Identification of Simple Product-Form Plumes Using Networks of Sensors With Random Errors

Nageswara S. V. Rao
Computer Science and Mathematics Division
Oak Ridge National Laboratory
Oak Ridge, TN 37831
raons@ornl.gov

Abstract -
We consider a class of simple, idealized plumes which are specified by a product of injection and distance decay terms. The plume propagates with a constant velocity, and its distance term decays exponentially with respect to distance in a planar region. If the intensity sensors are error-free, the difference triangulation method can identify the origin of plume both in time and space within a specified precision. In our case, the sensors are subject to random, correlated errors with unknown distributions in measuring the plume intensity. The sensors are available or in place to conduct controlled experiments and collect measurements. We present a training method that utilizes the plume equation together with controlled sensor measurements to identify the plume's origin with distribution-free probabilistic performance guarantees. The training consists of utilizing the measurements to compute a suitable precision value for the difference triangulation method to account for sensor distributions. We present a distribution-free relationship between the training sample size and the precision and probability with which plume's origin is identified.

Keywords: Distributed sensor network, plume detection and identification, difference triangulation, training, sample size.

1 Introduction

Sensor networks are proliferating into a wide variety of applications [3, 6] such as target tracking using a network of radar systems, vehicle signature identification using a seismic sensor network, and attack source isolation over computer networks using a collection of signature sniffers. In particular, networks of spatially dispersed sensors could be useful for the identification, detection, tracking and prediction of chemical, biological or nuclear spatio-temporal propagative phenomena. Within this application area, the problem of detecting and tracking plumes has been addressed using sensor networks for chemical dispersions in [9, 5] and nuclear radiation in [2].

A particular class of simple product-form plumes has been studied in [14] from an analytical and computational viewpoint. Such plumes were shown to be completely identifiable and tractable using a network of three ideal intensity sensors in several cases such as simple explosion models. For a product-form plume that decays exponentially in space and propagates with a constant velocity, the Difference Triangulation Method (DTM) of [14] localizes (or isolates or identifies) the plume origin to be within a precision region using measurements collected at three sensors. The size of this precision region is specified by the parameter $\gamma$, and the region itself can be computed with the computation complexity $O(\log^2 (1/\gamma))$.

A component of the above problem subsumes the classical source localization problem using Difference of Time-of-Arrival (DTOA) measurements, which has been solved using two general approaches: (a) intersection of hyperbolic curves [7], and (b) linear algebraic solution which typically involves matrix inversion and solving a quadratic equation [16, 11] (see [15] for a recent overview of network-based localization methods). In general the precision to which the source location is estimated is a complex function of the precision with which various numerical operations are implemented. DTM makes this dependence explicit which enables us to control its computational complexity as function of the precision. If DTM is directly applied to sensor measurements with random errors, the resultant precision region is no longer guaranteed to include plume source. However, we show in this paper that under sensor errors, the precision region of DTM can be associated with the probability of localizing the source, which can be controlled through a training process. Sensor errors can have drastic effects on the other DTOA localization methods as well such as the underlying quadratic equation having only imaginary roots thereby rendering the method incomplete, and matrix inversion becoming ill-conditioned.

In practical sensor networks, however, the sensors are often subject to random errors with complex distributions that are hard to estimate. Furthermore, the sensors errors are affected by plume intensities at the sensor sites, which in turn are coupled by the plume equation. Consequently, the sensor errors could be highly correlated which makes it challenging to localize the plume source by combining the sensor readings using traditional methods. Bayesian methods [20] are
difficult to apply since the minimization risk criteria requires a complete knowledge of the joint sensor distribution. We emphasize that because of the lack of statistical independence of sensor errors it is not sufficient to know just the individual (marginal) sensor distributions to apply these methods. Also, the additive independent Gaussian errors models for sensors such as in [4] are not applicable to this case. On the positive side, the sensors are available and are often deployed so that controlled experiments can be conducted to collect measurements. While there are several DTOA source localization methods that account for various types of errors [18, 17, 4], solutions that effectively exploit the availability of sensor measurements are needed for this case. Our main objective is to study the basic analytical tractability of the plume identification problem of [14] when the sensors are subject to random, correlated errors but are available to collect controlled measurements.

In multiple sensor systems subject to random sensor errors, training methods are often used to account for error correlations in computing the fused estimates [13] such as plume source estimator in our case. In the existing approaches, typically, the sensor system is exposed to a phenomena with controlled parameters, and the corresponding sensor measurements are collected. Then fused estimates are computed based on training measurements to predict the unknown parameter values. Within the context of isolating the plume’s origin, a direct application such approach is tantamount to exposing the sensor network to (artificially created) training plumes with known parameters. Such method is impractical in our case for a number of reasons: (a) training plumes must be generated within the deployment region which could be potentially hazardous, and (b) cost of generating and cleaning up the plumes that span large areas could be high even in controlled areas.

We present a hybrid training method that utilizes the plume equations and controlled laboratory measurements to facilitate the plume source localization:

(i) analytical plume equations are used to compute intensity values at the sensor sites;

(ii) in a controlled laboratory setting, the sensors are subject to the computed intensity levels and the corresponding measurements are collected; and

(iii) a precision value is computed from the measurements which is subsequently used in DTM to compute the precision region for the plume origin.

This method avoids exposing the deployment region to plumes and the cost of plume generation and clean up. We show that by controlling the sample size, the plume’s origin can be guaranteed to be within DTM precision region with a specified probability. In particular, we derive a distribution-free relationship between the training sample size, and the precision and probability with which plume’s origin is identified.

We present a product-form plume model and briefly describe the DTM for plume source isolation in Section 2. We present our training method and derive performance equations in Section 3 by considering random sensor errors. In Section 4 we describe simulation results to illustrate the applicability and performance of the proposed training method.

2 Difference Triangulation Method

We consider a simple and idealized plume of the product-form [14] with intensity

\[ I(r, t) = A(t)e^{-\gamma}, \]

where \( t \) is the time, \( r \) is the distance from the origin of plume, \( A(t) \) is the source injection term, and \( e^{-\gamma} \) is the distance decay term. For a simple explosion model we have \( A(t) = b e^{-at} \) such that the intensity decays exponentially in time and space starting with an initial strength of \( I(0, 0) = b \) at the origin. A simple constant source injection rate model can be represented by

\[ A(t) = \begin{cases} b & \text{for } t \in [0, T] \\ b e^{-c(t-T)} & \text{for } t > T. \end{cases} \]  

(1)

We want to emphasize that this is an extremely simplistic model, and much more complex models for plume have been quite common [10, 5, 8, 19].

We consider that the sensors monitor a finite monitoring region, within which the plume source must be identified based on the sensor measurements. This task involves localizing the plume source in space and time, and estimating \( A(t) \). In [14] such plumes are shown to be identifiable in several cases; for example, in the simple explosion model, based on source location, the rest of the plume parameters can be estimated using closed-form expressions. In this section, we describe DTM [14] for the ideal case wherein an ideal sensor located at a distance \( r \) from the origin provides an exact measurement of intensity \( I(r, t) \). In the next section, we consider sensors with random errors such that the sensor measurement \( M \) corresponding to intensity \( I \) is distributed according to an unknown distribution \( P_{M,I} \). This sensor characterization is simplified compared to practical chemical [9] or radiation sensors [2].

An efficient implementation of DTM is presented in [14] as a 2D geometric search on a suitably computed 2D algebraic curve. Let \( \hat{s} \) be the plume source predicted by DTM, and \( R_{\hat{s},\gamma} \) be the corresponding prediction region with a precision \( \gamma \). This method ensures that the source lies within the region \( R_{\hat{s},\gamma} \) which is a distorted \([-\gamma, \gamma] \times [-\gamma, \gamma]\)-box centered at \( \hat{s} \) as will be described in detail in next section. The sides of this region are formed by displaced hyperbolic curves as shown in Figure 5.

Consider two sensors \( S_i \) and \( S_j \) located at distances \( r_i \) and \( r_j \), respectively, from the plume’s origin. For ease of notation, we also denote by \( S_i \) its physical location in the plane. By utilizing two ideal measurements collected by \( S_i \) and \( S_j \) at the same time, we compute the following distance difference

\[ \delta_{ij} = r_i - r_j = \ln (I(r_j, t)/I(r_i, t)), \]
As \( x \) moves along \( S_3S_2 \) left-to-right, \( \Delta_{12}(x) \) monotonically increases from a negative value to a positive value. \( S_1 \) is located at \((0,0)\), \( S_2 \) is located at \((100,0)\), and \( S_3 \) is located at \((0,10)\), \((0,20)\), \((0,30)\) and \((0,40)\) for different plots.

The basic idea is to utilize this monotonicity. The basic idea is to utilize this monotonicity of \( \Delta_{12}(x) \) to support a binary search: move \( x \) along on the line segment \( S_iS_j \), \( \Delta_{ij}(x) \) varies monotonically and linearly from \(-d(S_1,S_2)\) to \(d(S_1,S_2)\), and crosses 0 at the bisector point. Similarly, when we move \( x \) along on the line segment \( S_3S_2 \) away from \( S_3 \), \( \Delta_{ij}(x) \) increases monotonically (and not linearly) as shown in Figure 1.

We consider the locus of points defined by

\[
L_{1,2}(\delta) = \{ x | \Delta_{12}(x) = \delta \}
\]

which is described by the algebraic equation \( d(x,S_i) - d(x,S_j) = \delta \). For \( \delta = 0 \), \( L_{1,2}(0) \) is simply the Voronoi bisector [12, 22] that divides the plane into two half-spaces (Figures 2 and 3). In Figure 2 we show three \( L_{12}(\delta) \) corresponding to \( \delta = -10,0,10 \). The first one \( L_{12}(-10) \) is closer to \( S_1 \) and curves around it, and the last curve \( L_{12}(10) \) with positive \( \delta \) curves away from it as shown in Figure 2. For \( \delta < 0 \), \( L_{12}(\delta) \) divides the plane into a convex region that contains \( S_1 \) and a concave region that contains \( S_2 \) as shown in Figure 3.

As we move \( x \) along \( L_{12}(\delta) \), \( \Delta_{13}(x) \) varies monotonically. The basic idea is to utilize this monotonicity of \( \Delta_{13}(x) \) to support a binary search: move \( x \) along \( L_{12}(\delta_{12}) \) until we reach \( x_r \) nearby \( x \in L_{12}(\delta_{12}) \) such that \( \| \Delta_{13}(x_r) - \delta_{13} \| \leq \gamma \). The binary search on \( L_{12}(\delta_{12}) \) is supported by probing at \( x \) and suitably halving the search region with each probe. This process is illustrated in the execution trace shown in Figure 4, wherein source \( s \) is in the middle of the rectangle.

Figure 1: Plots of \( \Delta_{12}(\cdot) \) for various \( S_3S_2 \) line segments.

Figure 2: Plots of \( L_{12}(\delta) \) for \( \delta = -10,0,10 \) left to right. \( S_1 \) is located at \((0,0)\) and \( S_2 \) is located at \((100,0)\).

Figure 3: For \( \delta < 0 \), curve \( L_{12}(\delta) \) forms a convex region that contains \( S_1 \) and a concave region that contains \( S_2 \).
Here, various locations of $x$ as the search progresses toward $s$ are indicated by * and are labeled 2, 3, ...; note that all these locations are on $L_{12}(\delta_{12})$. Computing each such location where a probe operation is performed itself requires a binary search (details can be found in [14]). There are altogether $O(\log(1/\gamma))$ such probe operations, and each such probe can be performed with complexity $O(\log(1/\gamma))$.

It is implicitly assumed that computational implementation of DTM is of much finer resolution than $\gamma$ to ensure the correctness of the search process. While it is not an issue for typical workstations, care must be taken if DTM is implemented on a low-power, low-precision nodes typical of several wireless sensor network nodes. In particular, the distance computations and comparisons of DTM must be shown to be robust to finite precision effects of the underlying computational device. Other localization methods based on DTOA are also subject to similar phenomenon perhaps in a somewhat more complex manner. For methods that involve matrix inversion [11, 16], a special attention must be paid to avoid ill-conditioning problem in low precision implementations.

When sensor measurements are subject to random errors, source isolation methods may become incomplete or inaccurate. The methods based on solving quadratic equations [11] may not yield an answer as the roots become imaginary. For DTM, $R_{\hat{s},\gamma}$ may no longer contain the source $s$ for a given $\gamma$.

### 3 Sensor Network Training

Given sensor measurements $M_1, M_2, M_3$ collected at same time, let $\hat{s} = D_{\Delta}(M_1, M_2, M_3; \gamma)$ be the estimator of plume source $s$ returned by the difference triangulation method at resolution $\gamma$. $R_{\hat{s},\gamma}$ denotes a distorted box of length $\gamma$ centered at $\hat{s}$ obtained by the
intersection of two annular regions represented as
\[ R_{s;\gamma} = \{ x \mid \| \Delta_{13}(s) - \Delta_{13}(x) \| \leq \gamma \} \]
\[ \cap \{ x \mid \| \Delta_{12}(s) - \Delta_{12}(x) \| \leq \gamma \}, \]
as shown in Figure 5. Recall that DTM guarantees that \( s \in R_{s;\gamma} \) when there are no sensor errors, but in presence of sensor errors \( s \) is not guaranteed to lie inside \( R_{s;\gamma} \) for any fixed resolution \( \gamma \). Let \( P_{M_1,M_2,M_3|R_{s;\gamma}}(R_{s;\gamma}) \) be probability that DTM fails to capture the plume source \( s \) within its predicted region based on the measurements \( M_1, M_2 \) and \( M_3 \). For simplicity we represent \( P_{M_1,M_2,M_3|R_{s;\gamma}}(R_{s;\gamma}) \) by \( P(R_{s;\gamma}) \). If \( \gamma \) is chosen to be large enough to include entire monitoring area, we have \( P(R_{s;\gamma}) = 0 \), and it approaches 1 as \( \gamma \to 0 \). Our objective is compute a suitable value for \( \gamma \) based on the measurements to bound \( P(R_{s;\gamma}) \) to be below a specified \( \epsilon \). Since the sensor error distributions are arbitrary, such guarantee cannot be provided with probability 1 when the training sample is finite.

Let \( l \) sets of training samples be collected by choosing the sources \( s_1, s_2, \ldots, s_l \) and corresponding times \( t_1, t_2, \ldots, t_l \). For each \( s_i \) let \( M_{1,j}, M_{2,j}, M_{3,j} \) represent the corresponding sensor measurements. Then \( P_{l} \prod_{i=1}^{l} M_{1,i,M_{2,i},M_{3,i}|s_i} \) represents the distribution of the sample assuming that the measurements collected at different times are independent across time but are correlated in space for any given \( s_i \). Then we consider the condition
\[ P_{l} \prod_{i=1}^{l} M_{1,i,M_{2,i},M_{3,i}|s_i} \{ P(R_{s;\gamma}) < \epsilon \} > 1 - \delta, \]
when sensor distributions are unknown. For simplicity we represent \( P_{l} \prod_{i=1}^{l} M_{1,i,M_{2,i},M_{3,i}|s_i} \) by \( P \), which simplifies the above condition to \( P(P(R_{s;\gamma}) < \epsilon) < \delta \) or equivalently \( P(P(R_{s;\gamma}) < \epsilon) > 1 - \delta \). By exploiting the Vapnik-Chervonenkis (VC)-dimension \([1]\) of the class of regions \( R = \{ R_{s;\gamma} \} \) we show in this section that the condition
\[ P \{ P(R_{s;\gamma_{\max}}) > \epsilon \} < \delta \]
can be guaranteed for arbitrary \( \epsilon, \delta \in (0,1) \) given a training sample of size
\[ l = \frac{64}{\epsilon^2} \left[ 8 \ln \left( \frac{12}{\epsilon} \right) + \ln \left( \frac{1}{4\delta} \right) \right], \]
independent of the sensor distributions and their spatial correlations. This condition guarantees that with probability no higher \( \delta \) that the precision region \( R_{s;\gamma} \) fails to contain the plume source with probability no higher than \( \epsilon \). Equivalently, with probability at least \( 1 - \delta \) it can be guaranteed that the precision region \( R_{s;\gamma} \) contains the plume source with probability \( 1 - \epsilon \). We will derive a general relationship between \( l, \epsilon, \delta \) and empirical error on the training data. The quantities \( l, \delta \) and \( \epsilon \) are referred to as the sample size, confidence and precision, respectively.

### 3.1 Training Procedure
The training procedure consists of a measurement collection part with the following steps:

(i) A set of \( l \) sources \( s_1, s_2, \ldots, s_l \) are arbitrarily chosen within the monitoring region along with the corresponding times \( t_1, t_2, \ldots, t_l \).

(ii) For each \( s_i \), we compute the distances \( r_{ij}, i = 1,2,3 \) to the sensors. Then we estimate the intensities at the sensors \( I(r_{ij}, t) \) using the plume equation.

(iii) Then each sensor \( S_i \) is subjected to intensity level \( I(s_j) \) in a controlled laboratory environment, and the corresponding sensor measurement \( M_{ij} \) is collected.

The second part of the training consists of estimating a suitable precision value for DTM. For each set of measurements \( M_{1,j}, M_{2,j}, M_{3,j} \) we carry out difference triangulation computation with a small precision \( \gamma \). Let \( \gamma_i = d_i(s, \dot{s}_j) \) where \( \dot{s}_j = D_{\Delta}(M_{1,j}, M_{2,j}, M_{3,j}; \gamma) \).

### 3.2 Performance Analysis
We consider that training procedure of previous section be applied to a sample of size \( l \), and let the ordered set of precisions \( \gamma_1, \gamma_2, \ldots, \gamma_l \) be represented by \( \gamma(1), \gamma(2), \ldots, \gamma(l) \) such that \( \gamma_{\min} = \gamma(1) \) and \( \gamma_{\max} = \gamma(l) \). We first present a specific result in Theorem 1 without proof that utilizes the precision value \( \gamma_{\max} \) to illustrate the performance guarantee. Its proof follows from a more general result we prove in Theorem 3.

**Theorem 1**
Let \( \hat{s} = D_{\Delta}(M_1, M_2, M_3; \gamma_{\max}) \) be the source estimated by DTM using sensor measurements \( M_1, M_2, M_3 \) corresponding to plume origin \( s \). Then we have
\[ P \{ P(R_{\hat{s};\gamma_{\max}}) > \epsilon \} < \frac{(\epsilon l)^4}{4} e^{-\epsilon^2 l / 32}, \]
for any \( \epsilon \in (0,1) \) where \( \gamma_{\max} \) is computed based on the training sample of size \( l \).

The above upperbound on the probability will be decided by the exponential term for large sample sizes, and can be made equal to \( \delta \) by appropriately choosing large enough sample size. This guarantee is independent of sensor error distributions, which could be arbitrarily correlated, and does not require the knowledge of the underlying sensor distributions or their correlations. The following Corollary presents a sample size estimate obtained using Part 3 of Lemma 4.4 of \([21]\) (a slightly different sample bound \( l = \frac{64}{\epsilon^2} (8 \ln(12/\epsilon) + \ln(4/\delta)) \) can be obtained using Theorem 4.3 of \([1]\)).

**Corollary 2**
Under the condition of Theorem 1, we have
\[ P \{ P(R_{\hat{s};\gamma_{\max}}) > \epsilon \} < \delta \]
for any \( \epsilon, \delta \in (0,1) \) given a training sample of size
\[ l = \frac{64}{\epsilon^2} \left[ 8 \ln \left( \frac{24}{\epsilon} \right) + \ln \left( \frac{1}{4\delta} \right) \right]. \]
For any two dimensional region $R_{s_i;\gamma}$, we define empirical error $\hat{P}(R_s;\gamma)$ as

$$\hat{P}(R_{s_i;\gamma}) = \frac{1}{l} \sum_{i=1}^{l} I_{\{s_i \in R_{s_i;\gamma}\}},$$

where $I_{s_i \in R_{s_i;\gamma}}$ is the indicator function (i.e. returns 1 if $s_i \in R_{s_i;\gamma}$, and 0 otherwise).

**Theorem 3** Let the sensor network be trained using sample of size $l$. Given the sensor measurements $M_1, M_2, M_3$ corresponding to plume origin $S$, and any $\gamma \in [\gamma_{\min}, \gamma_{\max}]$, we have

$$P\{P(R_{s_i;\gamma}) > \epsilon\} < \frac{(\epsilon l)^4}{4} e^{-(\epsilon - \epsilon l)^2/32}$$

where

$$\epsilon l = \frac{1}{l} \sum_{i=1}^{l} I_{\{s_i \in R_{s_i;\gamma}\}},$$

for $\hat{s} = D_{\Delta}(M_1, M_2, M_3; \gamma)$.

**Proof** The proof is fairly routine in the area of learning theory [1] and we will only provide an outline. Let $R = \{R_{s_i;\gamma}\}$ consist of all possible prediction regions returned by DTM. Each $R_{s_i;\gamma}$ is a real algebraic set that can be expressed as an intersection of four half-spaces each bounded by a hyperbolic curve, and is homeomorphic to a rectangle. Set of rectangles in plane has a VC-dimension of 4, and essentially the same arguments (for example see Example 4.4 of [21]) can be used to show that $VC - dim(R) = 4$. Using Theorems 3.7 and 4.3 of [1] we have

$$P\{P(R_{s_i;\gamma}) - \hat{P}(R_{s_i;\gamma}) > \epsilon\} < 4\frac{(\epsilon l)^4}{4} e^{-(\epsilon - \epsilon l)^2/32}.$$ 

The proof is complete by utilizing $\epsilon = \epsilon - \epsilon l$. □

Proof of Theorem 1 follows by noting that $\epsilon l = 0$ and $\hat{P}(R_{s_i;\gamma})$ for $\gamma = \gamma_{\max}$.

Following along the lines of Corollary 2, the condition of Theorem 3 is satisfied given a training sample of size

$$l = \frac{64}{(\epsilon - \epsilon l)^2} \left[ 8 \ln \left( \frac{24}{(\epsilon - \epsilon l)} \right) + \ln \left( \frac{1}{4\delta} \right) \right].$$

For a fixed $\delta$, this sample size corresponds to tighter (smaller) precision region compared to Theorem 1. However, to ensure same $\delta$ value, the sample size must be larger due to smaller $\epsilon - \epsilon l$ in the denominator compared to $\epsilon$.

### 4 Simulation Results

We simulated a network of three sensors on a $[0, 100000] \times [0, 100000]$ grid such that $S_1$ and $S_2$ are located at $(0, 0)$ and $(100000, 0)$ respectively. Location of $S_3$ is randomly generated on the line segment between $(0, 100000)$ and $(100000, 0)$. Each sensor measurement corresponds to $(1 + f)r$ where $r$ is the actual distance from the sensor to plume source, and $f$ is uniformly randomly generate in the interval $[0, F]$ for a fixed multiplicative factor $F$. While $f$ values are generated independently, note the sensor error is multiplicative; more precisely, the error magnitude is proportional to the distance from the sensor to plume origin. Due to the spatial relationships between the sensor locations, the sensor errors are correlated. For example, a plume source close to $S_3$ will generate small measurement error there, whereas the errors at both $S_1$ and $S_2$ are larger. This is a simple error model, but the sensor errors are not independent.

We implemented a linear algebra based method [11, 16] which proved to be incomplete since quadratic equation had imaginary roots for a small percentage of sources as shown in Table 1 based on simulation of 100,000 plume sources. The generated sources are shown in Figure 6(a) which are uniformly distributed across $[0, 100000] \times [0, 100000]$ grid. For the case $F = 5$ in Table 1, about 0.32% of the sources yielded imaginary solutions to the quadratic equation. Interestingly, the sources that yielded imaginary roots are concentrated around the sensors as shown in Figure 6. Note that a proximity to a sensor implies high degree of correlation between the measurements at the other two sources. The method of [4] requires the sensor measurements to be independent Gaussian and hence is not directly applicable to our simulation example.

Results of our training method are shown in Table 2 for sample of size 10, where $\gamma_{\min} = 7.856613$ and $\gamma_{\max} = 29.011267$. This method is complete and always returned the precision region for the training sample. The worst-case location error is of the order $0.0003\%$, which can be chosen as the final precision parameter of DTM. Choosing $\gamma = 29.011267$ for the future precision of DTM enables us to utilize the performance guarantees in Corollary 2. For smaller values of $\gamma$, Theorem 3 would be applied by appropriately choosing $\epsilon l$ value from Table 2, which typically requires a larger sample (for the same $\delta$) to counter balance the reduction in precision by $\epsilon l$. This method is implemented in C on Linux workstation and the typical execution times for dataset of this size are under one second.

### 5 Conclusions

We presented a simple and idealized plume detection problem and a solution using a network of three inten-

<table>
<thead>
<tr>
<th>$F$</th>
<th>imaginary roots percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>0.32</td>
</tr>
<tr>
<td>10</td>
<td>0.963</td>
</tr>
</tbody>
</table>

Table 1: Listing of percentage of imaginary solutions to the DTOA quadratic equation as a function of the multiplicative factor $F$. 


The sensors deployed in the field are generally more sophisticated, less precise, and error-prone compared to our ideal intensity sensors. Sensors with directional capabilities might help in estimating the plume origin. The sensors might themselves be probabilistically specified in that the measurements are probabilistically related to the plume intensity being measured. Furthermore, the sensors themselves may be mobile and may be actively steered to aid the plume detection.

Under more general plume and sensor characterizations, it might not always be possible to accurately detect and identify the plume, which makes the tracking and prediction much more challenging. In fact, multiple plume models may fit the sensor measurements, and further sensor monitoring may be needed to reduce the number of candidate models. Also, the detection and identification problems, even at less accurate levels, might be computationally intractable, thereby requiring approximate solutions. Also, the sensors need

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\gamma_i$</th>
<th>$\sigma = P(R_i, \gamma_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.011267</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>22.744946</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>18.004549</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>25.958393</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>7.856613</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>14.310971</td>
<td>0.8</td>
</tr>
<tr>
<td>7</td>
<td>27.373035</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>26.435724</td>
<td>0.1</td>
</tr>
<tr>
<td>9</td>
<td>25.848469</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
<td>23.103365</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 2: Empirical errors corresponding to different $\gamma$ values.
to be supported by an underlying communications network, which might have to be optimized to support efficient plume identification and tracking computations.

The case of more than three sensors has been extensively studied in the DTOA-based localization methods [15]. It would be interesting to study such extensions of the methods proposed in this paper.

Acknowledgments
This work is funded by the SensorNet program at Oak Ridge National Laboratory (ORNL) through Office of Naval Research. ORNL is managed by UT-Battelle, LLC for U.S. Department of Energy under Contract No. DE-AC05-00OR22725.

References


