Measurement Association for Emitter Geolocation with Two UAVs

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Abstract—Geolocation with three or more unmanned aerial vehicles (UAVs) based on time-difference-of-arrivals (TDOA) is possible but has implementation problems including UAV trajectory optimization, measurement association, and communication bandwidth limitations. The complexity of each of these problems is manageable with a simpler system of two netted UAVs that processes multiple TDOA measurements collected over time. Based on TDOA measurements and given a dense pulse environment, a pulse detected at one UAV may correlate with multiple pulses at the other UAV, giving rise to a TDOA measurement set consisting of true and false TDOA measurements at each stage. These measurements are then converted to 2-d measurement components using Gaussian sum approximation. A tracking algorithm that uses integrated track splitting and track pruning and employs a bank of Kalman filters is used to track the emitter. Results from this tracker are superior to those from an unscented Kalman filter, a traditional answer to tracking in the presence of nonlinearities.

Keywords: Tracking, TDOA, data association, Kalman filtering, estimation.

I. INTRODUCTION

In [1] the problem of emitter geolocation using a set of netted UAVs was investigated. The approach was adapted from emitter geolocation using multiple geostationary satellites [2]. It achieves geolocation by processing measurements from a single pulse that is visible to each of the UAVs. Based on this method, a minimum of three UAVs is required when the transmitter is known to be on the surface of the earth and four if the altitude of the transmitter is not known. While this concept is easy to demonstrate, there are a number of obstacles towards practical implementation. They include: (i) the need to determine UAV trajectories for optimal geolocation; (ii) the need for an association algorithm capable of grouping together measurements from a common pulse given the sheer number of pulses that are detected by each UAV; (iii) the need for a suitable low bandwidth communication network for measurement transfer. The complexity of each of these problems is prohibitive when three or more netted UAVs have to be deployed. In order to reduce the complexity of these problems, a simpler system consisting of two netted UAVs was developed and presented in [3]. In that paper, geolocation was achieved by processing multiple TDOA measurements collected over time by the UAVs as they traversed the surveillance region. When a pulse is detected by the two UAVs, a noisy range difference measurement is generated and based on Gaussian sum approximation, a processing algorithm then converts this measurement to a set of noisy Gaussian measurement components whose mean values lie on a hyperbola defined by the range-difference measurement and whose covariances are related to the width of the one-σ region around the measurement hyperbola. These measurement components were then used to compare the performance of a newly developed linear tracker that employs a bank of Kalman filters and is based on Gaussian sum approximation, to that of a benchmark tracker based on a bank of unscented Kalman filters.

In the benchmark tracker, each measurement component from the first TDOA was used to initiate an emitter track component. Subsequent updates of these track components were then carried out using actual range-difference measurements (rather than their components) modeled by an appropriate range-difference equation. In the new geolocation tracker, referred to later as the GSM tracker, track initiation was similar to that of the benchmark tracker, but maintenance of track components was carried out by updating each track component with each measurement component for each incoming range-difference measurement. This is integrated track splitting (ITS) and leads to an exponential growth in the number of track components. To counter this growth, a track pruning procedure was used to delete the least probable track components after each measurement update assuming these input measurement components were generated only from true (target originated) TDOA measurements. Track components not supported by the new incoming measurements tended to die out while the track components initiated near the emitter tend to survive and congregated in the vicinity of the emitter.

Results from the GSM tracker were found to be superior to those from the benchmark tracker. The above comparison was based on a sparse pulse environment in which a pulse detected at one UAV could correlate with only one pulse at the other UAV, giving rise to only one target originated range-difference measurement. In this paper, we compare the two tracking algorithms assuming a dense pulse environment in which a pulse detected at one UAV may correlate with more than one pulse at the other UAV, giving rise to a TDOA measurement
The emitter localisation problem considered here is based on two netted UAVs each of which is equipped with an electronic support (ES) sensor and a GPS receiver. Each platform therefore measures the leading edge time-of-arrival of each detected pulse and packages it with its GPS generated location before transmitting it to a common processing center through a low bandwidth communication channel. The vehicles, UAVi, i = 1 . . . 2 are located at \( s_i = (\eta_i, \zeta_i) \), i = 1 . . . 2, respectively. These locations are GPS-generated but for the sake of simplicity, we assume that they are noise free.

The emitter for this experiment is a stationary scanning radar that is located at \( x_k = (x_k, y_k) \). We assume one main beam and negligible side lobes. Furthermore, it can be configured to operate under two test scenarios; a low PRF scenario in which a single pulse at UAV1 can only correlate with at most one pulse arriving at UAV2, thus giving rise to at most one true TDOA measurement, and a high PRF scenario in which a single pulse at UAV1 can correlate with multiple pulses arriving at UAV2 giving rise to multiple TDOA measurements within each measurement set with only one of them being a true or target originated TDOA measurement. Results based on the first scenario were presented in [3]. In this paper we restrict ourselves to the high PRF scenario.

Each leading edge arrival time is based on a random standard deviation of \( \sigma_r \). The arrival time of the k-th pulse at the i-th UAV is then given by,

\[
t_k^i = t_k + r_k^i/c + n_k^i
\]

(1)

where \( t_k \) is the transmission time of the k-th pulse, \( r_k^i = \sqrt{(\eta_k^i - x_k)^2 + (\zeta_k^i - y_k)^2} \) is the direct distance between the emitter and the UAV, \( n_k^i \) is a zero-mean Gaussian measurement noise with standard deviation \( \sigma_t = \sigma_r/c \), and \( c \) is the propagation speed.

The emitter position is unknown and because this is a passive system, the time of emission of the signal \( t_k \), is also unknown. If a reference sensor is chosen and designated as sensor 1, time of arrival measurements from each other may be subtracted to remove the dependence of equation (1) on knowing the time of signal emission. The result is the time difference of arrival (TDOA), \( t_k^1 \), which is the difference between the TOA’s at each sensor with that of sensor 1.

\[
t_k^1 = t_k^1 - t_k = (r_k^1 - r_k^1)/c + n_k^1
\]

(2)

where \( n_k^1 \sim N(0, 2\sigma^2) \). For a single TDOA measurement from a pair of sensors, the noiseless measurement defines an hyperbola on which the emitter must lie and the additive measurement noise defines an uncertainty area around this hyperbola.

In the case of an arbitrary number of sensors, the emitter location is calculated by intersecting the hyperbolae from the TDOA measurements formed by pairing each sensor with the reference sensor. However in this paper, we restrict ourselves to the case of only two moving sensors. Because the TDOA is often relatively small, it is convenient to use the range difference of arrival (RDOA), calculated by multiplying the TDOA by \( c \). The measurement equation then takes on the form

\[
z_k = r_k^1 = c t_k^1 = (t_k^1 - t_k^1)c = r_k^1 - r_k^1 + w_k = h_k(x_k) + w_k
\]

(3)

where

\[
h_k(x_k) = \| x_k - s_k^1 \| - \| x_k - s_k^1 \|
\]

(4)

and \( w_k \sim N(0, 2\sigma^2) \).

The emitter considered in this paper is either stationary or uses a constant velocity model governed by

\[
x_k = F_{k-1} x_{k-1}
\]

(5)

where \( F \) is the state transition matrix. In the case of a stationary emitter, the emitter state is defined by the location of the emitter \( x_k = (x_k, y_k) \) and the state transition matrix is given by the two dimensional identity matrix, \( F_k = I_2 \).

For constant velocity emitter motion, the emitter state is defined by its position and velocity, \( x_k = [x_k \ ye_k \ y_k \ ye_k]^T \) and the state transition matrix is defined by \( F_k = \begin{bmatrix} I_2 & \Delta t_k \\ 0_2 & F_1 \end{bmatrix} \)

where \( F_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). Here \( \Delta t_k \) is the sampling interval of TDOA measurement \( z_k \) which depends on the emitter PRI geometry and emitter scanning patterns. The emitter periodically transmits a signal and if both UAVs are within the beamwidth of the emitter and are able to receive the same signal transmission, then a TDOA measurement can be generated. The sensors may use any motion model, provided that the sensor position is known at the arrival time of each detected signal.

At the fusion centre, TDOA measurements are generated by correlating samples of TOA measurements for the two UAVs, with the sampling interval selected to allow for sufficient UAV motion between sampling times. An outline of the process is presented in the next section.

Given TDOA measurement sets that have been generated over a number of stages, this paper compares two tracking algorithms in a high PRF or dense pulse environment. For a
stationary emitter, measures of performance for either tracker may include speed of geolocation and accuracy of the location estimates. Constraints for the tracking problem include communication bandwidth, UAV trajectories, and complexity of association algorithm.

III. MEASUREMENT ASSOCIATION AND TDOA GENERATION

For an ideal scanning radar with a narrow main beam and negligible side lobes, the main beam will envelope both UAVs over a small interval of time given by the intersection of the time intervals when the beam is over the individual UAVs. Thus, while each UAV can generate many measurements, only those measurements that fall within the intersection of the time intervals are relevant to the geolocation estimation problem.

Figures 1 and 2 show pulse trains under low and high PRF conditions, respectively, when both UAVs are within the radar beam. We assume that as the beam sweeps over the UAVs, the ES sensor on board each UAV collects time-of-arrival of the leading edge of each pulse detected as well as the GPS derived location of the UAV at that instant. These measurements are then packaged into data packets and transmitted to a common fusion centre where the task of generating the possible TDOA measurements is carried out.

Under both low and high PRF conditions, the UAVs may be able to collect data at a rate that is far beyond their capacity to transmit to the fusion centre. For the sake of simplicity, we assume that there is sufficient bandwidth and each measurement collected at each UAV is transmitted to the fusion centre.

The size of the region bounding the loitering UAVs is limited by the need to keep them within the beamwidth of the scanning beam (although this can be somewhat relaxed for a non-ideal beam where the sidelobes are not negligible), but within this region, the UAVs should be kept as far apart as possible to maximize the accuracy of the geolocation estimates. For any given pair of UAVs, the maximum magnitude for a candidate TDOA measurement is the time it takes a wave front to traverse the direct distance between the two UAVs. Based on this criterion, the process of TDOA measurement generation consists of picking a particular arrival time from one UAV and searching for possible arrival times of that pulse at the other UAV.

Figure 1 shows a pulse at measurement stage \( k \) whose arrival time is \( t_{k1}^{U1} \) and was sampled from the train of pulses arriving at UAV1. If the distance between the UAVs is \( d_{21} \) then any pulse that arrives at UAV2 within the interval \([t_{k1}^{U1} - \frac{d_{21}}{c}, t_{k1}^{U1} - \frac{d_{21}}{c}] \) could be the same pulse that was detected or seen by UAV1 at time \( t_{k1}^{U1} \). In this low PRF scenario, only one pulse at UAV2, the same pulse that was detected by UAV1, satisfies this condition and so only one TDOA measurement is generated. However, in Figure 2 where a similar situation is depicted for a high PRF scenario, up to four pulses, including the pulse that was detected at UAV1, can fall within this time interval giving rise to four candidate TDOA measurements with only one of them being a true TDOA measurement.

Results based on low PRF or sparse pulse environment were presented in [3]. In this paper we restrict ourselves to the high PRF or dense pulse environment.

IV. TDOA AND ASSOCIATED HYPERBOLA

TDOA is the difference in arrival times of a signal at two separate locations. For a network of two receivers, we have a single TDOA measurement that converts to a single range-difference measurement. Figure 3 shows a scanning radar E to be geolocated using a network of two ESM-equipped UAVs. Let \( U_i, i = 1, 2 \) be the UAVs and let \( D_{i,j} \), be the TDOA measured between \( U_2 \) and \( U_1 \). If \( c \) is the signal propagation speed, then

\[
r_{i+1,i} = cD_{i+1,i} = r_{i+1} - r_i; \quad i = 1
\]

where \( r_i \) denotes the distance between the transmitter and the \( i \)th UAV. Let \( U_i \) be at a known position \((x_k^i, y_k^i)\), \( i = 1, 2 \) and...
the transmitter unknown coordinates be \((x_k, y_k)\), then
\[
r_i^2 = (x_k - \eta_k^i)^2 + (y_k - \zeta_k^i)^2, \quad i = 1, 2
\] (7)
\[
r_{21} = cD_{21} = \sqrt{(x_k - \eta_k^1)^2 + (y_k - \zeta_k^1)^2 - (x_k - \eta_k^2)^2 + (y_k - \zeta_k^2)^2}
\] (8)

Now squaring until the square-root sign disappears, we obtain
\[
y_k^2(a_1 + a_2 + 4\zeta_k^1\zeta_k^2 - 2b - (\zeta_k^1 + \zeta_k^2)) + y_k(-2(\zeta_k^1a_2 + \zeta_k^2a_1) + 2b(\zeta_k^1 + \zeta_k^2) + a_1a_2 - b = 0
\] (9)

where
\[
a_1 = x_k^2 - 2x_k\eta_k^1 + \eta_k^1 + \zeta_k^{12}
\]
\[
a_2 = x_k^2 - 2x_k\eta_k^2 + \eta_k^2 + \zeta_k^{22}
\]
\[
b = x_k^2 - x_k(\eta_k^1 + \eta_k^2) + \frac{1}{2}(\eta_k^1 + \eta_k^2 + \zeta_k^{12} + \zeta_k^{22} - r_{21}^2).
\]
This is a quadratic equation in \(y_k\). Thus for a given range difference measurement and any value of \(x_k\) within the region of interest there are two possible target locations. Note that squaring the range difference destroys its sign and so this method generates two hyperbolae. The correct hyperbola is obtained by testing the generated points against range-difference using equation (8).

\[ \text{Figure 3. Emitter geolocation using a network of up to two ESM-equipped UAVs.} \]

**V. RANGE DIFFERENCE GAUSSIAN SUM APPROXIMATION**

For each range-difference measurement and its variance there is a unique hyperbola that defines the possible location of the emitter and a one-\(\sigma\) region around this hyperbola that represents the level of uncertainty of the possible emitter location. Figure 4 shows the hyperbola for a typical range difference measurement \(r_{21}\) given two netted UAVs and a radar emitter. Also shown is the one-\(\sigma\) region that is bounded by two hyperbolae based on \(r_{21} - \sigma_{21}/2\) and \(r_{21} + \sigma_{21}/2\). By segmenting the region bounded by the hyperbolae, it is possible to generate a set of 2-d measurement components that is a Gaussian sum approximation to the initial range-difference measurement. For each segment this conversion generates a Gaussian distribution whose mean is the centre of the segment and whose covariance is defined by a one-\(\sigma\) ellipse that bounds the segment. The weighted sum of these Gaussian distributions is the likelihood function. Let \(P_i, i = 1, \ldots, 4\) be the four points that define the \(j\)-th segment within the one-\(\sigma\) region and \(O_j\) be the centre of this segment, then the likelihood of \(x_k\) based on the Gaussian sum measurement approximation of \(z_k\) takes on the form
\[
p(z_k|x_k) = \sum_{j=1}^{M} \beta_j \mathcal{N}(x_k; \hat{x}_k^j, \mathbf{R}_k^j)
\] (10)

where \(\hat{x}_k^j\) is the centre of the \(j\)-th segment, \(\mathbf{R}_k^j\) is a covariance matrix derived from the \(j\)-th segment, \(\beta_k^j \propto \sqrt{|\mathbf{R}_k^j|}\) and \(\sum_{j=1}^{M} \beta_k^j = 1\), and \(M\) is the total number of components generated from measurement \(z_k\).

**VI. TRACKING ALGORITHMS**

The tracking algorithms presented here assume that the emitter is stationary but for the purpose of tracking it is modeled here as a constant velocity target with an additive zero-mean Gaussian process noise \(\nu_k\), with covariance \(\mathbf{Q}_k\).

**A. Tracking with unscented Kalman filter**

The UKF uses the unscented transform to deal with the nonlinear parts of the problem. Assume that an estimate of the target state \(\hat{x}_{k-1|k-1}\) and the covariance \(\mathbf{P}_{k-1|k-1}\) are available to the UKF at time \(k\). The state prediction at time \(k\):
is a linear process and takes on the form

\[ \dot{x}_{k|k-1} = F_{k-1}x_{k-1|k-1} \]
\[ P_{k|k-1} = F_{k-1}P_{k-1|k-1}F_{k-1}^T + Q_{k-1} \]

where \( F_k \) is the state transition matrix referred to in Section II and \( Q_k \) is the process noise covariance.

A set of sigma points, \( \chi^{(i)}_{k|k-1} \), are obtained from this prediction with weights \( W^{(i)} \) for \( i = 1, \ldots, 2N_z \), using the unscented transform described in [4], where \( N_z \) is the state dimension. The predicted measurement is given by

\[ \hat{z}_{k|k-1} = \sum_{i=0}^{2N_z} W^{(i)} h_k(\chi^{(i)}_{k|k-1}) \]

The measurement update proceeds as follows:

\[ \hat{x}_{k|h} = x_{k|k-1} + K_k(z_k - \hat{z}_{k|k-1}) \]
\[ P_{k|h} = P_{k|k-1} - K_k S_k K_k^T \]

where

\[ K_k = P_{zz}S_k^{-1} \]
\[ S_k = P_{zz} + R_k \]
\[ P_{zz} = \sum_{i=0}^{2N_z} W^{(i)} (\chi^{(i)}_{k|k-1} - \hat{x}_{k|k-1}) \]
\[ \times (h_k(\chi^{(i)}_{k|k-1}) - \hat{z}_{k|k-1})^T \]

The measurement pdf is split into fragments, where each fragment may introduce problems. One way of dealing with this problem is that each state estimate Gaussian sum element linearises the presentation [5] reduces the effects of measurement nonlinearity. The measurement Gaussian sum presentation avoids the limited memory [6].

\[ p(x_k|Z^{k-1}) = \sum_{c=1}^{C_k} \beta_k(c, g) \cdot p(x_k(c, Z^{k-1})) \]

\[ p(z_k|c, Z^{k-1}) = N(\hat{z}_k(c), S_k(c, g)) \]

\[ \hat{z}_k(c) = H\hat{x}_{k|k-1}(c) \]

\[ S_k(c, g) = HP_{k|k-1}(c)H^T + R_{k,g} \]

where \( C_k \) denotes the number of components, and \( \xi_{k-1}(c) \) denotes the probability that measurement history of component \( c \) is correct, given target existence, at the beginning of the scan update.

The \textbf{a priori} pdf of \( g \)th component of measurement \( z_k \) given target trajectory state estimate component \( c \) is given by

\[ p_g(z_k|c, Z^{k-1}) = N(\hat{z}_k(c), S_k(c, g)) \]

\[ \beta_k(c, g) = K^{-1} \gamma_g \xi_{k-1}(c) p_g(z_k|c, Z^{k-1}) \]

\[ \sum_{c,g} \beta_k(c, g) = 1 \]

and

\[ p(x_k(c, g)) = \sum \beta_k(c, g) \cdot p(x_k(c, g), z_k, Z^{k-1}) \]

\[ = N(\hat{x}_k(c), \hat{z}_{k|k}(c, g), P_{k|k}(c, g)) \]
where $\hat{x}_{k|k}(c, g)$, $P_{k|k}(c, g)$ are obtained by applying Kalman filter to state estimate prediction component $c$ using measurement component $g$. Each pair \{ existing component, measurement component \} generates a new component for the next scan, with probabilities $\xi_k(c)$ equal to the corresponding $\beta$ item from equation (30). The number of components at scan $k + 1$ equals

$$C_{k+1} = C_k \cdot G.$$  

(34)

The number of components grows exponentially, and their number must be controlled. A number of techniques [10], [11] exists to control the number of component; from pruning the components with low probability $\xi(c)$, to subtree pruning, to sophisticated merging [12]. Component control is not part of the ITS algorithm itself, however it is a necessity in any practical implementation.

VII. NUMERICAL RESULTS

The numerical results presented here are based on three netted UAVs each of which is equipped with an ES sensor and a GPS receiver. The range standard deviation of the ES sensor is $\sigma_r = 5.0$ m. The three vehicles, UAV1, UAV2, and UAV3, are initially located at (80,57) km, (96,55) km, and (112,50) km, respectively, with UAV2 used here as a reference. They travel at a constant speed of 300km/hr in a southerly direction.

The emitter is a stationary scanning radar that rotates at 24 revolutions per minute and is located at (20,20) km. We assume a main beam with a beam width of 8 degrees and negligible side lobes. Furthermore, we assume a high PRF or a dense pulse environment.

Under the high PRF or dense pulse scenario, it operate with a PRF of 37.5kHz or a PRI of $2.6667 \times 10^{-5}$ sec, which converts to wavefronts that are separated by 8 km. With a minimum separation of 16 km maintained between neighboring UAVs, and in the absence of clutter, a single pulse at UAV2 can correlate with up to 4 pulses arriving at UAV1. This yields 4 candidate TDOA measurements within each measurement set with only one of them being a genuine or target originated TDOA measurement. Thus at the fusion centre, samples of TDOA measurements are generated by picking an arrival time from UAV2 and searching for arrival times from UAV1 that could have been generated by the same pulse. They are then converted to a predetermined number of measurement components based the Gaussian sum approximation.

The numerical results that follow are based on two experiments. In Experiment 1 both trackers were allowed to process up to ten sets of TDOA measurements sampled over two rotations of the beam but based only on arrival time measurements from pair UAV1 and UAV2. This processing was achieved over the time interval [0.426279, 2.933612]. In Experiment 2, the trackers were restricted to samples that fell within the first rotation of the beam but were allowed to switch inputs. In the first half of the run, five TDOA measurements were provided from the pair UAV1 and UAV2 while in the second half, six TDOA measurements came from the pair UAV2 and UAV3. All 11 measurements were generated over the time interval [0.426279, 0.481399] seconds. Minimum sampling interval for both experiments was fixed at 1.5 milliseconds. Figure 5 shows 80 Gaussian components generated from the first set of four TDOA measurements. Using one-point initialisation this are also the initial track components for each of the trackers. Figure 6 shows target distribution in the x-y plane after the processing of a second set of TDOA measurements by the GSM tracker.

Clearly, after a sufficient number of TDOA measurements, all track components not in the vicinity of the emitter eventually get pruned out. Thus only tracks that are initiated in the vicinity of the emitter are able to survive and it is
their branches that congregate in the vicinity of the emitter after a sufficient number of TDOA measurement sets have been processed. Figure 7 shows the highest probability track component after the processing of ten TDOA measurement sets following two rotations of the beam. This is the final track estimate under Experiment 1. Note here that results from both experiments are identical over the first five measurement stages.

Figure 8 shows the highest probability track at stage $k = 5$ (just before the switch-over) and Figure 9 shows the highest probability track at $k = 6$ (just after the switch-over). Note the dramatic reduction in the size of the one-$\sigma$ ellipse following the switch-over. Figure 10 shows a collection of hyperbolae from all 11 TDOA measurement sets (five before and six after the switch-over) processed during Experiment 2.

For the bank of unscented Kalman filters, none of the track components initiated in the vicinity of the emitter attained a significant weight during Experiment 1. During the Experiment 2 however, track component 19 initiated near the emitter eventually attained the highest probability but only after the switch-over. Figure 11 shows all track probabilities or weights under Experiment 2.

Figure 12 and 13 show probability of convergence and root-mean-square errors, respectively. They were obtained from Monte-Carlo simulations of the GSM tracker for the two
Figure 11. Probabilities of all 80 track components after the processing of 11 TDOA measurements by the UKF based tracker under Experiment 2.

Figure 12. Monte-Carlo generated convergence probabilities of highest probability track component and highest probability cluster from the GSM tracker.

Figure 13. Monte-Carlo generated rms errors of highest probability track component and highest probability cluster from the GSM tracker.

VIII. CONCLUSIONS

Given the highly nonlinear problem of emitter localisation using cluttered TDOA measurements from a pair of netted UAVs, we have compared a nonlinear tracker that employs a bank of unscented Kalman filters to a new linear tracker that employs a bank of Kalman filters and is based on the Gaussian sum approximation. We have presented numerical results from both trackers based on measurements generated under Experiment 1, and Experiment 2. Results show that while Experiment 2 requires the presence of a strategicallyplaced third UAV, the numerical results are vastly superior to those obtained from Experiment 1. Furthermore, Experiment 2 results were obtained over a time interval of 0.055 seconds compared to 2.509 seconds for Experiment 1. Thus while the trackers presented here can only process measurements from a single UAV pair at any time, the presence of a second independent UAV pair can dramatically improve estimation accuracy and speed.

REFERENCES