Data association for PHD filter based on MHT

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Abstract - The main drawback of probability hypothesis density (PHD) filter is that it can’t identify the trajectories of the different targets. Data association for PHD filter based on Multiple Hypotheses Tracking (MHT) is presented to solve the problem. The track-oriented MHT is used to perform data association on the output of PHD filter. An adaptive Kalman filter based on “current” statistic model, combined with MHT, is implemented to track maneuvering targets. Two examples are given to test the performance of the new method. Monte Carlo simulation results show that this approach is computationally feasible and effective for associating multi-targets in dense clutter environments.

Keywords: data association, probability hypothesis density, track-oriented MHT

1 Introduction

Multiple targets tracking (MTT) is a challenging problem in point target tracking scenario. MTT techniques were classified as indirect estimation and direct estimation frameworks [1]. Indirect estimation is to use a data association technique to assign the correct measurement to each single target filter that assign to each target [1]. Three main approaches of data association techniques are global nearest neighbor (GNN) approach, joint probabilistic data association (JPDA) method [2] and multiple hypotheses tracking (MHT) method [3,4], respectively. The GNN approach assigns the most likely observations to existing tracks, which only works well in the case of widely spaced targets, accurate measurements, and few false alarms in the track gates. The JPDA method allows a track to be updated by a weighted sum of all observations in its gate. Both the GNN approach and the JPDA method increase the Kalman filter track covariance matrix to account for the association uncertainty. However, increasing the Kalman filter covariance matrix can exacerbate the problem since an increased covariance matrix leads to even more false observations in the track gate. Moreover, the JPDA method suffers from a problem that tracks on closely spaced targets will tend to come together [3].

Multiple hypotheses tracking (MHT) is the preferred method for solving the data association problem, and it is a deferred decision logic in which alternative data association hypotheses are formed whenever observation-to-track conflict situations occur. Rather than choosing the best hypothesis or combining the hypotheses as in the JPDA method, MHT propagate the hypotheses into the future in anticipation that subsequent data will resolve the uncertainty. The main drawback of MHT method is that it’s exhaustively search over all possible hypotheses can be very expensive. However, track-oriented MHT presented by Kurien [4] maintains the tracks that survived pruning and reforms the hypotheses each scan, it maintains less number of hypotheses and hence has less computational load.

Direct estimation techniques treat the problem of estimating the true observation-to-track association as unnecessary and instead attempt to directly estimate the multi-target state($x_1,\ldots,x_T$) as a whole [1]. This technique based on the random finite set (RFS) framework presented for the purposes of data fusion [1]. In the RFS formulation, the collection of individual targets and observations is treated as a set-valued states and observations. By using the RFSs in multiple targets dynamical model in the presence of clutter and association uncertainty, the problem of multiple-target estimation can be formulated in a Bayesian filtering framework [1]. Unfortunately, the RFS multiple-target Bayes filter is just a theoretical framework and computationally intractable since it involves multiple integrals. Probability hypothesis density (PHD) filter proposed by Mahler [1,5] began the development of tractable multi-target filter that can be implemented. Two main implementations for PHD filter are the sequential Monte Carlo PHD (SMC- PHD) filter or particle-PHD filter [6], and the Gaussian mixture PHD filter [7]. The PHD filter can give both the number of targets and their locations. The main drawback of the PHD filter is that it can’t identify the trajectories of different targets.

The method for solving the problem is to combine data association techniques with the PHD Filter. There have been two previous approaches of data association for the PHD Filter. Lin [8] described a data association technique for doing peak-track association for particle-PHD filter. The peaks of PHD were matched with tracks by using an optimization technique to minimize the cost of association. Panta et al. [9] used the SMC-PHD filter to remove unlikely measurements before inputting the data to a
Multiple Hypothesis Tracker (MHT) filter. The above methods have some shortcomings: firstly, since the particle-PHD filter needs large number of particles and be unreliability of clustering techniques for extracting state estimates, it is computationally intractable in practice. Secondly, the two above methods are just “abstract” targets states from particles which represent the multi-target posterior density. However, they have not yet identifying the trajectories of different targets.

The technique of data association for PHD filter in this paper is based on track-oriented MHT filter. The Gaussian mixture PHD filter is used as pre-filtering measurements filter, and the output of the PHD filter is defined as a new observation to a MHT association filter. Simulation result shows that this method can identify the trajectories of the different targets effectively and outperforms the PHD filter.

The structure of this paper is as follows. Section 2 presents a briefly review of the PHD filter and the data association algorithm for the GM_PHD filter based on track-oriented MHT filter, Section 3 presents a new method for evaluating performance of multi-target tracking algorithms, simulation results, which include comparisons of the proposed method with the standard PHD are presented in Section 4 and conclusions are given in Section 5.

2 Data association for PHD filter based on Track-oriented MHT

2.1 Random Finite Set (RFS) Multi-Target Bayes filter

The multi-target Bayes filter is an optimal Bayes estimation method that is generalization of the single-target Bayes filter. Let \( x_{k,1}, \ldots, x_{k,M(k)} \) be the multi-target states and \( z_{k,1}, \ldots, z_{k,N(k)} \) be the measurements at time \( k \). then the RFS of multi-target states and measurements be the set:

\[
X_k = \{ x_{k,1}, \ldots, x_{k,M(k)} \} \subset \mathcal{X} \\
Z_k = \{ z_{k,1}, \ldots, z_{k,N(k)} \} \subset \mathcal{Z}
\]  

where, the number of targets \( M(k) \) and measurements \( N(k) \) are all variable with the time \( k \). Uncertainty in a multi-target system is characterized by modeling multi-target states and multi-target measurements as RFSs \( X_k \) and \( Z_k \) on the state and observation spaces \( E_s \) and \( E_o \) respectively. Under the definition of RFS on multi-target states and measurements, the multi-target dynamics and observation can be described as follows.

Given a realization \( X_{k-1} \) of the RFS \( \Xi_k \) at time \( k-1 \), the RFS model for multi-target state at time \( k \) is formed as

\[
X_k = \bigg[ \bigcup_{x \in X_{k-1}} S_{k|k-1}(x-1) \bigg] \bigg[ \bigcup_{x \in X_{k-1}} B_{k|k-1}(x-1) \bigg] \bigcup \Gamma_k
\]  

where, the RFS \( S_{k|k-1}(x-1) \) denotes the RFS of targets in time \( k \) survives from time \( k-1 \), \( \Gamma_k \) denotes the RFS of spontaneous births, and \( B_{k|k-1}(x-1) \) denotes the RFS of targets spawned from \( x_{k-1} \in X_{k-1} \).

The RFS measurement model is formed as

\[
Z_k = K_k \left[ \bigcup_{x \in X_{k-1}} \Theta_k(x) \right]
\]  

where the measurement RFS \( \Theta_k(x) \) is generated by state \( x_k \in X_k \), it can take on either \( \{ z_k \} \) when the target is detected, or \( \emptyset \) when the target is not detected.

Assuming the RFS of multi-target states follows a Markov process with multi-target transition density:

\[
f_{k|k-1}(X_k | X_{k-1})
\]  

The Markov process is observed in the observation space as modeled by multi-target likelihood function:

\[
g_k(Z_k | X_k)
\]  

Let \( p_k(\cdot | Z_{1:k}) \) denote the multi-target posterior density. The multi-target Bayes filter can be formulated by

\[
p_{k|k-1}(X_k | Z_{1:k-1}) = \\
\int f_{k|k-1}(X_k | X) p_{k-1}(X | Z_{1:k-1}) \mu_X(dX)
\]  

\[
p_k(X_k | Z_{1:k}) = \\
\int g_k(Z_k | X_k) p_{k|k-1}(X_k | Z_{1:k-1}) \mu_X(dX)
\]  

where \( \mu_X \) is reference measure on the space of finite subsets of \( \mathcal{X} \). The above formulation involves multiple integrals on the space of finite sets, which are computationally intractable. The PHD filter proposed by Mahler [1,5] began the development of tractable optimal multi-target filter that can be implemented. Currently, there are two main effective techniques to implement the PHD filter. Sequential Monte Carlo (SMC) filter or particle-PHD filter method is the popular technique to implement the PHD filter. However, the particle-PHD method needs large number of particles and being unreliability of clustering techniques for extracting state estimates. Recently, a closed-form solution namely Gaussian Mixture probability hypothesis density (GM_PHD) filter is presented by Ba-Ngu Vo [7] to the PHD recursion for Gaussian models. The efficiency and simplicity in implementation of the GM_PHD filter ensure it practical application.
2.2 Flowchart of Track-oriented MHT method

Rather than maintaining hypotheses from scan to scan as hypothesis-oriented approach, the track-oriented MHT[3,4] maintains the tracks that survived pruning and then use the tracks and new observations to reform the hypotheses each scan, so it maintains less number of hypotheses tracks and has less computational load. The logic flowchart to track-oriented MHT [3] is given here.

Figure 1. Track-oriented MHT logic flowchart

The track-oriented MHT starts by forming tracks with new observations each scan. In our method, instead of using kalman filter to form new tracks, an adaptive filter based on “Current” model is used to form tracks for tracking Maneuvering Targets.

2.3 “Current” state model and an adaptive Kalman filter for Maneuvering Targets

“Current” model [10] is a nonzero mean-adaptive acceleration model that is modified from Singer Model for maneuvering targets. Rather than assuming that target acceleration \( a(t) \) is a zero-mean stationary first-order Markov process as Singer Model, the “Current” model restrict the acceleration of target within the neighborhood of the mean of current acceleration, i.e., \( a(t) = \bar{a}(t) + \alpha(t) \). where \( \bar{a}(t) \) is the zero-mean Singer acceleration process; \( \alpha(t) \) is the mean of the current acceleration, assumed constant over each sampling interval. The “Current” model satisfies follow equation

\[
\dot{a}(t) = -\alpha a(t) + \alpha \bar{a}(t) + w(t)
\]

The corresponding continuous-time state-space equation is

\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \bar{a}(t) + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} w(t)
\]

The corresponding discrete-time state-space equation is

\[
X_{k+1} = F_{\alpha} X_k + U_{\alpha} + w(k)
\]

where \( F_{\alpha} = \begin{bmatrix} 1 & T & (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ 0 & 1 & (1 - e^{-\alpha T})/\alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \)

\[
U_{\alpha} = \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix} \begin{bmatrix} (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ (1 - e^{-\alpha T})/\alpha \\ e^{-\alpha T} \end{bmatrix}
\]

The parameter \( \alpha = 1/T_m \) is the reciprocal of the maneuver time constant \( T_m \) and depends on how long the maneuver lasts. The “Current” model is more effective for maneuvering target tracking than the Singer mode. An adaptive Kalman filter for maneuvering targets can be obtained by combining the “Current” model and individual observation model with standard Kalman filter.

2.4 Data association for PHD filter

This approach takes the output of the PHD filter and performs data association on them. At each time step \( k \), the output of the PHD filter are Random Finite Set (RFS) of multi-target states and can be defined as a new observation that the estimate error is regarded as the noise. Thus, regardless of the fact that the observation process is non-linear, the data-association functionality is given a linear observation process. The method can be informally explained as follows:

**Step 1 Gaussian Mixture PHD Filter implementation for the PHD filter.**

Multi-Target model is assumed as Gaussian dynamics

\[
f_{k|k-1}(x \mid \varsigma) = N(x; F_{k-1}x, Q_{k-1}) \tag{12}
\]

\[
g_k(z \mid x) = N(z; H_kx, R_k) \tag{13}
\]

The survival and detection probabilities are assumed independent and constant. The PHD of the birth and spawn RFSs are Gaussian mixtures of the form

\[
\gamma_k(x) = \sum_{i=1}^{j} w_k^{(\gamma,i)} N(x; m_k^{(\gamma,i)}, P_k^{(\gamma,i)}) \tag{14}
\]

\[
\beta_{k|k-1}(x \mid \varsigma) = \sum_{j=1}^{j} w_k^{(\beta,j)} N(x; \varsigma + m_k^{(\beta,j)}, P_k^{(\beta,j)}) \tag{15}
\]

Under these assumptions, the PHD recursion for the linear Gaussian model could be implemented in the closed-form of the following two steps [7]:

**Prediction:**

Suppose that the posterior PHD at time \( k-1 \) is a Gaussian mixture of the form:

\[
v_{k-1}(x) = \sum_{i=1}^{j} w_{k-1}^{(i)} N(x; m_{k-1}^{(i)}, P_{k-1}^{(i)}) \tag{16}
\]

Then, the predicted PHD at time \( k \) is a Gaussian mixture of the form:

\[
v_{k|k-1}(x) = v_{k|k-1}^{(\gamma)} + v_{k|k-1}^{(\beta)} + \gamma_k(x) \tag{17}
\]
where
\[ v^{(i)}_{k+1}(x) = e^{J_{i+1}} \sum_{i=1}^{J_{i+1}} w^{(i)}_{k-1} N(x; F_{k+1}m_{k-1}^{(i)} + Q_{k-1} + F_{k-1}^T P_{k-1}^{(i)} F_{k-1}^T) \] (18)
\[ v^{(i)}_{k+1}(x) = \sum_{i=1}^{J_{i+1}} w^{(i)}_{k-1} N(x; m_{k-1}^{(i)} + m_{k-1}^{(i)} + P_{k-1}^{(i)} + P_{k-1}^{(i)}) \] (19)

Update:

Suppose that the predicted PHD at time \( k \) is a Gaussian mixture of the form:
\[ v_{k|k-1}(x) = \sum_{i=1}^{J_k} w^{(i)}_{k|k-1} N(x; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}) \] (20)

Then, the posterior PHD at time \( k \) is a Gaussian mixture given by
\[ v_k(x) = (1 - p_D) v_{k|k-1}(x) + \sum_{z \in z_k} v^D_k(x; z) \] (21)

where
\[ v^D_k(x; z) = \sum_{i=1}^{J_k} p_D w^{(i)}_{k|k-1} q^{(i)}_k(z) N(x; m^{(i)}_k(z), P^{(i)}_k) \] (22)
\[ q^{(i)}_k(z) = N(z; H_km^{(i)}_k + R_k + H_k P^{(i)}_k H_k^T) \] (23)
\[ m^{(i)}_k(z) = m^{(i)}_{k|k-1} + K^{(i)}(z - H_k m^{(i)}_{k|k-1}) \] (24)
\[ P^{(i)}_k = [I - K^{(i)} H_k] P^{(i)}_{k|k-1} \] (25)
\[ K^{(i)}_k = P^{(i)}_{k|k-1} H_k^T (H_k P^{(i)}_{k|k-1} H_k^T + R_k)^{-1} \] (26)

Step 2 defining a new observation model:

Suppose that the original observation set are zero-mean Gaussian process, then the output of the GM-PHD filter are Random Finite Set (RFS) of multi-target states \( X_k \) with variance \( P_k \) at time \( k \). For carrying out data association with track-oriented MHT conveniently, we define \( Y_k = X_k + W_k \) as a new observation Random Finite Set model, where \( W_k \) is zero-mean Gaussian noise with variance \( P_k \). Surely, the RFS \( Y_k \) is smaller than the original observation set. As a result, a much smaller number of track hypotheses are created at each time step and the computational cost of the data-association will be smaller as well.

Step 3 Implementation of data-association with track-oriented MHT method:

As described flowchart in Fig.1, the implementation of data-association using track-oriented MHT filter include the follow steps:

a) Track formation and maintenance

As new observations (i.e., the output of the GM-PHD filter) are received, standard gating method is used to determine the valid ones. Then, an adaptive Kalman filter based on “Current” model is used to update the existing tracks with all observations within the gates. Moreover, each observation is used to initiate a new track.

b) Track-level pruning and confirmation

Once tracks are updated and initiated, a track-level probability and score that is the log likelihood ratio for each track are calculated by Bayes’ rule, then a sequential probability ratio test is performed to compare the scores with a suitably chosen deletion threshold. The tracks that fail this test are deleted and the surviving tracks are propagated to next step and tested for confirmation.

c) Clustering

The process of clustering separates all tracks into clusters, each cluster only includes tracks that are linked by common observations directly or indirectly. The means of indirect tracks is that these tracks do not share observations directly but both share observations with a third track. Multiple hypotheses are generated within each cluster independently. By "decoupling" the tracks into clusters, the MHT generates less hypotheses each scan and has less computational load.

d) Hypotheses formation and pruning

Once the clusters are formed, hypotheses are generated within each cluster. Hypotheses are defined to be sets of consistent tracks that no two tracks within a hypothesis share observations [4]. One approach to Hypotheses formation is a straightforward search method that define one-track hypothesis and expands it by adding new tracks that consistent with any tracks in the hypothesis.

e) Global-level track pruning and merging

Once the hypotheses are generated, the score and probabilities of hypotheses can be calculated. The a posteriori probability of a given track is the sum of the probabilities of all hypotheses containing that track. Each track whose probability is below a deletion threshold is removed from the track file. Then merging logic is performed to determine which tracks are redundant representation of the same target and should be merged. Tracks that survive the above steps are predicted forward to next scan and also present to user.

f) Output

Since MHT system maintain several tracks for each true target each scan, most likely tracks is selected to provide to the user.

3 The performance for multi-target tracking algorithm

The performance for a multi-target tracking algorithm can be measured by a “distance” between two finite sets which represent the actual target states and the corresponding point estimates produced by the algorithm. Hoffman and Mahler [11] have shown that the Wasserstein distance is a good measure of the multi-target miss distance. Given the true multi-target state finite sets
$X = \{x_1, \ldots, x_n\}$ and the corresponding point estimates sets $Y = \{y_1, \ldots, y_n\}$, the Wasserstein distance $d_p^w$ is defined as:

$$d_p^w(X, Y) = \inf_C \left( \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} d(x_i, y_j)^p \right)$$

(27)

$$d_w^w(X, Y) = \inf_C \max_{1 \leq i \leq n, 1 \leq j \leq m} \tilde{C}_{i,j} d(x_i, y_j)$$

(28)

where the infimum is taken over all $n \times m$ “transportation matrices” $C_{i,j}$; $C_{i,j}$ satisfies the followings:

$$\sum_{i=1}^{n} C_{i,j} = \frac{1}{m}, \sum_{j=1}^{m} C_{i,j} = \frac{1}{n}, C_{i,j} > 0$$

(29)

And $\tilde{C}_{i,j}$ satisfy the followings: $\tilde{C}_{i,j} = 1$ if $C_{i,j} \neq 0$, and $\tilde{C}_{i,j} = 0$ otherwise.

The Wasserstein distance is preferred over the commonly used Hausdorff distance for the Hausdorff distance is insensitive to the different number of objects in two sets. The Wasserstein distances could provide useful and intuitively sensible measures of a multi-target tracker to correctly estimate the numbers and states of the targets in a multi-target scenario at any given time.

4 Simulation Results and Discussion

Two simulation examples are used to test the proposed method.

Example 1: Manoeuvring targets scenario:

Consider a two-dimensional two maneuverings targets scenarios, each target has a survival probability 0.99. Assume no spawning, and that the spontaneous birth RFS is Poisson with intensity:

$$\gamma_k(x) = 0.1 N(x; m_r^1, P_r) + 0.1 N(x; m_r^2, P_r),$$

where,

$$m_r^1 = [-1100, 500, 0, 0, 0]'\quad m_r^2 = [700, 1070, 0, 0, 0]'\quad P_r = diag([2500, 2500, 2500, 2500, (6 \times \pi / 180)^2])'$$

each target has a probability of detection $p_d = 0.99$.

An observation consists of bearing and range measurements $z_k = \begin{bmatrix} \arctan(r_y/r_x) \\ \sqrt{r_x^2 + r_y^2} \end{bmatrix} + \varepsilon_k$,

where, $\varepsilon_k \sim N(\varepsilon; 0, R_k)$ with $R_k = diag([\sigma_\theta^2, \sigma_r^2])$, $\sigma_\theta = 2 \times (\pi / 180)$ rad/s, $\sigma_r = 20$ m. The clutter RFS is uniformly distributed over the surveillance region $[-\pi / 2, \pi / 2] \times [0, 2000]$ rad \times m, the number of clutter

each scan follows the Poisson model with $\lambda = 8 \times 10^3 (\text{rad} \text{ m})^{-1}$, i.e., an average of 50 clutter returns on the surveillance region each scan.
Example 2: alignment targets scenario:
Consider a two-dimensional four alignment targets scenarios, each target move with constant velocity 400 m/s and has a survival probability 0.99. Assume no spawning, and that the spontaneous birth RFS is Poisson with intensity:

\[
\gamma_k(x) = 0.1N(x; m^1_r, P_r) + 0.1N(x; m^2_r, P_r) + 0.1N(x; m^3_r, P_r) + 0.1N(x; m^4_r, P_r)
\]

where

\[
m^1_r = [5000, 45000, 0, 0]'
\]
\[
m^2_r = [5000, 38000, 0, 0]'
\]
\[
m^3_r = [5000, 15000, 0, 0]'
\]
\[
m^4_r = [5000, 8000, 0, 0]'
\]
\[
P_r = \text{diag}([2500, 2500, 2500, 2500, (6 \times \pi / 180)^2])'.
\]

Each target has a probability of detection \( p_d = 0.99 \). An observation consists of bearing and range measurements

\[
z_k = \left[ \begin{array}{c}
\arctan \left( \frac{r_y}{r_x} \right) \\
\sqrt{r_x^2 + r_y^2}
\end{array} \right] + \epsilon_k,
\]

where \( \epsilon_k \sim N(0, R_k) \) with

\[
R_k = \text{diag}([\sigma_\theta^2, \sigma_r^2]), \quad \sigma_\theta = 2 \times (\pi / 180) \text{ rad/s}, \quad \sigma_r = 50 \text{ m}.
\]

The clutter RFS follows the uniform over the surveillance region \([- \pi / 2, \pi / 2] \times [0.52000, 52000] \text{ m} \), the number of clutter each scan follows the Poisson model with \( \lambda_c = 3 \times 10^{-4} (\text{rad} \text{ m})^{-1} \) (i.e., an average of 50 clutter returns each scan).
Since MHT method is computational intractable for scenarios of observations immersed in such dense clutter as the above two examples, we only compare the presented method with GM_PHD method. Figures 2 and 7 show the true targets trajectories and the estimated position of targets for PHD filter, Figures 3 and 8 show the tracks estimates of the PHD_MHT method, Figures 4, 5, 9 and 10 show x and y coordinate velocity estimates of PHD_MHT method against time step respectively, in scenario 2, since velocity of target 2 and 4 are equal to velocity of target 1 and 3 respectively, only velocity of target 1 and 3 are plotted in Figures 9 and 10, Figures 6 and 11 plot Wasserstein distance between the point estimate outputs and the true tracks for the PHD and PHD_MHT method. Evidently, the PHD_MHT method can correctly associate the same target all the iterations. Moreover, Figures 6 and 11 show that the PHD_MHT method has better performance over the PHD filter.

5 Conclusions

A data association method for the GM_PHD filter combined with track-oriented MHT implementation is proposed for multi-targets tracking. It uses the closed-form GM_PHD filter to get the estimated number and locations of the multi-targets from dense clutter. The output of the GM_PHD filter are the RFS of multi-target states, these RFS of multi-target states can be regarded as new “observations” with “sparse clutter”, a simple observation model is formed under these “observations”, track-oriented MHT is used to implement data association with the model and “observations”; two examples are given to compare the performance of the method with PHD filter. Simulation results demonstrated that the proposed method associates multi-targets effectively and with better performance than the GM_PHD filter for multi-targets tracking.
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