Abstract - The Probability Hypothesis Density (PHD) filter, which was derived from finite set statistics is a promising approach to multi-target tracking. An analytical closed-form solution for the PHD, named Gaussian Mixture PHD Filter, is given for linear Gaussian target dynamics with Gaussian births by B.Vo and W. Ma. Based on the Gaussian Mixture PHD filter, in this paper, without consideration of data association technique, a method using three passive sensors for multi-target location system is proposed, which can restrain greatly the false triangulations, calls ghosts, where the measurements of the Bearing-only Multi-target location system are spoiled by clutter.

Keywords: Random Finite Set; Probability Hypothesis Density Filter; Gaussian Mixture; Bearing-only; Multi-target Location

1 Introduction

The modeling of multi-target dynamics using Random Finite Sets (RFS) naturally leads to tracking algorithms which incorporate track initiation and termination, a procedure has mostly been performed separately in traditional tracking algorithms. The Probability Hypothesis Density (PHD) filter[1], which was derived by Mahler from finite set statistics, a reformulation of point process, is a promising approach to multi-target tracking. The Probability Hypothesis Density filter, which propagates only the first moment instead of the full multi-target posterior, still involves multiple integrals with no closed forms in general. Current PHD filter implementations have been studied based on Sequential Monte Carlo method[2][5], a numerical integration method that can accurately approximate the PHD recursions. Based on the Kalman Filter conception, an analytical closed-form solution, Named Gaussian Mixture PHD[3][6], is given for linear Gaussian target dynamics with Gaussian births by B.Vo and W. Ma, and the extension to nonlinear scenario[3] is discussed in detail. In addition, multi-peak extraction becomes an easy issue to the Gaussian mixture PHD filter.

The estimation of the position of an emitting source from passive angle measurements is a widely investigated problem [7]. Bearing measurements collected by two or more fixed direction finding sensors can be intersected to determine the emitting target location. Multi-target location from Bearing-only measurements spoiled by clutter is a classical navigation problem. Since the passive sensors can obtain bearing-only, a target location has to be estimated by triangulating bearing measurement from two sensors at least. However, multiple targets create a number of false triangulations, called ghosts [8][9][11] which cannot be resolved on the basis of measurements from just two sensors. And the number of sensors must be more than or equal to three. Mathematically, this formulation of the data association problem leads to a generalization of the three-dimensional assignment problem, which is known to be NP hard [10][11], i.e. the number of intersections increases exponentially with the number of sensors or measurements. In order to solve this crux, many algorithm based on data association technique for passive sensor were presented over past the decades[7][8][11-17]. Probability Hypothesis Density filter is a promising approach to multi-target tracking, which incorporate track initiation and termination, and without consideration of data association. In this paper, a different approach using three passive sensors is proposed to solve the false triangulations problem, where the Gaussian Mixture PHD filter corresponding to extended Kalman Filter is used. Simulations reveal that the false targets caused by the intersection of passive bearing measurements spoiled by clutter are restrained greatly.

The structure of this paper is as follows. Section 2 presents the Gaussian Mixture PHD filter for multi-target tracking. Section 3 describes briefly the passive multi-target location problem, and its NP hard crux with data association technique solution. PHD filter method for passive multi-target location is introduced in Section 4.
Section 5 gives the simulation of the proposed method. Conclusion is drawn in last section. Readers who are familiar with Gaussian Mixture PHD Filter may go straightly to Section 3.

2 The Gaussian Mixture PHD Filter

The PHD[1] filter propagates the first-order statistical moment associated with the multi-target posterior density[4], instead of the multi-target posterior density. The posterior intensity or PHD can be propagated in time through the following recursion.

$$v_{k|k-1}(x) = \int \phi_{k|k-1}(x,\zeta) v_{k-1}(\zeta) d\zeta + \gamma_k(x) \tag{1}$$

$$v_k(x) = (1 - p_D(x)) v_{k|k-1}(x) + \sum_{\zeta \in Z_k} \kappa_k(\zeta) \int \psi_{k,z}(\zeta) v_{k|k-1}(\zeta) d\zeta \tag{2}$$

Where

$$\phi_{k|k-1}(x,\zeta) = e_{k|k-1}(\zeta) f_{k|k-1}(x|\zeta) + \beta_{k|k-1}(x|\zeta)$$

$$\psi_{k,z}(\zeta) = p_D(x) g_k(z|x)$$

$$\gamma_k(x)$$ the intensity of the birth RFS at time $$k$$

$$\beta_{k|k-1}(x|\zeta)$$ the intensity of the RFS spawned at time $$k$$ by a target previous state $$\zeta$$

$$e_{k|k-1}(\zeta)$$ the probability that a target still exist at time $$k$$ given that its previous state is $$\zeta$$

$$p_D(x)$$ the probability of detection given a state $$x$$ at time $$k$$

$$\kappa_k(\zeta)$$ intensity of the clutter RFS at time $$k$$

Assume each target follows linear Gaussian dynamics, and the survival and detection probabilities are constants, which are commonly used in target tracking to make the problem more tractable. If the intensities of the birth and spawn RFSs are Gaussian mixtures of the form

$$\gamma_k(x) = \sum_{j=1}^{J_k} w_k^{(j)}(x) N(x; m_k^{(j)}, P_k^{(j)}) \tag{3}$$

$$\beta_{k|k-1}(x|\zeta) = \sum_{j=1}^{J_k} w_k^{(j)}(x) N(x; \zeta + m_k^{(j)}, P_k^{(j)}) \tag{4}$$

Where $$N(;m,P)$$ denotes a Gaussian density with mean $$m$$ and covariance $$P$$, $$w_k^{(j)}, m_k^{(j)}, P_k^{(j)}$$ are respectively the weight, mean and covariance of the $$j$$th Gaussian component of the birth intensity $$\gamma_k$$, and $$w_k^{(j)}, m_k^{(j)}, P_k^{(j)}$$ are respectively the weight, mean and covariance of the $$j$$th Gaussian component of the spawning intensity $$\beta_{k|k-1}(x|\cdot)$$ of target $$\zeta$$. And the Gaussian Mixture PHD[3][6] recursion are followings

i. Suppose that the posterior intensity at time $$k-1$$ is a Gaussian mixture of the form

$$v_{k-1}(x) = \sum_{i=1}^{J_k} w_{k-1}^{(i)} N(x; \hat{x}_{k-1}^{(i)}, P_{k-1}^{(i)})$$

Then, the predicted intensity at time $$k$$ is also a Gaussian mixture, and is given by

$$v_{k|k-1}(x) = v_{k|k-1}^{(x)}(x) + v_{k|k-1}^{(b)}(x) + \gamma_k(x)$$

Where

$$v_{k|k-1}^{(x)}(x) = e_k \sum_{i=1}^{J_k} w_{k-1}^{(i)} N(x; F_k x_{k-1}^{(i)}, Q_k + F_k P_{k-1} F_k^T)$$

$$v_{k|k-1}^{(b)}(x) = \sum_{i=1}^{J_k} \sum_{j=1}^{J_k} w_{k-1}^{(i)} w_k^{(j)} N(x; \hat{x}_{k|k-1}^{(i)}, P_{k|k-1}^{(i)})$$

$$\hat{x}_{k|k-1}^{(i)} = \hat{x}_{k-1}^{(i)} + \hat{x}_{k|k-1}^{(b)}$$

$$P_{k|k-1}^{(i)} = P_{k-1}^{(i)} + P_{k|k-1}^{(b)}$$

ii. Suppose that the predicted intensity at time $$k$$ is a Gaussian mixture of the form

$$v_{k}^{(x)}(x) = \sum_{i=1}^{J_k} w_{k}^{(i)} N(x; \hat{x}_{k}^{(i)}, P_{k}^{(i)})$$

Then, the posterior intensity at time $$k$$ is also a Gaussian mixture, and is given by

$$v_{k}(x) = (1 - p_D(x)) v_{k|k-1}(x) + \sum_{\zeta \in Z_k} \kappa_k(\zeta) \int \psi_{k,z}(\zeta) v_{k|k-1}(\zeta) d\zeta \tag{5}$$

$$v_{k}^{(d)}(x;z) = \sum_{i=1}^{J_k} p_D w_{k|k-1}^{(i)} q_k^{(i)}(z) N(x; \hat{x}_{k}^{(i)}(z), P_{k}^{(i)}) \tag{6}$$

$$\kappa_k(\zeta) + p_D \sum_{j=1}^{J_k} w_{k|k-1}^{(j)} q_k^{(j)}(z)$$

where

$$q_k^{(i)}(z) = N(z; H_k \hat{x}_{k|k-1}^{(i)}, R_k + H_k P_{k|k-1} H_k^T)$$

$$\hat{x}_{k}^{(i)}(z) = \hat{x}_{k|k-1}^{(i)} + K_k(z - H_k \hat{x}_{k|k-1}^{(i)})$$

$$P_{k}^{(i)} = (I - K_k H_k) P_{k|k-1}^{(i)}$$

$$K_k = P_{k|k-1}^{(i)} H_k^T (H_k P_{k|k-1}^{(i)} H_k^T + R_k)^{-1}$$

And the algorithm is extended to nonlinear target dynamics using approximation approach from extended and unscented Kalman filters[3].

3 Problem description

As aforementioned, bearing measurements collected by two or more fixed direction finding sensors can be intersected to determine the emitting target location. Multi-target location from Bearing-only measurements spoiled by clutter is a classical navigation problem. However, multiple targets create a number of ghosts [8][9][11] which cannot be resolved on the basis of measurements from just two sensors. And the number of
sensors must be more than or equal to three [11]. Mathematically, this formulation of the data association problem leads to a generalization of the three-dimensional assignment problem, which is known to be NP hard [10][11]. It is a crux for the passive multi-target location with the data association technique.

As shown in Fig. 1, if $T_1$ and $T_2$ are real target in the surveillance region, $T_1'$ and $T_2'$ are ghosts, caused by measurement intersections. If the number of measurements from one sensor is $n_k$, and suppose the average number of real clutter is $\rho_k$, then the number of targets is $n_k - \rho_k$, and the number estimated of real targets is $n_k^2 - n_k + \rho_k$, if two sensors is used. Suppose the number of measurements from those sensors in the system are equivalent each other.

4 Passive multi-target location

It has been shown that the Gaussian Mixture PHD filter is a promising approach to multi-target tracking without consideration of data association. If the ghosts of passive multi-target location system are disordered around like clutter, and the Gaussian Mixture PHD filter can be used directly to reduce those ghosts like reducing clutter.

And some method should be used to make those ghosts disordered around like real clutter in the surveillance region. In this paper, a method is proposed, which uses three sensors, and the burden of computation is same to standard GM-PHD filter. As shown in Fig. 4, measurements are composed of three sensors in a cyclic fashion, i.e. at time $k-1$ the measurements are composed of measurements from the first sensor and the second sensor, at time $k$ the measurements are composed of measurements from the first sensor and the third sensor, and at time $k+1$ the measurements are composed of measurements from the second sensor and the third sensor, and so on, by means of this way the location of those ghosts can be disorder around like real clutter.

5 Simulation

As shown in Fig. 1, is a target in the surveillance region, suppose $\alpha$ and $\beta$ are the measurement from sensors $O_1$ and $O_2$ respectively.

$$\alpha = \arcsin \frac{x}{\sqrt{x^2 + y^2}},$$

$$\beta = \arcsin \frac{x - L}{\sqrt{(x - L)^2 + y^2}}$$

and observation processes are nonlinear.

$$p(Z_k \mid X_k) = N[Z_k; \varphi(X_k), R_k]$$  \hspace{1cm} (8)

Where $\varphi(X_k) = \begin{bmatrix} \arcsin \frac{x}{\sqrt{x^2 + y^2}} \\ \arcsin \frac{x - L}{\sqrt{(x - L)^2 + y^2}} \end{bmatrix}$

$$R_k = \sigma^2_v \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Assume each target state follows linear Gaussian dynamics

$$p(X_k \mid X_{k-1}) = N[X_k; F_{k-1}X_{k-1}, P_{k|k-1}]$$  \hspace{1cm} (9)

Where

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q_{k-1}$$

$$X_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^T, \quad F_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
The state process noise and measurement process noise variance are given by 

\[ \sigma_e^2 = 0.25T^4 \begin{bmatrix} 0.5T^3 & 0 & 0 \\ 0.5T^3 & T^2 & 0 \\ 0 & 0 & 0.25T^4 \\ 0 & 0 & 0.5T^3 \end{bmatrix} \]

where \( T \) is the sample period, \( \sigma_e^2 \) and \( \sigma_\omega^2 \) are the variance of the state process noise and measurement process noise respectively, where \( T = 1s, \sigma_e = 5m/s^2 \) and 

\[ \sigma_\omega = \frac{\pi}{180} rad \] for simulations.

Assume that the spontaneous birth RFS is a Poisson RFS with intensity 

\[ \gamma_k(x_k) = 0.1N(x_k; \bar{m}_i, \bar{P}) + 0.1N(x_k; \bar{m}_2, \bar{P}) + 0.1N(x_k; \bar{m}_3, \bar{P}) \]  

(7)

Where 

\[ \bar{m}_i = [1000, 3, 1000]^T, \]

\[ \bar{m}_2 = [500, 3, 2000]^T, \]

\[ \bar{m}_3 = [2000, 3, 1500]^T; \]

\[ \bar{P} = \text{diag}([10, 1, 1])^T. \]

And suppose there are no spawning targets. The clutter RFS is modeled by a Poisson RFS with intensity 

\[ \kappa_k(z_k) = \lambda_k u(z_k) \]

Where \( u(\cdot) \) is a uniform distribution function over \( V = [0, 1.5km]^2 \) which is the surveillance region of the Multi-target location system with three passive sensors. 

\[ \lambda_k = n_k^2 - n_k + \rho_k \]

where \( n_k \) is the number of measurements from one sensor. The target survival probability is set to \( e_k = 0.99 \). The probability of detection is \( p_D = 0.98 \). The purpose of pruning is to reduce the number of Gaussian components propagated to the next time step \([3]\). Simulation with the truncation threshold \( T_r = 10^{-3} \) and the merging threshold \( U = 4 \) in the pruning procedure, the state estimates are obtained from the PHD filter by taking the means of the Gaussian components that have weights greater than 0.9, and the average number real clutter points in the surveillance region set to \( \rho_k = 0 \), i.e. there are no real clutter appeared in the surveillance region; \( \rho_k = 3 \) i.e. the average number real clutter points in the surveillance region is 3.

As show from Fig. 3 \( \rho_k = 0, J_{\text{max}} = 50 \), the maximum allowable number of Gaussian terms of Gaussian Mixture PHD filter) comparing with Fig. 2, we see that the ghosts can not be reduced properly, if only two sensors are used, this is because the track of those ghosts are not composed of real clutter but measurements intersections, this cause the motion type of those false intersection targets are similar to those real targets'. Simulations of the proposed method reveal that the ghosts are restrained greatly, as shown in Fig.5 \( \rho_k = 3, J_{\text{max}} = 50 \).

**Conclusion**

Gaussian Mixture PHD Filter is a promising approach for multi-target tracking, which can reduce the clutter properly during target tracking in the surveillance region. This paper proposed a method for multi-target location with three sensors, which make the location of ghosts disordered around like real clutter, and reduce them properly with Gaussian Mixture PHD. And simulations reveal that the ghosts caused by passive measurement
intersections are restrained greatly. The NP hard issue
discussed is avoid because of without utilization of
data association, and the burden of computation is same to
that of regular Gaussian Mixture PHD filter for multi-
target tracking.

Acknowledgements

This research was jointly supported by National Natural
Science Foundation of China(60674107), Hebei Natural
Science Foundation(2006000343), Basic Research
Project(A14200606161) and Project
(9140A01060606JW0305)

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