Multitarget Tracking Algorithm Based on Finite Mixture Models and Equivalent Measurement

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Abstract— In this paper the multitarget tracking (MTT) under a cluttered environment is considered. The proposed approach contains two steps: The first step is based on clustering algorithm of finite mixture models (FMM). The second step first obtain equivalent measurement (EQM) and then the EQM is used to estimate state of target. In fact, the first step is the parametric estimation of the FMM and the second step is the state estimation of the target. Compared with the traditional algorithm, the proposed approach has several characteristics. First, it dose not use validation gate. Second, it can deal with the uncertain number of targets, especially when the target number is large.

Keywords: Clustering algorithm, FMM likelihood, mixture models, EM Algorithm, multitarget tracking, uncertain target number, validation gate.

I. INTRODUCTION

Under a cluttered environment, traditional multitarget tracking (MTT) approaches have two classes, i.e., target-oriented and measurement-oriented. The prior includes the probabilistic data association (PDA) [1] and joint probabilistic data association (JPDA) [2], [3]. The latter includes Reid’s multiple hypothesis tracking (MHT) [4]. In the environment, the measurement-origin is uncertain. PDA and JPDA consider all measurements that lie within validation gate with different probability. The probability denotes the degree of association between measurements and targets. When target number is large, the computation of feasible event would become intractable. The MHT algorithm deals with all the measurements using different hypothesis. The main challenge is the management of all these hypothesis set.

Various studies have been given to the MTT that do not need the data association. Two popular approaches are symmetric measurement equations [5] and random sets theory [6]-[9]. Especially random finite set, many important results have been proposed [8], [9]. All these results are based on the PHD filter.

In this paper a measurement-oriented approach is proposed. The basic ideal is based on the clustering algorithm of finite mixture models (FMM). In other words, the correct (target measurement) and incorrect measurement (clutter measurement) have different distribution function. The first step is the parametric estimation of these distributions. The second step is the state estimation for the multitarget. The FMM has been studied in many areas for a long time, including biology, image, pattern recognize, AI, and so on. Its main objective is to solve the missing data problem. According to [11], the observation data, which was called the missing data, lose the connection information between the data and its origination (the distribution function). In fact, the measurement-origin uncertainty also belongs to the missing data. The PDAF (or JPDAF) is a class of the FMM with unknown weight, while in this study all the parameters of the FMM are unknown. Thus we research the MTT under the FMM paradigm.

The aim of the FMM is to estimate the parameters of the distribution function. There are two types of algorithms to solve the problem. The first is stochastic approach such as Markov Chain Monte Carlo (MCMC) algorithm [10]. The second is deterministic approach such as expectation-maximization (EM) algorithm [11]-[13]. In this paper we adopt the EM algorithm for it is relatively simple and quick speed compared with stochastic algorithm, especially those with uncomplicated distribution. Further, the number of the observation data which belong to certain target measurement distribution density can be derived by the missing data, and the number can be used to estimate the target number.

To derive the update measurements, the conventional approach uses the validation gate. Then the measurements that lie in the gate are used to update the target state. The validation gate may increase the computational efficiency. In contrast, it may require more computational power when the target number is large, and the validation gate may eliminate the proper measurements. In this paper the validation gate will not be used, and equivalent measurement (EQM) of the target is proposed to estimate the target state. In PDAF (JPDA) the equivalent measurements is the weighted sum of association probability of measurements that lie within the validation gate, while the proposed EQM use all the measurements and the association probability is substituted by the missing data. Further, the single target FMM algorithm would be extended to the multitarget case.

The multitarget system is described by the following dynamic and measurement equations:

\[ x_{k+1}^j = F_k^j x_k^j + G_k^j v_k^j \]  \hspace{1cm} (1)
\[ z_{k+1}^j = H_k^j x_{k+1}^j + w_k^j \]  \hspace{1cm} (2)

where superscript \( j \) denotes the \( j \)th target. \( v_k^j, w_k^j \) are Gaussian
with mean 0, covariance matrix $Q^j$ and $R^j$, respectively. Note that in this paper the maneuver is not considered.

The paper is organized as follows. Section II is about single target likelihood based on the FMM. Section III is the proposed EQM and state estimation based on the FMM framework. Section IV is algorithm implementation and a single target simulation is given. Section V is the proposed MTT algorithm, which is the extension of the proposed single target tracking algorithm. A two-target experiment is given in this section. Section VI is our conclusion.

II. SINGLE TARGET LIKELIHOOD BASED ON FMM

First, the single target is considered, and further the multi-target case will be extended to in section V.

A. Basic Assumptions

Our basic assumptions are the same as PDA (JPDA), i.e. A.1: No more than one measurement originates from each target.
A.2: Correct measurement follows Gaussian and clutter measurements follows uniform distribution.
A.3: False measurements are modeled as independent clutter points drawn from a Poisson distribution with parameter $\lambda$.
A.4: Detection of a target is an independent event at each sample time with probability $P_D$.

Remark 1: In our research, the assumptions A.2, A.3 can be relaxed, although the Gaussian and uniform distribution function are used, it is not limited to.

B. Finite Mixture Models

Let $Z \in \mathbb{R}^d$ be observation vector. For the realization data $z_k = \{z_k^i\}_{i=1}^{n_k}$ at time $k$, where $z_k^i \in \mathbb{R}^d$. Assume that these data follows a m-component mixture distribution, then the m-component FMM can be expressed as follows

$$f(z_k^i | \theta_k) = \sum_{j=1}^{m_k} \pi_k^j f_j^k(z_k^i | \theta_k^j) \quad i = 1, ..., n_k$$

where $\theta_k = \{\theta_k^1, ..., \theta_k^{m_k}\}$, $\theta_k^j$ is the mixture component parameter. $m_k$ is the number of the components. $\pi_k^j$ is mixing weight. The missing data $e_k^j$ is a $m_k$ dimensional vector that indicates to which component $z_k^i$ belongs, so the complete data is $(z_k^i, e_k^j)$. The mixing weight is defined as follows

$$\pi_k^j = P(e_k^j = 1 | \theta_k^j) \quad i = 1, ..., n_k, j = 1, ..., m_k$$

where $e_k^j = 1$ denotes that the $i$th observation data belongs to the $j$th component and meets with $e_k^j \in \{0, 1\}$ and $\sum_{j=1}^{m_k} e_k^j = 1$. It is missing part of the observation, and the missing part can be estimated by the following Bayesian equation

$$E(e_k^{ij} | z_k) = p(e_k^{ij} | z_k) = p(z_k | e_k^{ij}) p(e_k^{ij}) / p(z_k)$$

If the correct measurements follow Gaussian with different parameters $\theta_k^j = (\mu_k^j, \Sigma_k^j)$, then the FMM can be rewritten as follows

$$f(z_k^i | \theta) = \sum_{j=1}^{m_k} \pi_k^j \mathcal{N}(z_k^i | \mu_k^j, \Sigma_k^j)$$

$\{z_k^i\}_{i=1}^{n_k}$ are assumed to be independent, thus the likelihood $f(z_k | \theta_k)$ is

$$p(z_k | \theta_k) = \prod_{i=1}^{n_k} \sum_{j=1}^{m_k} \pi_k^j \mathcal{N}(z_k^i | \mu_k^j, \Sigma_k^j)$$

C. FMM Likelihood of Single target

Fig. 1 shows that the correct measurements and the clutter measurements follow Gaussian and uniform distribution, respectively. These measurements distribution can be modeled by the FMM. According to the assumptions A.1-A.4 and the FMM, the corresponding FMM is:

$$f_k(z_k^i | \theta_k) = \pi_k^0 c_k(z_k^i) + \pi_k^1 g_k(z_k^i | \theta_k)$$

where $c_k(\cdot), g_k(\cdot)$ are the clutter (incorrect) and correct measurement distribution function, respectively. $\pi_k^0 = p(e_k^{10} = 1 | \theta_k)$ is the weight of the incorrect measurement distribution and $\pi_k^1 = p(e_k^{10} = 1 | \theta_k)$ is the weight of the correct measurement.

D. EM Algorithm for FMM

The EM algorithm includes two steps: E step and M step. In general, the E step is to compute the following Q-function:

$$Q(\theta_k) = \log p(z_k | \hat{\theta}_k) = \sum_{i=1}^{n_k} \log \left( \sum_{j=1}^{m_k} \pi_j^* \mathcal{N}(z_k^i | \hat{\mu}_k^j, \hat{\Sigma}_k^j) \right)$$

The M-step is to maximum the Q-function

$$\theta_k^{(t+1)} = \arg \max_{\theta_k^{(t)}} Q(\theta_k^{(t)})$$

where $t$ denotes the iterative step. The algorithm is an iterative procedure until the parameter $\{\theta_k^{(t)}\}$ is stable.

Remark 2: EM have two drawbacks, i.e., initialization dependence and local maxima. Many works have been proposed to deal with these two problems. The first is the stochastic EM (SEM) method [12]. The second is the deterministic annealing EM (DAEM) algorithm [13]. As for the first drawback, in fact,
if the targets are separate and the initial point is around the true target position, then under the Gaussian and single target condition, it could converge the true point. On the other hand, if the initial point is too far from the true position, it usually denotes that the target has been lost. Under the multitarget condition, it may converge to the local maximum point, which denotes that the tracker may catch the wrong target. As for the second drawback, because of the target separate and the initial point, so it will converge to the global maxima with a large probability.

Remark 3: The EM algorithm would become complex when the targets coalesce. If the initial positions of the targets are too close, then it may converge to the different local maximum points. Thus the target may be lost. This problem also occurs in JPDAF, MHT algorithm [14]. In this paper we will not consider this.

E. Equivalent Measurement

Although the likelihood can be derived, still there are some other problems. First, in the PDA algorithm the likelihood is measurement given state \( x_k \), while in the FMM it is the measurement given parameters, in other words, the measurement mean and covariance matrix. Second, in the PDA algorithm, the validation gate is used to to get the correct measurement, but for the FMM the validation gate dose not exist. For the first problem, when the dynamics and the measurement function are all linear Gaussian, \( g_k(z_k|x_k) \) is equal to \( f_k(z_k|\mu_k, \Sigma_k) \), because \( x_k \) and \( \mu_k, \Sigma_k \) contain the same information. As mentioned above, if the correct measurement can be derived, then validation gate can be ignored completely. It shows that we need certain measurement that contains the observational information of the target. To solve this problem we propose the equivalent measurement (EQM) for the target.

Definitions 1: The EQM is defined as follows

\[
\mu_k^i = \mathbb{E}(z_k^i|\hat{e}_k^i, i = 1, ..., n_k)
\]  

where \( \mu_k^i \) is the measurement mean of the jth target given jth missing variable \( \hat{e}_k^i \). Under single target case, \( j = 0 \) denotes clutter distribution and \( j = 1 \) denotes target distribution.

The EQM differs from the true measurement in that it is the fusion result of all these measurements, but not the true measurements. We will use it to estimate the state of the target. Here we need not the validation gate. We deal with all the measurements to derive the EQM. Although we need to consider all the measurements, it avoid the validation gate, cluster and efficient measurements. In contrast, the proposed algorithm may be more simple and cost less time. In particular the target number is large (we have simulated 10-targets motion, whether in computational power, time cost, tracking precision, or tracking lost rate. The proposed algorithm is better than the conventional algorithm. For space limitation, the experiment is omitted).

Besides, the convention data association algorithms use less distribution characteristic of all the measurements, whether for target or clutter. But under the FMM framework it consider this naturally, because the FMM could approximate the distribution function of the measurements.

III. STATE ESTIMATION FOR THE SINGLE TARGET UNDER THE FMM FRAMEWORK

In this section, the state estimation for the single target is proposed. In order to be consistent, the observation data in the FMM and measurement data in target tracking are all denoted by \( z_k \).

A. The FMM Parameter Estimation

The procedure of FMM parameter estimation is shown as follows

1) The distributional parameters are derived by the EM algorithm iteratively.
2) The EQM is derived.
3) Using the EQM to estimate the state of the target.
4) According to the estimates of the target, the FMM initial parameters is given.

B. The FMM Component Parameters

For the FMM, there are several types of parameters: the missing variable \( e_k^i \), the mixing weight \( \pi_k \), mean (or EQM) \( \mu_k \) and covariance \( \Sigma_k \).

1) The Missing Variable \( \hat{e}_k^i \): The missing variable is unobservable. According to (5), the following estimation equation is proposed:

\[
\hat{e}_k^i = \frac{\pi_k^i p_j(z_k^i|\mu_k, \Sigma_k)}{\sum_{j=0}^{n} \pi_k^j p_j(z_k^i|\mu_k, \Sigma_k)} \quad i = 1, ..., n_k
\]

where \( p_0(.) \) is the distribution function of the incorrect and correct measurements, respectively.

2) The Mixing Weight \( \pi_k^i \): According to the definition of mixing weight

\[
\{ \pi_k^0, \pi_k^1 \} = \left\{ \frac{l_k^0}{l_k^0 + l_k^1}, \frac{l_k^1}{l_k^0 + l_k^1} \right\}
\]

where \( \pi_k^0 + \pi_k^1 = 1 \), \( \pi_k^i \geq 0 \). \( l_k^i \) is the number of measurements that belong to the jth-component.

3) The EMQ \( \hat{\mu}_k \): If a measurement originates from the target, then its prior distribution is Gaussian and \( \hat{\mu}_k = E(z_k|\hat{e}_k^i) \). It will be estimated by the following equation

\[
\hat{\mu}_k^{(t+1)} = \int z_k f(z_k|\hat{e}_k^i) dz_k = \int z_k N(z_k|\hat{\mu}_k^{(t)}, \hat{\Sigma}_k^{(t)}) dz_k
\]

where \( t \) denotes the iterative step. Under linear Gaussian condition, the EQM can be derived by

\[
\hat{\mu}_k = \sum_{i=1}^{n_k} \frac{z_k^i \hat{e}_k^i}{\sum_{i=1}^{n_k} \hat{e}_k^i}
\]

If \( l_k^i \) is too small than a threshold \( \lambda_i \), for example \( \lambda_i = 1.0 \times 10^{-5} \), which implies that the true measurement may not be detected. Under this condition, the EQM will not be used. Instead, we use the prediction state as the estimates.
TABLE I
THE PROPOSED EM ALGORITHM WITH STATE ESTIMATION

<table>
<thead>
<tr>
<th>Step</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initialization:</td>
<td>( \hat{x}^{(0)}_k, \Sigma^{(0)}_k, t = 1 )</td>
</tr>
<tr>
<td>2. Computing</td>
<td>( e_{1t}^{(1)} ) and ( e_{2t}^{(1)} ) according to (12) and (14).</td>
</tr>
<tr>
<td>3. Computing</td>
<td>( \hat{x}^{(1)}_k ) according to (16).</td>
</tr>
<tr>
<td>4. Computing</td>
<td>( \hat{x}^{(t)}_k ) according to (17).</td>
</tr>
<tr>
<td>5.</td>
<td>( t = t + 1 ).</td>
</tr>
<tr>
<td>6.</td>
<td>Repeat the above steps (2)-(5) until parameter is stable</td>
</tr>
</tbody>
</table>

4) Covariance \( \Sigma_k \): Here \( \Sigma_k \) is obtained by:
\[
\Sigma_k = S_k = R_k + H_k P_{k|k-1} H_k^T
\]
where \( S_k \) is the new information covariance. At each iteration, \( S_k \) is a constant. In fact, the iteration just includes the EQM and mixing weight.

C. State Estimation

After the EQM is obtained, it will be used to estimate the state of the target. The Kalman filter (KF) is used.
\[
\hat{x}_k = \hat{x}_{k|k-1} + K_k (\hat{x}_k - H_k \hat{x}_{k|k-1})
\]
\[
\hat{x}_{k|k-1} = F_{k-1} \hat{x}_{k-1}
\]
\[
S_k = R_k + H_k P_{k|k-1} H_k^T
\]
\[
P_{k|k-1} = G_k Q_{k-1} G_k^T + F_{k-1} P_{k-1} F_{k-1}^T
\]
\[
P_k = P_{k|k-1} + K_k H_k P_{k|k-1}
\]
\[
K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}
\]

In order to be clear, the step is denoted as follows
\[
\hat{x}_k = ES(\mu_k) \quad \text{or} \quad \hat{x}_k = F_k \hat{x}_{k-1}
\]

where the second equation is used for \( l_k^t < \lambda_l \).

D. The Proposed EM algorithm for FMM

The iterative parameters include the mean \( \mu_k^{(t)} \), covariance matrix \( \Sigma_k^{(t)} \) and mixing weight \( \pi_k^{(t)} \), where \( t \) denotes the iterative step. The proposed EM algorithm is presented in Table I

IV. ALGORITHM IMPLEMENTATION

In this section the implementation process of the proposed algorithm is given. The process contains two stages, i.e., the EM and State Estimation (SE) stage.

A. EM Stage

Assume that the initial position information is known, in other word the prior information \( x_0, Q_0, R_0 \) are known, then \( \hat{x}_0 = z_0 \) can be derived by measurement equation (2).

B. SE Stage

In this paper, the KF is used to estimate the target state (For the FMM, the corresponding first and second order moment are measurement estimates \( \hat{x}_k \) and new information covariance matrix \( S_k \), respectively). Otherwise, other nonlinear algorithm should be used.

C. Experiment 1: Single target

In this subsection, a single target CV motion is given to test the proposed algorithm. Assume that the track has been initialized. The surveillance region is \([-1000, 1000] \times [-1000, 1000] \text{m}^2\). State vector is \( X = [x, \dot{x}, y, \dot{y}]^T \). The target initial position is at \((0, 0)\). The dynamic and measurement equations are
\[
X(k + 1) = AX(k) + Bu(k)
\]
\[
Z(k + 1) = CX(k + 1) + v(k)
\]
where
\[
A = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \end{bmatrix}
\]
\[
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
\]

Detection probability \( P_D = 0.98 \). Clutter density \( \lambda = 2 \times 10^{-5} \text{m}^{-2} \), which hints there are about 80 clutter points. State and measurement covariance matrix \( Q_0 = \text{diag}(25, 25), R_0 = \text{diag}(100, 100) \). Target initial follow Gaussian \( X_0 \sim N(x; x_0, B_0) \), where \( x_0 = [0, 3, 0, -3]^T, B_0 = \text{diag}(10, 5, 10, 5) \).
be given as follows
\[
\begin{align*}
  f_k(z^c_k|\theta_k) & = f^c_k(z^c_k|\theta^c_k) + f^f_k(z^f_k|\theta^f_k) + f^g_k(z^g_k|\theta^g_k) \\
  f_k(z^f_k|\theta_k) & = \pi_k^{c} c_k(z^c_k) + \pi_k^{f} f_k(z^f_k|\theta^f_k) + \pi_k^{g} g_k(z^g_k|\theta^g_k) \\
  f_k(z^g_k|\theta_k) & = \pi_k^{f} f_k(z^f_k|\theta^f_k) + \pi_k^{g} g_k(z^g_k|\theta^g_k)
\end{align*}
\]
(25)
where \( f^c_k(\cdot), f^f_k(\cdot), f^g_k(\cdot) \) are the clutter, survival, birth and spawn target FMM, respectively. They have the following form
\[
\begin{align*}
  f^c_k(z^c_k|\theta^c_k) & = \pi_k^{c} c_k(z^c_k) \\
  f^f_k(z^f_k|\theta^f_k) & = \pi_k^{f} f_k(z^f_k|\theta^f_k) + \pi_k^{s} S_k p(z^f_k|\theta^f_k) \\
  f^g_k(z^g_k|\theta^g_k) & = \pi_k^{g} g_k(z^g_k|\theta^g_k) + \pi_k^{b} B_k p(z^g_k|\theta^g_k)
\end{align*}
\]
(26)  (27)  (28)
where the mixing weight meets with
\[
\begin{align*}
  \pi_k^0 & = \sum_{j=1}^{S_k} \pi_k^{s,j} + \sum_{j=1}^{B_k} \pi_k^{b,j} + \sum_{j=1}^{A_k} \pi_k^{a,j} = 1
\end{align*}
\]
(29)
where \( S_k \) is the number of the survival targets. \( B_k \) is the number of the targets birth distribution. \( \pi_k^{b,j} \) and \( p(z^f_k|\theta^f_k) \) are the weight and distribution for the targets birth. Correspondingly, \( A_k \) is the number of distribution for the spawning targets. \( \pi_k^{a,j} \) and \( p(z^g_k|\theta^g_k) \) are weight and distribution for the spawning target. Under the linear Gaussian condition, the multistate FMM is given as follows
\[
\begin{align*}
  f_k(z_k|\theta_k) & = \pi_k^0 c_k(z_k) + \pi_k^1 N(z_k; \mu_k^1, \Sigma_k^1) + \ldots + \pi_k^n S_k N(z_k; \mu_k^n, \Sigma_k^n) + \pi_k^{b,j} B_k N(z_k; \mu_k^{b,j}, \Sigma_k^{b,j}) + \pi_k^{a,j} A_k N(z_k; \mu_k^{a,j}, \Sigma_k^{a,j}) + \ldots + \pi_k^{a,i} A_k N(z_k; \mu_k^{a,i}, \Sigma_k^{a,i})
\end{align*}
\]
(30)
Similar as in single target case, we assume that the measurements are independent.

B. The Multitarget FMM Parameter Distribution

The unknown variables are the missing variable, mixing weight, mean and covariance of the corresponding component.

1) The Multitarget Missing Variable:
\[
\begin{align*}
  \epsilon_k^{i,j} & = \frac{\pi_k^i p_j(z_k|\mu_k^i, \Sigma_k^i)}{\sum_{j=0}^{T_k} \pi_k^i p_j(z_k|\mu_k^i, \Sigma_k^i)} \\
  p_0(\cdot) & = c_k(z_k) \\
  p_j(\cdot) & = N(z_k; \mu_k^j, \Sigma_k^j), j = 1, \ldots, m_k \\
  i & = 1, \ldots, n_k
\end{align*}
\]
(31)

2) Mixing Weight \( \pi_k^i \): The mixing weight meets with the Dirichelet distribution, i.e.
\[
\begin{align*}
  \{\pi_k^0, \ldots, \pi_k^{m_k}\} & = \left\{ \frac{p_k^0}{\sum_{j=0}^{m_k} p_k^j}, \ldots, \frac{p_k^{m_k}}{\sum_{j=0}^{m_k} p_k^j} \right\} \\
  p_k^j & = \sum_{i=1}^{n_k} \epsilon_k^{i,j} \\
  j & = 0, \ldots, m_k
\end{align*}
\]
(32)  (33)
where \( \sum_{j=0}^{m_k} \pi_k^j = 1, \pi_k^j \geq 0 \).
3) The Multitarget Equivalent Measurement: The multitarget EQM is defined as follows

**Definitions 2:** The EQM of the jth target is defined as

$$\mu^j_k = E(z_k|e^j_k, i = 1, ..., n_k)$$

(34)

Similar, under Gaussian it can be derived by

$$\hat{\mu}^j_k = \frac{1}{n_k} \sum_{i=1}^{n_k} e^j_k$$

(35)

The mean prior is proposed as follows

$$f(\mu^j_k) = \mathcal{N}(\mu^j_k; e^j_k, \Sigma^j_k)$$

(36)

4) Covariance $\Sigma^j_k$: Here $\Sigma^j_k$ can be derived by the following equation

$$\Sigma^j_k = S^j_k = R^j_k + H^j_k P^j_{k|k-1} H^{j^T}_k$$

(37)

C. State Estimation for the Multitarget

Using the multitarget EQMs, the state estimation for the multitarget is the same as the single target.

$$\hat{x}^j_k = \hat{x}^j_{k|k-1} + K_k (\hat{P}^j_k - H^j_k \hat{x}^j_{k|k-1})$$

(38)

$$\hat{x}^j_{k|k-1} = \hat{x}^j_{k-1}$$

(39)

$$S^j_k = R^j_k + H^j_k P^j_{k|k-1} H^{j^T}_k$$

(40)

$$P^j_{k|k-1} = G_k Q^j_{k-1} G^{j^T}_k + F^j_{k-1} P^j_{k-1} F^{j^T}_{k-1}$$

(41)

$$P^j_k = P^j_{k-1} + K_k^j H^j_k P^j_{k|k-1}$$

(42)

$$K^j_k = P^j_{k|k-1} H^{j^T}_k (H^j_k P^j_{k|k-1} H^{j^T}_k + R^j_k)^{-1}$$

(43)

**Remark 4:** There seem no any data association, in fact, the indicator variable $e^j_k$ plays the role of association. Also, $e^j_k$ includes the information of measurement distribution.

D. Target Birth and Death

With the increase of the time step, the number of components in (25), (30) will also increase, so the component management technique is adopted, which is the same as the MHT [4], Gaussian sum filter [15] and Gaussian components terms in Gaussian mixture PHD [9]. Because of false alarms and the miss detection, it is difficult to determine whether a target birth or death through one step measurements. Therefore, multiple step measurements must be considered. Here the number for the birth step threshold $n_b$ and death step threshold $n_d$ is proposed as follows

$$n_b = \left\lfloor \frac{k_{\text{birth}}}{\alpha_{\text{birth}}(1 - P_F)} \right\rfloor, n_d = \left\lfloor \frac{k_{\text{death}}}{\alpha_{\text{death}} P_D} \right\rfloor$$

(44)

where $\lfloor \cdot \rfloor$ denotes rounding to the nearest integer. $P_F$ is the probability of false alarms. $\alpha_{\text{birth}}$ and $\alpha_{\text{death}}$ are coefficients of birth and death, which meet with $0 < \alpha_{\text{birth}}, \alpha_{\text{death}} \leq 1$. They show the belief degree for these two probabilities. If $\alpha_{\text{birth}}$ and $\alpha_{\text{death}}$ equal to 1, then they have complete belief level. $k_{\text{birth}}, k_{\text{death}} \geq 1$ are the step coefficients.

The management approach is proposed as follows

- If the number of the measurements that belong to certain component is less than a threshold $\beta_{\text{death}}$ in a continuous $n_d$ steps, then the track is deleted.
- If the number of measurement of new component is large than a threshold $\beta_{\text{birth}}$ in a continuous $n_b$ steps, then a new track is built.

E. Estimation of the Target Number

Each target state follows different distribution (or with different parameters). It provides the information of the target number. According to the prior assumption, a target gives no more than one measurement, so the average number of measurements which belong to the target measurement distribution can give an estimation for the target number. Thus we could use the missing data $e^j_k$ to estimate the number of targets.

$$T_k = E(\sum_{i=1}^{n_k} e^j_i | z_k) = \sum_{i=1}^{n_k} E(e^j_i | z_k) = \sum_{i=1}^{n_k} \hat{e}^j_i$$

(45)

The above equation can also be expressed as follows

$$T_k = \sum_{j=1}^{m_k} \sum_{i=1}^{n_k} e^j_i = \sum_{j=1}^{m_k} \hat{e}^j$$

(46)

where $\hat{e}^j_k$ is the number of the measurements which belong to the jth target measurement distribution.

**Remark 5:** Under the assumptions A.1, $\hat{e}^j_k$ can be used to estimate the number of the targets, because the value usually is not integer, so the thresholds are proposed to judge whether a target exists. For example, if $\hat{e}^j_k > \beta_{\text{birth}} = 0.9$, then a target exists. If $\hat{e}^j_k < \beta_{\text{death}} = 0.1$, then a target does not exist.

F. Experiment 2 - Multitarget Experiment

In this experiment, we focus on the target birth, death and the estimation of the target number. A two target-CV motions is considered. The surveillance region is $[0, 1000] \times [-500, 500]$ and the clutter density is $5 \times 10^{-5}$m$^{-2}$. The detection probability $P_D$ and false alarms rate $P_F$ are 0.98 and 0.001, respectively. $\alpha_{\text{birth}}$ and $\alpha_{\text{death}}$ have the same value 0.9. $\beta_{\text{birth}}$ and $\beta_{\text{death}}$ are the 2 and 3, respectively. Thus the $n_b, n_d$ are all 2. $\beta_{\text{birth}}$ is 0.9 and $\beta_{\text{death}}$ is 0.1. The covariance of state noise is $Q \sim \mathcal{N}(0, \text{diag}[0.5, 0.3])$ and the covariance $R^j_k$ of measurement noise is diag[10, 10], $i = 1, 2$. The targets born from two places $(0, 120)$, (0, 0) with initial states $x^i_0 = (0, 16, 0, 0), x^j_0 = (0, 10, 120, -5)$. The corresponding birth and death time are at 1s, 40s for target 1, and 10s, 50s for target 2. According to (25), at time 1 the multitarget FMM is as follows

$$f_1(z^1_1|\theta_1) = \pi^0_{\text{c1}} c_1(z^1_i) + f^1_1(z^1_i|\theta_1)$$

where

$$f^1_1(z^1_i|\theta_1) = \pi^{b,1}_{1} \mathcal{N}(z^1_i; \mu^{b,1}_{1}, \Sigma^{b,1}_{1}) + \pi^{d,1}_{1} \mathcal{N}(z^1_i; \mu^{d,1}_{1}, \Sigma^{d,1}_{1})$$

in which $\mu^{b,1}_{1} = HA x^b_0, \mu^{d,1}_{1} = HA x^d_0, \Sigma^{b,1}_{1} = S^b_1, \Sigma^{d,1}_{1} = S^d_1$

At time 2 the multitarget FMM is as follow

$$f_2(z^1_2|\theta_1) = \pi^{0}_{2} c_2(z^2_i) + f^2_2(z^2_i|\theta_1) + f^2_2(z^2_i|\theta_2)$$

$$f^2_2(z^2_i|\theta_1) = \pi^{b,2}_{2} \mathcal{N}(z^2_i; \mu^{b,2}_{2}, \Sigma^{b,2}_{2}) + \pi^{d,2}_{2} \mathcal{N}(z^2_i; \mu^{d,2}_{2}, \Sigma^{d,2}_{2})$$

in which $\mu^{b,2}_{2} = HA x^b_0, \mu^{d,2}_{2} = HA x^d_0, \Sigma^{b,2}_{2} = S^b_2, \Sigma^{d,2}_{2} = S^d_2$
\[
f_2^1(z_1^1 | \theta_1) = \pi_{2}^{s,1} \mathcal{N}(z_1; \mu_{2}^{s,1}, \Sigma_{2}^{s,1}) + \pi_{2}^{s,2} \mathcal{N}(z_1; \mu_{2}^{s,2}, \Sigma_{2}^{s,2})
\]

in which initial parameter \( \mu_{2}^{s,1} = H A x_{1} \), \( \mu_{2}^{s,2} = H A x_{2} \), \( \Sigma_{2}^{s,1} = S_{1} \), \( \Sigma_{2}^{s,2} = S_{2} \).

\[
f_2^2(z_2^1 | \theta_2) = \pi_{2}^{b,1} \mathcal{N}(z_1; \mu_{2}^{b,1}, \Sigma_{2}^{b,1}) + \pi_{2}^{b,2} \mathcal{N}(z_1; \mu_{2}^{b,2}, \Sigma_{2}^{b,2})
\]

Fig. 5 and Fig. 6 are the track processes for the target 1 and 2. Fig. 7 shows the comparison between the true and estimation velocity. Fig. 8 illustrates the number of targets against time.

Fig. 5 shows the number of targets against time. The estimated number of targets against time, which is the same as the true number at the most time points show the effects of miss detection, false alarms (clutter) and system error (process and measurement). At time 2 and 3, the true measurement is not detected, so the \( l_{k}^{1} \) is approximated to zero. The two targets cross at time 23, because the two targets measurements are more approximate to target 1, so \( l_{k}^{1} \) is larger than \( l_{k}^{2} \). The total number is correct. At time 26, target is effected by the error and the \( l_{k}^{1} \) shows fluctuation. At the time 46, the target 1 suggests effects of the false alarms. These results suggest that the missing variable \( e_{k}^{1,2} \) and the number of measurements \( l_{k}^{2} \) can be used to estimate the targets number and shows the effects of the miss detection, false alarms and system error.

Fig. 10 illustrates the true measurement and EQM process against time. In the most time EQM is equal to true measurement except few points, such as the points at time 2, 22 and 23. The first points is because miss detection and the true measurement is not derived. The other two points are because of the coalescence of these two targets.
In this research, we assume that no more than one measurement originates from a target and the multitarget system are linear Gaussian. When the multiple measurements are from a target and the system are nonlinear, further research works will be needed. Similar to JPDA, the target may be lost when target coalesces. Much proposed works can be used to deal with the problem.

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VI. CONCLUSION

The key to multitarget tracking is the association between the measurement and target. For the traditional approach, there are several limitations. First, when the target number is large, the computational load of association probability, clusters and the hypothesis set are intractable, thus the computational time increasing rapidly. Second, the validation gate is to improve the speed, under some condition. In contrast, validation gate may increase computational load and reduce the speed.

In this paper we propose a multitarget tracking clustering algorithm based on the FMM and EQM, compared with the traditional algorithm, the proposed algorithm has the following characteristics: First, it does not need validation gate. Second, it can deal with the uncertain number of targets. And with the increase of the target number, the computational load is just linear increase.

Fig. 9. \( l_j^t \) against time (\( l_j^t \) denotes the number of measurements which belongs to the \( j \)th component or target)

Fig. 10. The EQM track against time

\[
\begin{align*}
T_{\text{time stop}} & \quad T_{\text{TARGET 1}} & \quad T_{\text{TARGET 2}} \\
0 & \quad 100 & \quad 200 \\
-60 & \quad -40 & \quad 0 \\
0 & \quad 20 & \quad 40 \\
-60 & \quad 0 & \quad 60 \\
0 & \quad 0 & \quad 0 \\
100 & \quad 120 & \quad 0 \\
0 & \quad 0 & \quad 0 \\
10 & \quad 12 & \quad 14 & \quad 16 & \quad 18 & \quad 20 \\
-20 & \quad 0 & \quad 20 & \quad 40 & \quad 60 & \quad 80 & \quad 100 \\
0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
11 & \quad 13 & \quad 15 & \quad 17 & \quad 19 & \quad 21 & \quad 22 & \quad 24 & \quad 26 & \quad 28 & \quad 30 & \quad 32 & \quad 34 & \quad 36 & \quad 38 & \quad 40 & \quad 42 & \quad 44 & \quad 46 & \quad 48 & \quad 50 \\
-60 & \quad -40 & \quad -20 & \quad 0 & \quad 20 & \quad 40 & \quad 60 & \quad 80 & \quad 100 \\
0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
0 & \quad 10 & \quad 20 & \quad 30 & \quad 40 & \quad 50 & \quad 60 & \quad 70 & \quad 80 & \quad 90 & \quad 100
\end{align*}
\]