta} \int_{\min(\beta, r)}^{\sqrt{r^2 - \rho^2}} \int_{-\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} (y - \rho) dy dx \right\}$$ (48)

For simplicity we substitute the limits of integration along the $x$-axis as follows:

$$a = \max(\alpha, -r)$$

$$b = -\sqrt{r^2 - \rho^2}$$

$$c = \alpha(1 - \delta_{\alpha}) - \alpha \sqrt{r^2 - \rho^2}$$

$$d = \beta(1 - \delta_{\beta}) + \beta \sqrt{r^2 - \rho^2}$$

$$e = \sqrt{r^2 - \rho^2}$$

$$f = \min(\beta, r)$$ (49)

which yields

$$P_{GW}' = \frac{2}{L^2 W} \left\{ \delta_{\rho} \delta_{\alpha} \int_{a}^{b} \int_{-\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} (y - \rho) dy dx ight.$$}

$$+ \int_{c}^{d} \int_{\rho}^{\sqrt{r^2 - x^2}} (y - \rho) dy dx$$

$$+ \left. \delta_{\rho} \delta_{\beta} \int_{e}^{f} \int_{\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} (y - \rho) dy dx \right\}$$ (50)

$$= \frac{1}{L^2 W} \left\{ \rho \sqrt{c - z^2} \left( \arcsin \frac{c}{r} - \arcsin \frac{d}{r} \right) + c^3 - d^3 ight.$$}

$$+ 2 \rho \delta_{\rho} \delta_{\alpha} \left( a \sqrt{r^2 - a^2} - b \sqrt{r^2 - b^2} \right) - c(r^2 + r^2)$$

$$+ 2 \rho \delta_{\beta} \delta_{\rho} \left( e \sqrt{r^2 - e^2} - f \sqrt{r^2 - f^2} \right) + d(r^2 + r^2)$$

$$+ 2 \rho \delta_{\beta} \delta_{\rho} \bar{r}^2 \left( \arcsin \frac{a}{r} - \arcsin \frac{b}{r} \right) + \rho \sqrt{r^2 - c^2}$$

$$+ 2 \rho \delta_{\beta} \delta_{\rho} \bar{r}^2 \left( \arcsin \frac{e}{r} - \arcsin \frac{f}{r} \right) - \rho d \sqrt{r^2 - d^2} \right\}$$

**REFERENCES**


