Comparison of Data Association Algorithms for Bearings-only Multi-sensor Multi-target Tracking

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Abstract—In multi-sensor multi-target bearings-only tracking we often see false intersections of bearings known as ghosts. When the bearing measurements from each sensor have been associated to form sequences termed threads, the problem is to associate pairs of threads to identify the true target intersections. In this paper we present two algorithms: (i) Classical Bayesian Thread Association (CBTA) and (ii) Monte Carlo Thread Association (MCTA), for this problem. The performance of these algorithms is compared using Monte Carlo simulations. Furthermore, we also compare their performance against the Rao-Blackwellised Monte Carlo Data Association (RBMCDA) algorithm, which uses unthreaded measurements, in order to ascertain the benefits of using thread information. Simulations show that MCTA is superior to CBTA, and that there is significant benefit in using thread information in this class of problems.

Keywords: Data Association, Multi-target Tracking, Ghost Elimination

I. INTRODUCTION

In multi-sensor multi-target bearings-only tracking, before we can fully exploit the presence of more than one sensor, we must be able to associate the measurements from different sensors. This is an important problem because having measurements from multiple sensors allows us to use triangulation to localise the targets faster than is possible using a single sensor, where observability is a significant problem.

Several researchers have investigated the data association problem for both active and passive tracking. Some of these algorithms include the Joint Probabilistic Data Association (JPDA) [10], Multiple Hypothesis Tracking (MHT) [11] and S-D assignment algorithms [8], [9]. The JPDA and MHT based algorithms have been used mostly in single sensor (active and passive) and multi-sensor active cases. Although the S-D assignment based algorithms have been used in multi-sensor passive tracking, their use has been restricted to cases of more than two sensors. Recently a Monte Carlo data association approach termed Rao-Blackwellised Monte Carlo Data Association (RBMCDA) was proposed in [1], which was able to handle the difficult case of two sensor multi-target passive data association.

There are many situations in which the sensors are able to associate their own measurements between successive scans, thereby producing sequences of associated measurements termed threads. These threads consist of measurements which are known to originate from just one target. In such cases, attempting to triangulate these threads will often lead to false targets (also referred to as ghost targets). The data association problem in this case reduces to choosing pairs of threads that correspond to particular targets.

In this paper we propose two algorithms to solve the thread association problem for multi-sensor bearings-only tracking: (i) Classical Bayesian Thread Association (CBTA) and (ii) Monte Carlo Thread Association (MCTA). The CBTA algorithm is based on a Bayesian data association approach, which recursively computes the probabilities of the thread association hypotheses. The MCTA algorithm is a Monte Carlo based approach similar to RBMCDA [1], and it represents the target states and association hypotheses within a particle filter framework. Both of these algorithms make use of single target bearings-only state estimators such as the Extended Kalman Filter (EKF) and the Shifted Rayleigh Filter (SRF) [2], [3] to calculate measurement likelihoods.

We first compare the performance of the above data association algorithms to ascertain the benefits of using a Monte Carlo based approach over a classical Bayesian data association method. We then compare the performance of these algorithms with that of the RBMCDA algorithm in order to assess the advantage of using the thread information.

The organisation of the paper is as follows. Section II describes the data association problem for multi-sensor multi-target tracking. Section III reviews single target bearings-only algorithms based on EKF and SRF. In Section IV, data association algorithms for threaded data are presented, while Section V reviews the RBMCDA algorithm for unthreaded data. Simulation results are then presented in Section VI, followed by some concluding remarks in Section VII.

II. PROBLEM FORMULATION

In this section we present the data association problem for the case of multi-target tracking with two bearings-only sensors. Though in theory the principles described in Section IV can be applied to an arbitrary number of targets, due to computational constraints, the algorithms presented can only be implemented for a small number of targets. Furthermore, it is likely that approaches based on only two sensors would be insufficient for a large number of targets, leading to the
need for more sensors. This gives rise to an N-dimensional assignment problem (where N is the number of sensors), which requires a different class of algorithms to solve. For this reason, and to simplify explanation of the problem, we restrict our attention to the case of two constant velocity targets, as shown in Figure 1. Suppose a target, located at coordinates \((x_k, y_k)\) at time \(t_k\) (will be referred to as time \(k\) in the sequel), moves with a nearly constant velocity vector \((\dot{x}_k, \dot{y}_k)\). We define its state to be \(x_k = (x_k, y_k, \dot{x}_k, \dot{y}_k)\), which evolves according to the dynamics

\[
x_k = A x_k + v_k
\]

(1)

where \(v_k\) is a \(4 \times 1\) i.i.d process noise vector with \(v_k \sim \mathcal{N}(0, Q)\). Here \(\mathcal{N}(m, P)\) denotes a Gaussian density with mean \(m\) and covariance \(P\). The state transition matrix \(A\) and process noise covariance matrix \(Q\) are

\[
A = \begin{bmatrix}
1 & 0 & T_k & 0 \\
0 & 1 & 0 & T_k \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
Q = \begin{bmatrix}
\frac{\sigma^2}{4} & 0 & \frac{\sigma^2}{2} & 0 \\
0 & \frac{\sigma^2}{4} & 0 & \frac{\sigma^2}{2} \\
\frac{\sigma^2}{2} & 0 & T_k & 0 \\
0 & \frac{\sigma^2}{2} & 0 & T_k \\
\end{bmatrix}
\]

where \(T_k = t_k - t_{k-1}\) is the time between the previous and current measurement, and \(\tilde{q}\) is a scalar continuous time process noise intensity. Let the position of sensor \(j\) be defined as \(s^j = (x^j, y^j)\), which is assumed to be known exactly. The measurements from sensor \(j\) are modeled as

\[
z_k = h(x_k, s^j) + w_k
\]

(2)

where \(w_k\) is a zero-mean independent Gaussian noise with variance \(\sigma^2\) and

\[
h(x_k, s^j) = \arctan \left( \frac{x_k - x^j}{y_k - y^j} \right)
\]

(3)

is the true bearing. In the simulations carried out, we assume a clutter-free case and only consider target-generated measurements given by (3). In most parts of this paper, we assume that for each sensor, measurements from a particular target have been associated and this sequence is termed a thread. That is, we have four threads (two from each sensor) and the measurement at time \(k\) from thread \(\beta_k\) is denoted \(z_{k}^{\beta_k}\). Thus, each measurement is tagged with its thread number \(\beta_k\), \(\beta_k \in \{1, \ldots, 4\}\) and the measurements up to time \(k\) is denoted

\[
z_{1:k} = \{z_{1}^{\beta_1}, \ldots, z_{k}^{\beta_k}\}. \tag{4}
\]

We wish to triangulate the positions of the targets, but in order to do this we first need to overcome the ghost phenomenon, which means that there are more intersection points than targets. Without additional information, it is impossible to judge which intersection points correspond to real targets, and which ones are ghosts. To an extent it is possible to overcome this problem by adding more sensors, because a false crossing of three or more bearings is quite unlikely [8]. However it is still possible for ghosts to appear, and it may be desirable to deploy only a minimum number of sensors.

In this scenario there are two possible hypotheses defined as follows:

\[
\mathcal{H}_1 \triangleq 1 \leftrightarrow 3 \text{ and } 2 \leftrightarrow 4
\]

\[
\mathcal{H}_2 \triangleq 1 \leftrightarrow 4 \text{ and } 2 \leftrightarrow 3
\]

where \(r \leftrightarrow s\) indicates that thread \(r\) is associated with thread \(s\), that is, they both represent measurements from the same target. In order to choose the correct hypothesis, we wish to compute the probabilities \(P(\mathcal{H}_n|z_{1:k})\), \(n = 1, 2\) for these hypotheses.

Algorithms developed for the above case (associated measurements) will also be compared with the case of unassociated measurements. For this case, the measurement sequence is denoted by \(z_{1:k} = \{z_1, \ldots, z_k\}\) without the thread numbers. However, note that each measurement has an implicit tag giving its sensor number.

III. SINGLE TARGET BEARINGS-ONLY TRACKING

All three data association algorithms presented in this paper require tracking individual targets using bearing measurements, so in this section we review two methods for doing so, the Extended Kalman Filter (EKF) and the Shifted Rayleigh Filter (SRF). Section VI contains the results of using both the EKF and the SRF within the three data association algorithms.

A. Extended Kalman Filter

The EKF is a general algorithm for non-linear tracking, and is the most traditional method for tracking a single target using bearings-only measurements. The algorithm falls into the class of moment matching filters, and models the state distribution as a Gaussian, the estimated mean and covariance of which are calculated on the arrival of each new measurement. The non-linearity in the measurement equation is dealt with by linearizing about the predicted state estimate. As well as generating estimates of the state distribution, we can also use the EKF to generate measurement likelihoods, which
are needed to calculate the hypothesis probabilities for data association. Suppose the state estimate and its covariance at time \( k-1 \) are given by \( \mathbf{x}_{k-1} \) and \( \mathbf{P}_{k-1} \) respectively. Then the EKF for bearings only tracking is defined by the following equations.

**Prediction:**

\[
\begin{align*}
\hat{\mathbf{x}}_{k|k-1} &= \mathbf{A}\mathbf{x}_{k-1|k-1} \\
\mathbf{P}_{k|k-1} &= \mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^T + \mathbf{Q}
\end{align*}
\]

**Update:**

\[
\begin{align*}
\mathbf{z}_{k|k-1} &= \mathbf{h}(\mathbf{x}_{k|k-1}, \mathbf{s}^T) \\
\mathbf{H}_k &= \frac{\partial \mathbf{h}(\mathbf{x}, \mathbf{s}^T)}{\partial \mathbf{x}}|_{\mathbf{x} = \mathbf{x}_{k|k-1}} \\
\mathbf{S}_k &= \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R} \\
\mathbf{G}_k &= \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1} \\
\mathbf{v}_k &= \mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1} \\
\mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{G}_k\mathbf{H}_k)\mathbf{P}_{k|k-1}
\end{align*}
\]

**Measurement Likelihood:**

\[
P(\mathbf{z}_k|\mathbf{x}_{k|k-1}, \mathbf{P}_{k|k-1}) = \frac{1}{\sqrt{2\pi|\mathbf{S}_k|}} e^{-\frac{1}{2}v_k^T\mathbf{S}_k^{-1}\mathbf{v}_k} \tag{14}
\]

**B. Shifted Rayleigh Filter**

The SRF is an algorithm designed especially for the bearings-only tracking problem. Like the EKF, it is a moment matching filter which generates a Gaussian approximation to the state distribution that is updated with each new measurement. As such, it requires similar computation time to the standard EKF. However, it has been shown to outperform the EKF in certain scenarios, as it exploits the particular structure of the non-linearities encountered in bearings only tracking. The algorithm is summarised by the following equations, and the reader is referred to [2, 3] for more details.

**Prediction:**

\[
\begin{align*}
\hat{\mathbf{x}}_{k|k-1} &= \mathbf{A}\hat{\mathbf{x}}_{k|k-1} \\
\mathbf{P}_{k|k-1} &= \mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^T + \mathbf{Q}
\end{align*}
\]

**Update:**

\[
\begin{align*}
\mathbf{Q}_k^n &= \sigma^2 \left( ||\hat{\mathbf{H}}\hat{\mathbf{x}}_{k|k-1}||^2 + \sigma_x^2 + \sigma_y^2 \right) \mathbf{I}_{2\times2} \\
\mathbf{V}_k &= \hat{\mathbf{H}}\mathbf{P}_{k|k-1}\hat{\mathbf{H}}^T + \mathbf{Q}_k^n \\
\mathbf{K}_k &= \mathbf{P}_{k|k-1}\hat{\mathbf{H}}^T\mathbf{V}_k^{-1} \\
\mathbf{b}_k &= \left[ \sin(\zeta_k) \cos(\zeta_k) \right]^T \\
\zeta_k &= \beta_k^n - 2\zeta_k \rho(\zeta_k) + \rho(\zeta_k)^2 \\
\rho(\zeta_k) &= \sqrt{2\pi(\zeta_k^2 + 1)}F_{\text{normal}}(\zeta_k) \\
\gamma_k &= (\mathbf{b}_k\mathbf{V}_k^{-1}\mathbf{b}_k)^{-\frac{3}{2}}\rho(\zeta_k) \\
\delta_k &= (\mathbf{b}_k\mathbf{V}_k^{-1}\mathbf{b}_k)^{-1}(2 + \zeta_k\rho(\zeta_k) - \rho(\zeta_k)^2) \\
\mathbf{x}_{k|k} &= (\mathbf{I} - \mathbf{K}_k\hat{\mathbf{H}})\hat{\mathbf{x}}_{k|k-1} - \mathbf{K}_k\mathbf{b}_k + \gamma_k\mathbf{K}_kb_k \\
\mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k\hat{\mathbf{H}})\mathbf{P}_{k|k-1} + \delta_k\mathbf{K}_kb_k\mathbf{b}_k^T\mathbf{K}_k^T
\end{align*}
\]

In the above, \( F_{\text{normal}} \) is the cumulative distribution function of a standard Gaussian variable, \( u_k^n = (-x_k^n, -y_k^n) \) gives the sensor position, \( \sigma_x^2 \) and \( \sigma_y^2 \) are the one-step prediction variances of \( \hat{x}_{k|k-1} \) and \( \hat{y}_{k|k-1} \) respectively, and

\[
\hat{\mathbf{H}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.
\]

The measurement likelihood calculation is the same as the EKF, that is, we feed \( \mathbf{x}_{k|k-1} \) and \( \mathbf{P}_{k|k-1} \) calculated by the SRF into (14).

**IV. ASSOCIATION ALGORITHMS FOR THREADED DATA**

**A. Classical Bayesian Thread Association**

This section describes an algorithm that recursively computes the probabilities of hypotheses \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) defined in Section II. Suppose that we have the hypothesis probabilities \( P(\mathcal{H}_n|z_{1:k-1}, n = 1, 2) \) at time \( k-1 \). When a new threaded measurement \( z_{k}^{\beta_k} \) arrives, it is used to update the hypothesis probabilities using Bayes recursion as follows.

\[
P(\mathcal{H}_n|z_{1:k}) = \frac{P(z_{k}^{\beta_k}|\mathcal{H}_n)P(\mathcal{H}_n|z_{1:k-1})}{\sum_{i=1}^{2} P(z_{k}^{\beta_i}|\mathcal{H}_i)P(\mathcal{H}_i|z_{1:k-1})} \tag{27}
\]

We now describe how the likelihoods \( P(z_{k}^{\beta_k}|\mathcal{H}_n) \) are computed. Suppose we have four filters \( \mathcal{F}_\ell, \ell = 1, \ldots, 4 \), each with an initial position corresponding to the intersection of two threads. These filters could be EKFs or SRFs. Two of these filters estimate target states corresponding to hypothesis \( \mathcal{H}_1 \), while the other two target states corresponding to hypothesis \( \mathcal{H}_2 \). By making the simplifying assumption that initially we obtain measurements from all four threads simultaneously, we can use them to initialise the filters as follows.

Let \( \chi(r, s) \), denote the intersection of measurements from threads \( r \) and \( s \), and let \( \mathbf{x}_{r,s}^{\beta_k} \) denote the position estimate of filter \( \mathcal{F}_\ell \). Then \( \mathbf{x}_{r,s}^{\beta_k} \) is set as follows.

\[
\mathbf{x}_{r,s}^{\beta_k} = \begin{cases} 
\chi(1, 3), & \ell = 1 \\
\chi(2, 4), & \ell = 2 \\
\chi(1, 4), & \ell = 3 \\
\chi(2, 3), & \ell = 4 
\end{cases}
\tag{28}
\]

We also define another function \( \Psi \) which maps a particular hypothesis and thread number to one of the filters defined above.

\[
\Psi(\mathcal{H}_n, \beta) = \begin{cases} 
1 & n = 1, \text{ and } (\beta = 1 \text{ or } \beta = 3) \\
2 & n = 1, \text{ and } (\beta = 2 \text{ or } \beta = 4) \\
3 & n = 2, \text{ and } (\beta = 1 \text{ or } \beta = 4) \\
4 & n = 2, \text{ and } (\beta = 2 \text{ or } \beta = 3)
\end{cases}
\tag{29}
\]

Given these definitions, the likelihoods can be computed as

\[
P(z_{k}^{\beta_k}|\mathcal{H}_n) = P(z_{k}^{\beta_k}|\mathbf{x}_{k|k-1}^{\beta_k}, \mathbf{P}_{k|k-1}) \tag{30}
\]

Here the right hand side of (30) is the likelihood of measurement \( z_{k}^{\beta_k} \) computed using filter \( \mathcal{F}_\ell \), where \( \ell = \Psi(\mathcal{H}_n, \beta_k) \).
B. Monte Carlo Thread Association

This algorithm is similar to that proposed in [1], but instead of treating each measurement independently, we now use measurement threads which have already been associated with a particular target.

Every particle represents one of the hypotheses $H_1$ or $H_2$ defined earlier. The definition of a particle is similar to [1], except that we include the hypothesis number that will be used in identifying the filter that needs updating when a threaded measurement $z_k$ arrives. This hypothesis number is set during the initialisation and remains fixed for each particle. Thus, the $i$-th particle, denoted $\Upsilon^{(i)}_k$, has the form

$$\Upsilon^{(i)}_k = \left\{ m^{1,(i)}_k, P^{1,(i)}_k, m^{2,(i)}_k, P^{2,(i)}_k, w^{(i)}_k, H^{(i)}_k \right\}$$  \hspace{1cm} (31)

where $H^{(i)}_k \in \{1, 2\}$. The means and covariances of each particle are initialised as follows. Let the notation

$$(r \leftrightarrow s) \mapsto (m, P)$$

denote that the state estimate $m$ and its covariance $P$ are generated using the intersection of threads $r$ and $s$, a method for which is described in [6]. Then, the means and covariances of the $i$-th particle are initialised according to:

Hypothesis $H^{(i)}_k = 1$

$$(1 \leftrightarrow 3) \mapsto (m^{1,(i)}_0, P^{1,(i)}_0)$$

$$(2 \leftrightarrow 4) \mapsto (m^{2,(i)}_0, P^{2,(i)}_0)$$

Hypothesis $H^{(i)}_k = 2$

$$(1 \leftrightarrow 4) \mapsto (m^{1,(i)}_0, P^{1,(i)}_0)$$

$$(2 \leftrightarrow 3) \mapsto (m^{2,(i)}_0, P^{2,(i)}_0)$$

If we have no prior information about the target locations, then each of the two association hypotheses are equally likely and we initialise half of the particles to $H_1$ and the other half to $H_2$. If either hypothesis is initially more likely than the other, we would adjust the ratio of $H_1$ and $H_2$ particles accordingly. One recursion of the particle filter for this problem is described below.

Suppose we have a set of particles $\Upsilon_{k-1}^{(i)}$ that characterise the states of the two targets and the hypotheses, given measurements up to time $k - 1$. When a new measurement $z_k$ arrives, the particles are updated as follows:

- For each particle $\Upsilon^{(i)}_{k-1}$, compute the filter number $d \in \{1, 2\}$ that needs updating with measurement $z_k$:

$$d = \text{mod}(\Psi(H^{(i)}_k, \beta_k) + 1, 2) + 1$$  \hspace{1cm} (32)

- Compute the unnormalised particle weights:

$$w^{(i)'}_k = w^{(i)'}_{k-1} \times P(z_k^{\beta_k} | m^{d,(i)}_k, P^{d,(i)}_k)$$  \hspace{1cm} (33)

where $P(z_k^{\beta_k} | m^{d,(i)}_k, P^{d,(i)}_k)$ is the measurement likelihood associated with filter $d$ (which corresponds to target $d$).

- Update the mean and covariance of filter $d$ with $z_k^{\beta_k}$

$$m^{d,(i)}_k = \frac{w^{(i)'}_k m^{d,(i)}_{k-1} + z_k^{\beta_k} \sigma_{\text{est}}^{d,(i)}(z_k^{\beta_k})}{w^{(i)'}_k}$$

- Update the means and covariances of each particle using the EKF or SRF.

- Normalise the particle weights so they sum to unity.

$$w^{(i)}_k = \frac{w^{(i)'}_k}{\sum_{i=1}^{N} w^{(i)'}_k}$$  \hspace{1cm} (34)

- Calculate the effective number of particles.

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} (w^{(i)}_k)^2}$$  \hspace{1cm} (35)

- If the effective number of particles is less than some predetermined threshold, resample the particles and reset all the weights to $\frac{1}{N}$.

Note that this algorithm can be regarded as a special case of the RBMCD algorithm described in Section V. In particular, given a threaded measurement, the measurement to target association for each particle is determined by the particle’s hypothesis number, instead of being sampled from an importance distribution.

V. AN ASSOCIATION ALGORITHM FOR UNTHREAD DATA

In this section we describe a data association algorithm for the case of unthreaded data. The algorithm is termed Rao-Blackwellised Monte Carlo Data Association (RBMCD), and is described in detail in [1]. We review the key features of this algorithm below.

A. Rao-Blackwellised Monte Carlo Data Association

This algorithm was proposed in [1] to solve the joint data association and tracking problem in which all measurements are a priori independent, i.e. there are no measurement threads. The number of targets is assumed to be known, and each particle consists of the state mean and covariance for each target, as well as the particle weight. The means and covariances are maintained using a single target tracking algorithm such as the EKF or SRF. The problem is therefore split into two subproblems, data association and single target tracking.

The data association problem is solved by particle filtering, and the single target tracking problem is solved conditional on the data associations by either EKF or SRF. This procedure, in which part of the problem is solved by closed form filtering and part by Monte Carlo sampling, is called Rao-Blackwellisation. Here, the state distribution is represented as a mixture of Gaussians, which can be made more accurate than a finite set of discrete points. A brief description of the algorithm as applied to our two target, two sensor scenario is given here, and the interested reader is referred to [1] for more detail.

Each particle maintains two filters, one for each target, and the $i$-th particle at time $k$ can be represented as

$$\left\{ m^{1,(i)}_k, P^{1,(i)}_k, m^{2,(i)}_k, P^{2,(i)}_k, w^{(i)}_k \right\}$$

where $m^{1,(i)}_k$ and $P^{1,(i)}_k$ are the mean and covariance of the filter output corresponding to the $t$-th target, and $w^{(i)}_k$ is the associated weight. The particles are initialised based on the
bearing intersections as described in the previous section. Since we know nothing about the target locations a priori, half the particles correspond to ghost target intersections, and the weights are initialised to \( \frac{1}{N} \), where \( N \) is the number of particles.

When a new measurement \( z_k \) arrives, we update the particles using Sequential Importance Resampling (SIR) as follows.

- Predict the mean and covariance for all targets forward to the measurement time \( k \):
- Calculate the normalised optimal importance distribution for the measurement to target association using the measurement likelihood function and the prior association probability. The variable \( c_k \) represents an association event which takes on the value 1 if the measurement is associated with target 1, and 2 if the measurement is associated with target 2.

\[
p(c_k^i | z_{1:k}, c_{1:k-1}) \propto p(z_k | c_k, z_{1:k-1}, c_{1:k-1})p(c_k)
\]  

(37)

- Sample an association for the new measurement from the optimal importance distribution.

\[
c_k^i \sim p(c_k^i | z_{1:k}, c_{1:k-1})
\]  

(38)

- Update the associated target’s mean and covariance using the new measurement.
- Calculate the new particle weight.

\[
w_k^i \propto w_{k-1}^i \times \frac{p(z_k | c_k^i, z_{1:k-1}, c_{1:k-1})p(c_k^i | c_{1:k-1})}{p(c_k^i | z_{1:k}, c_{1:k-1})}
\]  

(39)

The particle weights are then normalised, and the effective number of particles is calculated.

\[
N_{eff} = \frac{1}{\sum w_k^i (w_k^i)^2}
\]  

(40)

If the effective number of particles is less than some threshold, we perform resampling and reset the particle weights to \( \frac{1}{N} \).

VI. SIMULATION RESULTS

In this section we compare the performance of these data association algorithms on three scenarios of varying levels of difficulty. All scenarios contain two stationary bearing-only sensors, such as sonobouys, and two constant velocity targets. The difficulty of a given scenario can be characterised by plotting the aggregate acceleration of the ghost intersections under zero sensor noise conditions. If one or both ghosts have large accelerations, then a good algorithm should easily eliminate them. On the other hand, if the ghosts appear to have near constant velocity, the algorithm has no basis on which to distinguish between the ghosts and the real targets. The target and sensor parameters for the three scenarios are given in Table I.

Scenario 1 is a relatively easy scenario in which the ghost tracks exhibit quite a large acceleration as shown in Figure 2. Scenario 2 is of moderate difficulty as the accelerations of the ghost tracks are smaller (see Figure 4), and scenario 3 is very difficult with the ghost tracks showing almost constant velocity (see Figure 6).

We compare the results of the three data association algorithms; CBTA, MCTA and RBMCDa, and we run each using the EKF and SRF as the single target state estimator, giving a total of six algorithms. For the particle filter based algorithms, the number of particles is set to 100, and in all cases the prior hypothesis probabilities are set to 0.5.

To evaluate the performance of the six algorithms, the probability of correct association is calculated at the end of each scan, and averaged over 100 Monte Carlo runs. For CBTA, this is simply the probability \( P(\mathcal{H}_1) \), which is a direct output of the algorithm. For RBMCDa, we record the measurement to target associations at every time step, and the probability is the sum of the weights of the particles that have the correct associations. For the MCTA, the probability is given by the sum of the weights of the \( \mathcal{H}_1 \) particles. The results for the three scenarios are shown in Figures 3, 5 and 7 respectively.

For the easy scenario, we find that all algorithms perform about the same, locking on to the correct association hypothesis after about 300 seconds and maintaining that hypothesis for the rest of the scenario.
For the moderate scenario, all algorithms take longer to converge on the solution due to the lower acceleration of the ghost tracks. In this case, the MCTA algorithm performs substantially better than RBMCDA, so the benefits of using the thread information can be clearly seen. The advantage of using the particle filter methods is also apparent, as these tend to exhibit more consistent performance than CBTA, particularly at the beginning of the scenario. The CBTA-EKF algorithm favours the incorrect solution for the first half of the scenario, but then finds the correct solution once more data is obtained. On the other hand, MCTA never favours the incorrect solution but instead shows a slow but steady rise in probability from the beginning. In the context of the CBTA algorithm, the SRF performs significantly better than the EKF, rivalling the performance of the more expensive MCTA. This demonstrates the advantages of using the SRF to obtain the sufficient statistics for classical data association, however, the benefits are quite marginal in the case of the particle filter methods.

For the difficult scenario, MCTA shows a very slow rise in probability and only reaches about 70\% by the end of the scenario. The RBMCDA algorithm shows no rise in association probability, suggesting that it is incapable of distinguishing between the real targets and ghost targets in this scenario. This is expected due to the very low acceleration of the ghost tracks as illustrated by Figure 6. The CBTA algorithm also fails in this scenario, although it shows slight improvement towards the end as more measurements become available.

These results show that the choice of single target state estimator appears to make very little difference when using particle filtering to solve the data associations, but in the classical approach using the SRF may have a significant benefit depending on the type of scenario.

A note on the computational complexity of these algorithms is in order here. The CBTA algorithm is by far the cheapest of the three as it uses only a small number of moment-matching tracking filters to feed the hypothesis probability calculations. The RBMCDA algorithm is the most expensive, and in the case of 100 particles was about 58 times slower than CBTA due to the large number of filters that need updating within the Monte Carlo framework. The MCTA algorithm fell in between, taking 26 times longer than CBTA due to the Monte Carlo sampling, but only half as long as RBMCDA because of the simplified logic in selecting the measurement to target association for each particle.

VII. CONCLUSIONS

In this paper we presented two algorithms to solve the thread association problem for multi-sensor multi-target bearings-only tracking: (i) Classical Bayesian Thread Association (CBTA), and Monte Carlo Thread Association (MCTA). These algorithms made use of single target bearings-only state estimators such as the Extended Kalman Filter and the Shifted Rayleigh Filter, to calculate measurement likelihoods. The
Simulations show that the MCTA algorithm outperforms the CBTA algorithm, particularly in moderate to difficult scenarios, demonstrating the benefits of using particle filtering to solve this problem. Furthermore, the MCTA algorithm was superior to RBMCDA in difficult scenarios, which suggests that there are significant performance gains to be made if thread information is available. The SRF based algorithms outperformed the EKF based algorithms, with the most significant difference being seen in the context of CBTA. In this case, using the classical data association algorithm with the SRF to estimate the individual targets can potentially give much better performance than the EKF, depending on the type of scenario.

REFERENCES