A Novel Framework for the Network-wide Distributed Detection Problem

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Abstract - This paper presents a new framework for distributed target detection in wireless sensor networks (WSNs). In our previous work, for multiple networked sensors collaboratively detecting the presence or absence of a target in the sensor field, every sensor uses an identical threshold for local decision-making. In this paper, we propose a framework where the sensors in the network collaboratively decide and select non-identical thresholds to improve network-wide detection performance in a dynamic manner. This threshold selection scheme is based on a new statistical metric called False Discovery Rate (FDR). Assuming a signal attenuation model, where the received signal power decays as the distance from the target increases, various performance indices like system level probability of detection and probability of false alarm are studied. Analytical and simulation results are provided for system level probability of false alarm and probability of detection. Performance comparison between the proposed approach and the classical identical local sensor threshold approach is provided to demonstrate the effectiveness of this scheme.

Keywords: Wireless sensor networks, distributed detection, decision fusion, false discovery rate.

1 Introduction

Recent technological advances have enabled the deployment of a large number of networked sensors to continuously observe an event area for reliable estimation/detection/tracking of events. One particularly interesting problem for wireless sensor networks (WSNs) is distributed target detection. In distributed detection, each sensor instead of sending its raw data sends quantized data to the fusion center to minimize power and bandwidth consumptions. The literature on distributed detection is quite extensive and numerous papers exist which deal with various aspects of distributed detection [1,2]. For the conventional distributed detection problem, where multiple sensors, e.g., radars, observe a single target and receive noisy versions of the same target signal, optimum fusion rule has been derived under the conditional independence assumptions [1,3]. Fusion rules under correlated sensor observations have been studied in [4,5,6].

Decision fusion for target detection in WSNs, where the received signal power decays as the distance from the target increases, has been investigated in [7,8,9,10]. In this problem, as the probability of detection of each sensor is unknown, due to unknown target location, the optimal Chair-Varshney fusion rule cannot be used. A counting rule that uses the total number of detections (1’s) reported by the local sensors as the decision statistic has been proposed in these papers. In this formulation, all the local sensors use the same threshold and the system level probabilities of false alarm and detection have been derived. The proposed scheme was also shown to be asymptotically optimal. In this paper, we break away from the identical local sensor threshold paradigm and explore the possibility of improved detection performance via non-identical sensor thresholds determined in a dynamic manner.

A new statistical metric, the false discovery rate (FDR), and an algorithm to control FDR for multiple comparison problems was proposed by Benjamini and Hochberg [11]. A preliminary investigation of the application of FDR for WSNs has been conducted by Ermis and Saligrama [12]. Their work studied the application of FDR for localization under communication constraints and also proposed a distributed algorithm that controls the FDR. In this paper, we develop a FDR based approach to set local sensor thresholds and study the performance of the resulting distributed target detection system. Under the Neyman-Pearson formulation, the relative performance of equal sensor thresholds and FDR based unequal sensor thresholds determined in a dynamic manner for detection in WSNs is investigated.

The remainder of the paper is organized as follows. Section 2 discusses the target and signal models adopted. Section 3 introduces the problem of distributed target
detection in WSNs and describes previous work. Section 4 provides the motivation behind non-identical thresholds and also introduces the concept of FDR. Section 5 introduces the proposed framework of non-identical and dynamic threshold selection using FDR and also presents analytical results for various performance metrics. Section 6 provides simulation results, and concluding remarks are provided in Section 7.

2 Problem Formulation

A total of N sensors are randomly deployed in the region of interest (ROI), which is assumed to be a square with area $b^2$. The locations of the sensors are assumed to be unknown to the fusion center, but the position coordinates are i.i.d. and follow a uniform distribution:

$$f(x_{i}) = \begin{cases} \frac{1}{b} & -\frac{b}{2} \leq x_{i} \leq \frac{b}{2} \\ 0 & \text{Otherwise} \end{cases}$$ (1)

$$f(y_{i}) = \begin{cases} \frac{1}{b} & -\frac{b}{2} \leq y_{i} \leq \frac{b}{2} \\ 0 & \text{Otherwise} \end{cases}$$ (2)

for $i = 1, ..., N$ where $(x_{i}, y_{i})$ are the coordinates of sensor $i$.

We assume that the noises at local sensors are i.i.d. and follow the standard

$$n_{i} \sim N(0,1)$$ (3)

For a local sensor $i$, the binary hypothesis testing problem is:

$H_1: s_i = a_i + n_i$ (4)

$H_0: s_i = n_i$ (5)

where $s_i$ is the received signal, and $a_i = \sqrt{P_i}$ is the signal amplitude due to the presence of target.

We assume that the signal power emitted by the target decays as the distance from the target increases. An isotropic signal power attenuation model is adopted here [9]:

$$P_i = g(d_i)$$ (6)

where $P_i$ is the signal power measured at sensor $i$ located at $(x_i, y_i)$, and $d_i$ is the distance between the target located at $(x_t, y_t)$ and local sensor $i$.

$$d_i = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2}$$ (7)

In this paper, the following $g(x)$ has been adopted,

$$g(x) = \begin{cases} P_0 & 0 \leq x \leq d_0 \\ P_0 \frac{d_0^n}{x^n} & x > d_0 \end{cases}$$ (8)

where $P_0$ is the signal power measured at a reference distance $d_0$, $n$ is the signal decay exponent and varies with the medium of propagation (air, water etc.) and surrounding environment (indoor, outdoor etc.).

The model adopted above is very general, and can be used for modeling targets that emit isotropically radiating electromagnetic or acoustic waves. The sensors are assumed to be passive sensors. Some potential application areas are battlefield surveillance and environmental monitoring.

3 Distributed Target Detection in WSNs: Identical Threshold Case

In the classical distributed detection scheme, due to bandwidth and energy constraints, each local sensor sends a hard decision, i.e., a single bit, to the fusion center. In this paper, it is assumed that the communication channels between the local sensors and the fusion center are perfect. We denote the binary data from local sensor $i$ by $I_i = \{0,1\}$ where $I_i$ takes the value 1 when there is a detection and 0 otherwise. The data from all the sensors are denoted by $I = \{I_i: i = 1, ..., N\}$. When the fusion center has complete knowledge of the $p_{d_i}$ and $p_{f_a_i}$ of each local sensor, the optimal decision fusion rule is the Chair-Varshney fusion rule [1, 3], and its decision statistic is given by:

$$\Lambda_i = \sum_{i=1}^{N} \left[ I_i \ln \frac{p_{d_i}}{p_{f_a_i}} + (1-I_i) \ln \frac{1-p_{d_i}}{1-p_{f_a_i}} \right]$$ (9)

But for the problem being considered here, it is very difficult to calculate the probability of detection of the $i^{th}$ sensor ($p_{d_i}$), as $p_{d_i}$ depends on the distance between the sensor and the target. In the work by Niu and Varshney [7,8,9], a ‘counting rule’ which uses the total number of detections (1’s) reported by the local sensors, as the decision statistic, has been proposed. The ‘counting rule’ is described below:
After collecting data $I$, the fusion center makes a final decision about a target’s presence or absence using the fusion rule,

$$ H_1 \quad \Lambda = \sum_{i=1}^{N} I_i > T \quad H_0 \quad \Lambda < T $$

where $T$ is the system level threshold.

In [7,8,9], all the sensors employ an identical threshold $\tau$, which fixes the $p_{fa}$ at each sensor to $p_{fa} = Q(\tau)$, and the system-wide $P_{FA}$ to:

$$ P_{FA} = \sum_{i=1}^{N} \binom{N}{i} p_{fa}^i (1 - p_{fa})^{N-i} $$

For a large number of sensors, using the Laplace-DeMoivre approximation, we have

$$ P_{FA} = Q \left( \frac{T - Np_{fa}}{\sqrt{Np_{fa}(1-p_{fa})}} \right) $$

In this problem, different local sensors have different $p_{d_i}$, which is a function of $d_i$. Hence under hypothesis $H_1$, the total number of detections no longer follows a Binomial distribution, and the system-wide probability of detection $P_D$ is difficult to derive analytically. Hence $P_D$ may be approximated by using the Central Limit Theorem (CLT) [7].

$$ P_D = Q \left( \frac{T - N\bar{p}_d}{\sqrt{N\sigma^2}} \right) $$

where, $\bar{p}_d$ and $\sigma^2$ are defined in [7]. This work was also extended to the case when the number of sensors is governed by Poisson distribution [8]. In [9], exact performance of the system has been derived without the assumption of a large number of sensors.

### 4 Motivation for Non-identical and Dynamic Thresholds

For the network-wide detection problem where the sensors in the network are collaboratively trying to detect the presence or absence of a target whose location is unknown, the distances between the target and the sensors are not the same resulting in the sensors having unequal signal-to-noise ratios (SNRs). Hence, intuitively network-wide performance improvement may be achieved by using non-identical thresholds obtained in a dynamic manner than a single fixed threshold used by all the sensors.

In this paper, we depart from the equal and static local sensor threshold paradigm of [7,8,9], and adopt a dynamic local sensor threshold determination scheme where this dynamic set of non-identical thresholds are collaboratively determined by all the sensors in the network. To this end, we propose a FDR based local sensor threshold selection method and assume a single-bit per sensor communication/broadcast capability. Next, we provide a brief introduction to FDR and also describe the original algorithm [11] to control it and a distributed realization of it.

#### 4.1 False Discovery Rate (FDR)

In a breakthrough paper, Benjamini and Hochberg (1995) [11] introduced a new approach to multiple hypothesis testing that controls the false discovery rate (FDR), defined as the fraction of false rejections among those hypotheses rejected. This procedure is very appealing as it controls a metric, which is often of more practical importance than the conventional family wide error rate (FWER). Table 1 provides details regarding the definition of FWER and FDR.

<table>
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<tbody>
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<td>$H_0$ True</td>
<td>$U$</td>
<td>$V$</td>
</tr>
<tr>
<td>$H_1$ True</td>
<td>$T$</td>
<td>$S$</td>
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<tr>
<td>Total</td>
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Family wide error rate (FWER) is defined as the probability of committing any type 1 error or false alarm, $P(V \geq 1)$. For this problem, it is the probability of one or more sensors reporting the presence of a target (false alarm) when no target is present. The complement, $(1 - P(V \geq 1))$, is the probability that no sensor reports the presence of target, when no target is present.

False discovery rate (FDR) is defined as the expected ratio of the number of observations falsely classified into alternate hypotheses (declared $H_1$ when $H_0$ is true) to the total number of observations classified into the alternate hypotheses ($H_1$). The proportion of errors committed by falsely rejecting null hypotheses (declared $H_1$ when $H_0$ is true) can be viewed through the random variable, $L = \frac{V}{V+S}$

FDR (Le) is defined to be the expectation of $L$, $L_e = E(L) = E\left(\frac{V}{V+S}\right) = E\left(\frac{V}{R}\right)$

$L_e$ is defined to be zero when $V+S=0$, as no false rejections may be committed.

#### 4.2 Algorithm to Control False Discovery Rate (FDR)

Consider testing $n$ observations based on the corresponding p-values $p_1, p_2, ..., p_n$, where the p-value for an observation $Y$ is defined as,
\[ p(Y) = \int_{0}^{\infty} f_{0}(t) dt \]  

(16)

where \( f_{0}(t) \) is the probability density function (pdf) of the observations under \( H_{0} \).

The algorithm by Benjamini and Hochberg [11] which keeps the FDR below a specified value \( \gamma \), is provided below:

Calculate the p-values of all the observations and arrange them in ascending order.

Let \( k \) be the largest \( i \) for which \( \frac{i \gamma}{N} \leq p_{(i)} \).

Declare all observations corresponding to \( p_{(i)}, i=1,2,...,k \) as \( H_{1} \).

Under the assumption of independent test statistics this procedure controls the FDR at \( \gamma \).

![Fig 2: FDR based decision system](image)

### 4.3 Decentralized FDR Control Algorithm

As evident from the algorithm provided above, the FDR decision making system sets its decision boundary by using the last crossing (from the left) of the p-values under the line shown in Fig 2. Let \( p_{1}, p_{2},...,p_{N} \) be the p-values of the observations. Let \( p_{(i)} < p_{(i+1)} < ... < p_{(N)} \) denote the ordered p-values, with \( p_{(0)} = 0 \), and \( l_{i} = \frac{i \gamma}{N} \), then the last crossing (from the left) or the first crossing (from the right) is the point \( D \) such that,  

\[ D = \max \{0 \leq i \leq N : p_{(i)} \leq l_{i}\} \]

The FDR procedure classifies as \( H_{1} \) all observations for which \( p_{i} \leq p_{(D)} \).

As ordering of p-values is required for the FDR control procedure described above, the procedure conventionally needs centralized processing. A decentralized procedure has been proposed by Ermis and Saligrama [12], which uses one bit for communicating with the fusion center and one-bit for in-network communication among the sensors. But this procedure is actually designed to find the first crossing from the left rather than the last one. Also the total communication cost for this algorithm may be large as a sensor may respond to a threshold test multiple times and hence may need to communicate more than one-bit. We provide an alternate decentralized FDR algorithm. The assumptions on the capability of each sensor and of the fusion center are the following:

1. The fusion center knows the total number of sensors in the sensor field.
2. Each sensor can compute the p-value of its observation and perform simple threshold tests.
3. Each sensor can do some simple counting operations.
4. Each sensor can remember whether it responded positively or negatively to the last threshold test it executed. Thus each sensor uses one bit to store this information.
5. Each sensor can broadcast a single bit that the fusion center and all other sensors in the network can hear.

The decentralized FDR control algorithm is now provided below:

1. Each sensor compares the p-value of its observation \( (p_{(0)}) \) to \( \gamma \).
2. The \( l \) sensors having \( p_{i} > \gamma \), broadcast this decision using a single bit to the fusion center and to the entire network (using a suitable communication protocol).
3. All the sensors update their thresholds to \( \frac{N - l}{N} \).
4. Only those sensors that did not broadcast their decisions earlier perform the threshold test. Let \( k \) be the number of sensors that have p-values greater than the present threshold. These \( k \) sensors broadcast their decision using a single bit to the fusion center and the entire network.
5. All the sensors update their thresholds to \( \frac{N - l - k}{N} \).
6. Steps 4 and 5 are repeated until there are no more sensors reporting p-values greater than the current threshold.
7. If \( S = l + k + ... \), is the total number of sensors which broadcast their decisions to the fusion center, the total number of decisions which are classified as \( H_{1} \) is given by \( N - S \).

This decentralized FDR algorithm proposed above requires only one-bit communication capability for each sensor. The maximum communication cost for the entire network is less than or equal to \( N \) bits per detection round, where \( N \) is total number of sensors in the network. But it is a sequential process with inherent latency. Some preliminary results on latency are presented in Section 6.
A detailed study on average number of iterations required will be part of our future work.

5 Distributed Target Detection using FDR based Local Sensor Threshold

As mentioned in Section 4, as different sensors have unequal SNRs, a local sensor threshold selection scheme that takes this fact into account is likely to provide performance gain. We propose control of the False Discovery Rate (FDR) for the entire family of observations (i.e., observations from the entire sensor field) to a value less than or equal to $\gamma$. The value $\gamma$ and the system level threshold $T$ are related to the system-wide probability of false alarm $P_{FA}$ which will be derived shortly. To control FDR to $\gamma$, the local sensor thresholds have to be set following the algorithm provided earlier. Setting the thresholds in this manner is an adaptive procedure and the thresholds depend both on the number of sensors in the event region and on the observations of the local sensors, i.e., the sensor with the largest p-value is compared to $\gamma$ and subsequent sensors are compared to $\frac{(N-i)\gamma}{N}$. Thus, sensors with lower p-values (i.e., higher-valued observations) are compared to lower p-value threshold (i.e. higher observation threshold).

5.1 Performance Analysis: System level $P_{FA}$

At the fusion center, the probability of false alarm is given by,

$$P_{FA} = \Pr \left( \Delta = \sum_{i=1}^{N} I_i \geq T | H_0 \right)$$

(17)

where, $\Delta = \sum_{i=1}^{N} I_i$ denotes the total number of sensors which report the presence of the target, when no target is present (false alarms) for the entire sensor field. The sensor observations under such conditions are i.i.d. and normally distributed $N(0,1)$. Hence the corresponding p-values are i.i.d. and uniformly distributed $U(0,1)$. The probability that the graph of an empirical distribution of uniformly distributed random variables on $[0,1]$ intersects a certain straight line at a certain height for the first time is given by Dempster’s formula [13]. For the general case, for arbitrary number of observations, the expected number of false alarms has been derived by Finer and Roters [14] by applying Dempster’s formula for barrier crossing distributions,

$$P(V = i) = \sum_{i=1}^{N} \left( 1 - \gamma \right) \left( \frac{i}{N} \right)^i \left( 1 - \frac{i}{N} \right)^{N-i-1}$$

(18)

Also,

$$\lim_{N \rightarrow \infty} P(V = i) = \frac{i^i \gamma^i}{i!} \exp(-i\gamma)$$

(19)

The probability of false alarm $P_{FA}$ at the fusion center, for a system level threshold $T$ may be shown to be,

$$P_{FA} = \sum_{i=T}^{N} P(V = i)$$

(20)

From Equations (18) and (20), the probability of false alarm $P_{FA}$ at the fusion center is given by,

$$P_{FA} = \sum_{i=T}^{N} \left( \frac{N}{i} \right) \left( 1 - \gamma \right)^i \left( 1 - \frac{i}{N} \gamma \right)^{N-i-1}$$

(21)

which is the system-wide $P_{FA}$ for any number of sensors. For large $N$ (large number of sensors) from Equation (19),

$$P_{FA} \approx \sum_{i=T}^{N} \frac{i^i \gamma^i}{i!} \exp(-i\gamma)$$

(22)

Using Stirling’s formula,

$$P_{FA} = \sum_{i=T}^{N} \sqrt{2\pi i} \frac{i^n}{i^n} \left( 1 - \gamma \right)^i \exp(-i\gamma)$$

(23)

which is the system-wide $P_{FA}$ for a large number of sensors.

5.2 Performance Analysis: System Level $P_D$

The conditional density of the observation of a sensor located at $(x_i, y_i)$ conditioned on a target being present at $(x_t, y_t)$ is given by, $N(\varphi,1)$ where, $\varphi$ is the received signal amplitude $a_i$, given by, $\varphi = \frac{P_0}{\sqrt{d_i}}$ (if $d_i > d_0$) or $\varphi = \sqrt{P_0}$ (if $d_i \leq d_0$).

For a Gaussian random variable of the form $N(\varphi,1)$ the density of the p-value $f_{\varphi}(u)$ is given by,

$$f_{\varphi}(u) = \exp(-\varphi^2/2) \exp(\varphi Q^{-1}(u)), 0 \leq u \leq 1$$

(24)

And the cumulative distribution function $F_{\varphi}(u)$ is given by,

$$F_{\varphi}(u) = \varphi Q^{-1}(u) - \varphi$$

(25)

The density function of the p-values is $U(0,1)$ only when $\varphi = 0$. But when a target is present, all the sensors receive non-zero signals. Hence when a target is present in the event region, the p-values corresponding to each sensor are no longer i.i.d. $U(0,1)$. Hence, obtaining the probability of detection analytically as described earlier in Section 5.1 is no longer possible.

5.2.1 Asymptotic Results on FDR

Genovese and Wasserman [15] have shown that asymptotically the Benjamini-Hochberg (BH) method corresponds to classifying as $H_i$ all p-values that are less
than a particular p-value threshold $u^*$, where $u^*$ is the solution to the equation $F(u) = \beta u$ and

$$\beta = \frac{1}{\gamma - A_0} \frac{1 - A_0}{1 - A_0}$$

(26)

Here, $F(u)$ is the distribution of the p-values under alternate hypothesis (H1), $\gamma$ is the FDR parameter and $A_0$ is the fraction of true H0’s.

### 5.2.2 Asymptotic Results for System Level Probability of Detection ($P_D$)

We next apply the above result to obtain the asymptotic system level probability of detection for the FDR based threshold selection scheme. This result is based on the assumption that the p-values are independent. But in the presence of the target, the p-values of the sensor observations are correlated since they are all functions of the target location, whose coordinates are random variables. If we assume that the region of interest (ROI) is large and/or the signal decay exponent is very large then within a very small region of the ROI the received signal power is sufficiently larger than zero. Under such conditions, ignoring the border effect [9], the assumption of the independence of the p-values of the sensor observations is reasonable, and they may be regarded as invariant to the target location (which may be assumed to lie at the origin). When a target is present in the event region, every sensor receives a signal from the target and the density of the p-values conditioned on the target being present is given by Eqn (24). The distribution of the p-values of each sensor is given by Eqn (25). The marginal distribution $F(u)$ of the p-values of sensor observations in the presence of target (H1) may be obtained as,

$$F(u) = \int_0^{b^2} Q^{-1}(u) - \frac{P_0}{d^2_i} f_T(t) dt + \int_{b^2}^{\infty} Q^{-1}(u) - \frac{P_0}{d^2_i} f_T(t) dt$$

(27)

where, $t = d^2_i = x_i^2 + y_i^2$.

Assuming that target is located at the origin ($x_i = y_i = 0$), and that $x_i$ and $y_i$ are i.i.d. and follow a uniform distribution within the interval $[-b/2, b/2]$, the density function $f_T(t)$ of $t = d^2_i$ can be shown to be [9]:

$$f_T(t) = \begin{cases} \frac{\pi}{b^2} & 0 < t \leq b^2/4 \\ \frac{2}{b^2} \arcsin \left( \frac{b^2}{2t} - 1 \right) / 2t & b^2/4 < t \leq b^2/2 \\ 0 & \text{Otherwise} \end{cases}$$

(28)

Now, using the asymptotic results on FDR, the asymptotic p-value threshold $u^*$ may be obtained from the solution of the equation,

$$F(u) = \beta u$$

where $\beta = \frac{1}{\gamma - A_0} \frac{1 - A_0}{1 - A_0} = \frac{\gamma}{\gamma'}$, since $A_0 = 0$ due to the fact that when a target is present there are no true H0 as every sensor receives a non-zero signal from the target. Once $u^*$ is evaluated, the results provided in Section 3 may be applied to obtain the probability of detection as now this scheme is analogous to each sensor using an identical threshold, $\zeta = Q^{-1}[u^*]$.

For $P_0 = 20, b = 10, n = 2, \gamma = 0.4$ and $N = 10$, the evaluation of $u^*$ is shown in Fig 3.

![Graph](image)

**Fig 3. Evaluation of $u^*$**

FDR procedure involves comparing the ordered p-values of the observations to a set of linearly decreasing p-value thresholds starting from $\gamma$ to $\gamma / N$. Asymptotically, this procedure is equivalent to comparing the p-values of the observations to a fixed p-value threshold $u^*$ where $\gamma / N \leq u^* \leq \gamma$ and $u^*$ may be obtained as shown in Fig 3.

### 6 Performance Comparison

Under the Neyman-Pearson formulation, to design the optimal detector, we seek to maximize the probability of detection $P_D$ for a specified value of $P_{FA}$.

Hence, the problem formulation is,

**Maximize $P_D$ subject to the constraint $P_{FA} = \alpha$**

To obtain the optimal detector for the two schemes, for a specified system-wide probability of false alarm $P_{FA}$ the pair of values $(\zeta, T)$ for the identical threshold scheme and $(\gamma, T)$ for the dynamic threshold scheme, which maximizes the probability of detection $P_D$ are obtained by exhaustive search here. Note that $\zeta$ is the local sensor threshold (for the identical threshold scheme), $\gamma$ is the FDR parameter (for the dynamic threshold scheme) and $T'/T$ is the system-wide threshold.
In our simulation results we have chosen the following parameters: \( P_0 = 10, b = 10, d_0 = 1, n = 3 \) and \( N = 15 \). The simulation results are based on \( 10^6 \) Monte Carlo runs. The sensors were randomly deployed in an event region of size \( 10 \times 10 \). A target was placed at a random location in the event region during each run.

**Table 2: Theoretical versus Simulated \( P_{FA} \)**

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<th>Theoretical ( P_{FA} )</th>
<th>Simulated ( P_{FA} )</th>
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<td>0.0100</td>
<td>0.0104</td>
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<tr>
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<tr>
<td>0.3000</td>
<td>0.2997</td>
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</table>

Table 2 shows the simulated and theoretically obtained system-wide probability of false alarm \( P_{FA} \) for the FDR based scheme. For this simulation, corresponding to different values of \( P_{FA} \), ranging from 0.01 to 0.3, a pair of \((\gamma, T)\) was obtained by numerically solving Equation (21). The simulation results show that the analytical results obtained for the system wide \( P_{FA} \) closely match the simulation results.

Also Figure 4 shows that under the NP formulation, the FDR based threshold selection scheme shows better detection performance compared to the identical threshold scheme. In most practical situations, we are interested in small values of \( P_{FA} \). In such cases the detection performance of the FDR based threshold selection scheme is substantially better than the identical threshold scheme.

For \( P_0 = 10, b = 10, d_0 = 0.1, n = 3 \) and \( N = 15 \), Fig 5 provides the histogram of delay/latency for the distributed FDR algorithm per detection round for \( 10^4 \) Monte Carlo runs. The delay (number of iterations required) per detection round is an integer number. For the simulations carried out, the average delay is 4.1424 when no target is present in the ROI and is equal to 3.53 when a target is present. The reason for the smaller delay when a target is present is because the distributed FDR algorithm starts with the largest p-value threshold (i.e. from the right end of Fig 2) and the first crossing of the line from the right occurs on the average earlier when there is a target (because there are more sensors which detect the presence of the target in this case, i.e., decide \( H_1 \)) compared to when there is no target.

For \( P_0 = 10, b = 10, d_0 = 0.1, n = 3 \) and for the target placed at the center of the event region, the average number of sensors in the event region which detect the presence of the target for the FDR based threshold selection scheme and for the case where every sensor uses identical threshold (equal to the asymptotic p-value threshold \( u^* \) as discussed in Section 5.2.2) are shown in Fig 6 as a function of the number of sensors. Fig 6 shows that as the number of sensors in the event region (ROI) increases the performance of the FDR based threshold selection scheme becomes identical to the scheme where every sensor uses the identical observation threshold \( \tau = Q^{-1}(u^*) \). Hence under the assumption of a very large sensor network and asymptotically large number of sensors, the detection performance of the FDR based threshold selection scheme becomes identical to the identical threshold scheme with threshold \( \tau = Q^{-1}(u^*) \) and the results discussed in Section 3 may be applied to obtain the system level probability of detection.
Figure 6. Average number of detections for the entire sensor field in presence of target for FDR based threshold scheme and for every sensor using the identical threshold $\xi = Q^{-1}\{\alpha^*\}$

7 Conclusion

In this paper, a FDR based local sensor threshold selection methodology has been developed for distributed target detection in WSNs. The system wide probability of false alarm has been derived. Also, the asymptotic system-wide probability of detection has been derived for large sensor networks. Simulation results for detection performance under the NP formulation are provided for both the FDR based threshold selection scheme and the identical threshold scheme. The results obtained in this paper indicate that non-identical and dynamic local sensor thresholds provide significant performance improvement for target detection in sensor networks.

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