Abstract - In this paper, the problem of target classification from multiple High Range Resolution (HRR) radars data is studied. The use of multi-sensor angle-diverse data aims at shortening of the required time before a decision is made, as compared to using single-sensor data. In order to avoid the high-dimensional HRR profile databases, involved in the classic Automatic Target Recognition (ATR) approaches, the classification is based on 2D spatial target profile matching. Stochastic target modeling is further applied, as a more robust way to deal with more realistic target scattering behaviors at variant aspect angles. Raw video measurements are used, with the objective to circumvent problems due to missed detections and false alarms in the thresholded profiles. This paper proposes a new method, where the above elements are combined. Some first encouraging simulation results are given.

Keywords: high resolution, radar network, 2D spatial target model, stochastic target model, MAP classification, particle filtering.

1 Introduction

The problem of moving extended radar target classification, investigated throughout this paper, can be formulated as a Dynamic Estimation Classification Problem. Here, multiple HRR radars data are used and a Maximum A posteriori Probability (MAP) solution is proposed, based on 2D spatial target model matching. However there are various approaches to joint Automatic Target Recognition (ATR) and Tracking appearing in the literature so far. Some of the most representative are listed below.

In [1], matching sequences of measured HRR range profiles to library range profiles is proposed. Though, the high variability of the range profiles for small changes in target orientation leads to the need for detailed target data libraries and very accurate knowledge of the target aspect angle. Efficient applicability of such systems is consequently questioned.

In [2], [3] alternatively, the Hidden Markov Model (HMM) formalism has been introduced as a way to fuse heterogeneous multi-aspect wideband scattering data. By exploiting the statistical dependence of a sequence of HRR profiles -with the angular step in consecutive measurements known- target feature statistics are extracted. Multi-aspect target classification relies on target modeling and not on detailed HRR profile libraries. However, it is based on modeling of the range profile changes due to the target structure, rather than on a 2D spatial target reflection model.

In [4], [5] a 2D target spatial model is used for joint target classification and tracking. The interesting concept of global and local motion is initiated there. However, this work considers exclusively the 2D rigid body target model and target structural information is assumed to be hidden in the peaks of multi-aspect range profiles. The validity of these main assumptions is doubtful yet, especially when wide-angle separated data are used. Finally, while working with a single radar, the acquirement of multi-aspect data highly depends on the target kinematics.

At this point, the main features of the proposed approach are highlighted:

- Use of multiple HRR radars data
- Model-based classification, with a target model of the below main characteristics:
  - 2D spatial
  - Stochastic
- Use of raw radar video measurements

The multi-radar multi-aspect data used are treated as wide-angle separated samples of a 2D spatial profile of the target. A stochastic approach is adopted and the classification is based on matching diverse-angle 1D reflected power distributions -raw HRR video data- to a 2D stochastic target reflection model. Once data from multiple radars at multiple time steps are jointly processed, the moving target -variant in time- aspect angle needs to be known. This is estimated in a point target tracking procedure using a particle filter.

2 System Model

2.1 HRR Radar Network

The system considered includes several identical spatially separated monostatic HRR radars, which exchange the information of their measurements, working in a network.

Time-evolving radar scenes are assumed and then the moving targets scatter power back to the radars at time-varying angles.
As depicted in Fig. 1 below, the radar network is considered asynchronous in the general case. This means that the multi-radar measurements may be available at different moments in time, under the condition that this time is known.

At that case, in order to constructively combine the fused multi-radar data in a unique multi-aspect target classification algorithm, the target needs to be tracked. Then the target aspect angles $\phi_m$, $m=1:M$ are estimated at each time step for the M radars and multi-radar 1D HRR data that is available at that moment can be related to a 2D spatial target structure.

### 2.2 HRR Measurement Data Model

The target is illuminated with a wideband waveform by the multiple HRR radars. In each target echo, multiple ($N_f$) samples of the target frequency signature are thus present. In the matrix $f$ below, the $N_f$ frequencies used are given:

$$f = \begin{bmatrix} f_1 & f_2 & \ldots & f_{N_f} \end{bmatrix}$$

with:
- $f_c$: the carrier frequency and $B = N_f \cdot \Delta f$ : the bandwidth

of the wideband signal the HRR radars work with.

The multi-radar data used are raw video data, represented with the following formula for the $m$ radar at the $k$ time step:

$$z_{m,k} = A_{m,k} \cdot q + n$$

with $A_{m,k}$ the data matrix for the $m$ radar at the $k$ time step:

$$A_{m,k} = \begin{bmatrix} e^{-j2\phi_{m,k}/c} & \ldots & e^{-j2\phi_{m,M}/c} \\ \vdots & \ddots & \vdots \\ e^{-j2\phi_{k}/c} & \ldots & e^{-j2\phi_{k,N}/c} \end{bmatrix}_{N_f \times N}$$

In the above formula:

- $f_i$, $i=1 : N_f$ are the $N_f$ frequencies used,
- $r_{m,k}^n = r_{0m,k} + (x_n \cdot \cos \phi_{m,k} + y_n \cdot \sin \phi_{m,k})$, $n=1 : N$ are the range distances from the $m$ radar to the $N$ target scattering centers at the $k$ time step, with:
  - $r_{0m,k}$: the range distance from the radar to the target centroid,
  - $\phi_{m,k}$: the radar aspect angle of the target
- $(x_n, y_n)$, $n=1 : N$: the 2D positions of the $N$ target scattering centers. The x-axis of the reference coordinate system corresponds to the target orientation and its center corresponds to the target centroid.

- $c$: is the speed of light

In (3), $n$ is used to model the measurement noise as additive complex Gaussian:

$$n \sim \mathcal{C}(0, \sigma_n^2 I_{N_f})$$

while $q$ is used to model the target scatterers’ radar echoes.

The simplest and widely used approach for modeling of extended radar targets is the 2D rigid body model. This model derives from the Geometrical Theory of Diffraction (GTD) and represents the extended radar target with a number of electrically isolated scattering centers [6]. In such a case and if scatterers of equal normalized strength are further assumed:

$$q = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}^T_{N_{c1}}$$

However, when the target view angle changes -at wide angular steps for the multi-radar measurements- the constellation of target scattering centers seen by the different radars also drastically changes due to various scattering mechanisms’ effects. The rigid body model may then prove to be inappropriate.

In order to better take into account such effects, one extension of this deterministic model to a stochastic target model is considered [6], [7]. For the stochastic model used, each scattering center is not any particular geometric point on the extended target, but rather represents a combination...
of scattering elements which return a complex Gaussian signal [7]

\[ q \sim CN(0, \sigma^2 I_N) \]  

with \( \sigma = 1 \) chosen here.

### 2.3 Library data

As already mentioned, in the proposed algorithm the classification decision is based on 2D spatial target model matching. Thus, each possible target class \( p_c \) corresponds to one such \( N \)-scatterers spatial model:

\[ p_c = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \\ y_1 & y_2 & \cdots & y_N \end{bmatrix} \]  

which is assumed known and stored in a library.

The model for the correct class the target belongs to parameterizes the measurement \( z_{m,k} \), by getting involved in the calculation of the ranges \( r_{m,k}^n \) in (5).

### 2.4 Target aspect angle estimation

From (5), it is evident that in order to match the 1D HRR measurements \( z_{m,k} \) to a 2D target profile \( p_c \), both the range distance from the radar to the target centroid \( r_{0m,k} \) and the radar aspect angle of the target \( \phi_{m,k} \) need to be known.

For all the further presented simulation results, \( r_{0m,k} \) is assumed accurately known in the algorithm, while \( \phi_{m,k} \) is estimated using a tracking filter.

The angle \( \phi_{m,k} \) can be calculated as a function of the radar 2D position \( (x_m, y_m) \), together with the target 2D center position \( (x_k, y_k) \) and velocity \( (v_{x_k}, v_{y_k}) \), using the formula:

\[ \phi_{m,k} = \text{azimuth} - \arctan(\frac{v}{y}) \]
\[ = \arctan\left(\frac{y - y_m}{x - x_m}\right) - \arctan\left(\frac{y_{k} - y_{m}}{x_{k} - x_{m}}\right) \]  

if the target orientation coincides with the target course.

While the positions of the M radars \( (x_m, y_m), m = 1:M \) in a common coordinate system are assumed known, the target kinematics state vector:

\[ s_k = \begin{bmatrix} x_k & v_{x_k} & y_k & v_{y_k} \end{bmatrix}^T \]  

is recursively estimated using a particle filter. The filter follows one update and one prediction step alternatively, making use of the following measurement and system dynamics model correspondingly.

#### Measurement model

Low resolution range, radial speed and bearing measurements from one of the radars are used, such that the target is considered to be a point target for the tracking procedure.

The measurement vector at the \( k \) time step is then expressed as:

\[ m_k = \begin{bmatrix} r_{m,k} \\ d_k \\ b_k \end{bmatrix} = h(s_k) + n_k \]  

where:

\[ h(s_k) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ x_k \cdot \dot{x}_k + y_k \cdot \dot{y}_k \\ \arctan\left(\frac{y_k}{x_k}\right) \end{bmatrix} \]  

with the radar providing the measurements, positioned at \( (x_1, y_1) = (0,0) \) and

\[ n_k \sim CN(0, R) \]

is additive white Gaussian measurement noise with covariance matrix:

\[ R = \text{diag}\{\sigma_r^2, \sigma_d^2, \sigma_b^2\} \]

In (15) \( \sigma_r, \sigma_d \) and \( \sigma_b \) are the standard deviation values for the radar range, radial speed and bearing measurement respectively.

#### System Dynamics model

Linear constant velocity target motion is assumed. Then:

\[ s_{k+1} = f(s_k) + w_k \]

where:

\[ f(s_k) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot s_k \]  

is the system dynamics matrix with \( T \) the interval between consecutive time steps and

\[ w_k \sim N(0, W) \]
is the system process noise, which is white Gaussian with covariance matrix:

$$W = \begin{bmatrix} T^3 & T^2 & 0 & 0 \\ \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & T^3 & \frac{T^2}{2} \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix} \sigma_1^2$$  \hspace{1cm} (19)

In (19), $\sigma_1$ is used to model the amount of deviation from the perfect linear motion model.

3 Dynamic Estimation Classification Problem

Throughout this section, the mathematical formulation of the studied dynamic estimation classification problem is given.

3.1 Maximum A posteriori Probability (MAP) Classification

For the 2D target profile matching and the decision on which class the target is most probable to be a member of the Maximum A posteriori Probability (MAP) solution is used.

Applying a recursive form of the Bayes rule and given all the available measurements till each time step $t_k$:

$$Z_k = [z_1 \ z_2 \ \ldots \ z_k]$$  \hspace{1cm} (20)

the a posteriori probability for the target to belong to class $p_c$ is:

$$p(p_c / Z_k) = \frac{p(Z_k / p_c) \cdot p(p_c / Z_{k-1})}{p(z_k / Z_{k-1})}$$  \hspace{1cm} (21)

$$= \frac{p(z_k / p_c, Z_{k-1}) \cdot p(p_c / Z_{k-1})}{p(z_k / Z_{k-1})}$$

with:

$$p(z_k / Z_{k-1}) = \sum_{p_c=1}^{N_c} p(z_k / p_c, Z_{k-1}) \cdot p(p_c / Z_{k-1})$$  \hspace{1cm} (22)

The a posteriori probabilities for the different target classes are sequentially estimated for $k = 1: N_k$, where $N_k$ is the total number of time steps and $N_c$ is the number of possible target classes.

Starting with equal prior probability for all the target classes:

$$p(p_c / Z_0) = \frac{1}{N_c} \ \forall p_c$$  \hspace{1cm} (23)

what is required in order to get an estimate of $p(p_c / Z_k)$ at each time step using (21) is an estimate of the likelihood function $p(z_k / p_c, Z_{k-1})$ at the specific time step.

However, the likelihood function is not explicitly known in this form, since the measurements directly depend on the unknown target kinematics state vector $s_k$, given in (11). Instead, $p(z_k / p_c, Z_{k-1})$ is only implicitly known via:

$$p(z_k / p_c, Z_{k-1}) = \int p(z_k / s_k, p_c) \cdot p(s_k / Z_{k-1}) ds_k$$  \hspace{1cm} (24)

where the target kinematics predictive density $p(s_k / Z_{k-1})$ is assumed to be independent of the target class $p_c$.

In the next paragraph, the calculation of the likelihood function $p(z_k / s_k, p_c)$ is given, while the estimation of $p(s_k / Z_{k-1})$ is developed in paragraph 3.3.

3.2 Likelihood function Calculation

Considering the HRR measurements of the $M$ radars in the network independent and assuming further that all the radars have a measurement available at each time step $k$:

$$p(z_k / s_k, p_c) = \prod_{m=1}^{M} p(z_{m,k} / s_k, p_c)$$  \hspace{1cm} (25)

Adopting the modeling in paragraph 2.2 and under the assumption of deterministic (rigid body) target model, $z_{m,k}$ is sampled from a multivariate Complex Gaussian distribution of mean:

$$\mathbf{z}_{m,k} = \left[ z_{m,k}^1 \ z_{m,k}^2 \ \ldots \ z_{m,k}^{N_f} \right]^T,$$

$$\mathbf{z}_{m,k}^i = \sum_{n=1}^{N_e} e^{-i \pi n} \frac{z_{m,k}^n}{c}, \ i = 1: N_f$$

and covariance:

$$Q = 2\sigma_n^2 \cdot I_{N_f}$$  \hspace{1cm} (27)

Then:

$$p(z_{m,k} / s_k, p_c) = \prod_{i=1}^{N_f} \left\{ \frac{1}{2\pi\sigma_n^2} \exp \left( -\frac{z_{m,k}^i - \mathbf{z}_{m,k}^i}{2\sigma_n^2} \right) \right\}$$  \hspace{1cm} (28)

Following again the modeling of paragraph 2.2 but under the assumption of stochastic target model, $z_{m,k}$ is distributed according to a multivariate complex Gaussian distribution of zero mean and covariance:

$$Q = E \left( \mathbf{z}_{m,k} \cdot \mathbf{z}_{m,k}^H \right)$$

$$= \left( A_{m,k} \cdot E(q \cdot q^H) \cdot A_{m,k}^H + E(n \cdot n^H) \right)$$

$$= 2\sigma_n^2 \cdot A_{m,k} \cdot A_{m,k}^H + 2\sigma_n^2 \cdot I_{N_f}$$  \hspace{1cm} (29)

In this case, the likelihood function is given by:
The target kinematics predictive density $p(s_k / Z_{k-1})$ is recursively estimated using a particle filter [8]. By implementing a recursive Bayesian filter with a particle filter, what is acquired at each time step is a discrete weighted approximation to the true posterior distribution $p(s_k / Z_k)$:

$$p(s_k / Z_k) = \sum_{j=1}^{N_p} w_k^j \cdot \delta(s_k - s_k^j)$$

(31)

where $\{s_k^j, j = 1, \ldots, N_p\}$ is a set of $N_p$ support points with associated weights $\{w_k^j, j = 1, \ldots, N_p\}$. By selecting a large number of particles $N_p$, the approximation in Eq. (31) becomes an equivalent representation of the true $p(s_k / Z_k)$. The prediction step of the particle filter produces an approximation of the predictive density $p(s_k / Z_{k-1})$ and the likelihood function $p(z_k / p_c, Z_{k-1})$ in (24) can then be calculated as:

$$p(z_k / p_c, Z_{k-1}) = \sum_{j=1}^{N_p} p(z_k / s_k^j, p_c) \cdot w_k^j$$

(32)

### 4 Simulation Setup

In this section, the simulated scenario is described and the selected values for the key parameters are given.

#### 4.1 Geometry

A network of four HRR radars positioned at:

- $HRR_1 : (0, 0)$
- $HRR_2 : (-707, 293)$
- $HRR_3 : (-10^3, 10^3)$
- $HRR_4 : (707, 293)$ [m]

in a common coordinate system is considered.

The single target present in the radar scene is moving linearly -along the x-axis of the coordinate system- with constant velocity $50 \text{m/sec}$. $N_k = 10$ time steps are used, with $T = 1 \text{sec}$ time interval between the consecutive time steps. The target is moving from the initial position $(x_0, y_0) = (0,10^3) [m]$ to the final position $(x_{10}, y_{10}) = (500,10^3) [m]$. HRR$_1$ is the tracking radar.

#### 4.2 Multi-aspect HRR data

The wideband waveform that the HRR radars work with has a bandwidth $B = N_f \cdot \Delta f = 320 \text{MHz}$, corresponding to a range resolution: $\Delta R = 0.47 \text{m}$. Complex Gaussian noise with zero mean and standard deviation $\sigma_n = 0.1$ is added to data signal of normalized power.

#### 4.3 Library data for the target classes

Two target classes are assumed. The template structures $p_c$ stored in the system library consist for both of them of $N = 8$ scatterers. The real target is of Class 1.

#### 4.4 Particle filter specifications

Process noise standard deviation: $\sigma_i = 5 \text{m/s}^{3/2}$
Measurement noise standard deviation:

$\sigma_{ni} = 5 \text{m} , \sigma_d = 5 \text{m/s} \text{ and } \sigma_p = 0.005 \text{rad}$

Number of particles: $N_p = 2 \cdot 10^4$

### 5 Simulation Results

All the results presented in this section are average (with 10 Monte Carlo runs) classification curves. In all the figures below, the blue curves correspond to the real class the target belongs to (Class 1), while the red curves correspond to the competitor target Class 2.
With the stochastic target model being the main option, all simulations have been performed under the assumption of deterministic target model as well. The objective was to compare the classification algorithm performance for the two distinct target model cases.

5.1 Added-value of multi-radar HRR data fusion

First, simulation results when the deterministic (rigid body) target model is used are presented below. In Fig.4, the classification result when data only from HRR\(_1\) are used is depicted, while in Fig.5 the result with use of data from all four radars is given.

![Figure 4: Data from HRR\(_1\) only are used and the deterministic target model is applied](image1)

![Figure 5: Data from all radars are used and the deterministic target model is applied](image2)

The upgrade of the system performance, in terms of quicker convergence to the correct decision, with use of multiple radars’ data is obvious. In Fig. 6 and Fig. 7 below, the corresponding results for the case in which the stochastic target model is applied instead are given.

![Figure 6: Data from HRR\(_1\) only are used and the stochastic target model is applied](image3)

![Figure 7: Data from all radars are used and the stochastic target model is applied](image4)

Some unexpected additional performance upgrade, with use of the stochastic model is observed. This has not been analyzed in depth so far, however we expect that it can be ascribed to the randomness in the reflection model. For each single radar, the discrimination capability between the two target classes highly depends on the specific scatterers constellations for the two classes, with respect to the radar view angle. Concluding, the important gain from joint processing of multi-radar data is in any case a more stable system performance. This is due to averaging over various discrimination capabilities at widely separated view angles.

5.2 Stochastic Model Robustness

For the results presented in this paragraph, a function which simulates shadowing of some of the scatterers at each view angle has been applied to the data generation.
In Fig. 9 and 10 below, average classification curves with use of data from all radars are given for the case that the deterministic or the stochastic target model is used respectively.

In either case, the shadowing effect is not explicitly taken into account by the estimator. As illustrated, this leads to failure of the estimator that works under the deterministic target model assumption. In the case of stochastic target modeling, the system performance is only slightly degraded instead.

The different impact of this real scattering mechanisms mis-modeling on the two estimators performance is expected. While the deterministic model assumes constant non-zero scatterers strength, the stochastic model takes into account random decrease of the scatterers reflected power to zero.

This is of course just one example of radar target variant scattering behavior at variant aspect angles. The objective at this point was to highlight the robustness of the stochastic modeling approach to such -probably unpredicted- effects, when the deterministic model is prone to failure.

6 Conclusions

In this paper, a new algorithm has been presented, where raw radar video data from a HRR radar network are jointly processed, with the objective of extended target classification in a dynamic system. The main novel concept is the relation of the multi-radar measurements to a 2D RCS distribution of the target. This distribution may physically be the result of various scattering mechanisms and is mathematically formulated as a stochastic process which is modulated by a deterministic 2D spatial target template, stored in a library.

The above simulations are indicative of the added value of the angular diversity in multi-radar HRR data. Additionally, difference in the system performance with use of stochastic, as opposed to deterministic (rigid body), target model has been found out.

Finally, while in this work the classification result comes from 2D target library template (\(p_c\)) matching, considered future extension is the use of the proposed stochastic approach for sequential retrieval of the 2D target stochastic profile itself, from limited angular samples.

Acknowledgement

This project has received research funding from the Early Stage Training action in the context of the European Community’s Sixth Framework Programme. The paper reflects the authors’ view and the European Community is not liable for any use that may be made of the information contained herein.

References


