A Track Before Detect Approach for Extended Objects

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Abstract - This paper deals with target tracking for extended objects in a track before detect context. In the scope of this paper a target is called extended if its physical size is large enough to occupy multiple (radar) resolution cells, e.g. in range and/or azimuth. We show how the existing track before detect approach can be amended in order to deal with extended targets. The algorithm, that we propose, will jointly estimate on-line both the standard kinematic parameters of the target, i.e. position and velocity, as well as the size or extent of the target. The estimation is performed by means of a particle filter. It is shown that the extended target approach is significantly superior in terms of performance to a point target approach in case the target is extended.

Keywords: Tracking, extended objects, track before detect, particle filter, radar.

1 Introduction

In this paper we focus on an extended target tracking application. The application under consideration here deals with tracking on the basis of unthresholded measurements, as opposed to tracking on the basis of thresholded measurements, plots. This method is sometimes also referred to as Track Before Detect (TBD), see chapter 11 of [1] for an excellent overview. An efficient method to implement such a TBD processing is provided by a particle filter, see the seminal papers [2] and [3]. Quite some subsequent work in this area has been performed recently, see e.g. [1], [4], [5] and [6].

Most of the times in a target tracking application, targets are assumed to be point targets. Often this is quite a good approximation. However, there also exist situations in which this not the case. If in such situations the target is still treated as a point target performance degradation or worse failure or divergence of the algorithm might occur. In the context of this paper, we call a target extended whenever the target extent is larger than the sensor resolution. Thus, whether or not a target is considered to be extended does not only depend on the actual physical size of the target, but on the size relative to the sensor resolution.

Recent work on extended target tracking on a plot basis has been performed in [7], where a diffuse spatial distribution over the target extent has been assumed to exist. Here we will follow a similar approach, but unlike in [7], we do not have to deal with data association hypotheses, because we consider the raw data here, i.e. without applying a threshold, see also [4], [5] and [6]. Another difference with the approach of [7] is that we do not assume knowledge of the target extent. In this paper the target extent is to be inferred from the data. Furthermore, we also model the presence or absence of a target through a Markov process and we treat this discrete variable as a mode. Thus, in fact we use a multiple model filtering approach.

In the remainder we will present a system setup, a model for the target extent, comparative simulations and conclusions.

2 System Model

Consider a general nonlinear jump-Markov system evolving according to

\[ s_{k+1} = f(s_k, m_k, w_k), \quad k \in \mathbb{N}, \]  

\[ \text{Prob}(m_{k+1} = i | m_k = j) = \Pi_{ij}, \]  

\[ z_k = h(s_k, m_k, v_k), \quad k \in \mathbb{N}. \]  

where

- \( s_k \in \mathbb{R}^n \) is the state of the system
- \( m_k \in \mathbb{N} \) is the modal state of the system
- \( z_k \in \mathbb{R}^p \) is the measurement
- \( w_k \) is the process noise
• $v_k$ is the measurement noise
• $f$ is the system dynamic function
• $h$ is the measurement function
• $\Pi$ is the Markov matrix, including all modal states transitions.

The system defined by equations (1), (2) and (3) is referred to as a jump Markov system, see e.g. [8] and [9]. The key feature of such a system is that it involves both continuous as well as discrete state variables. The continuous state variables represent kinematic information, e.g. position and velocity. The discrete state variable describes the modal state. In this application the modal state is an indicator of target absence/presence.

2.1 Dynamical Model

In the dynamical model that we use here both the target’s center dynamics as well as its extent are modelled within the dynamical model.

In our application we will consider a target that is extended in only one dimension, i.e. a 'stick like' target. However the method can be used or adopted to deal with targets that are extended in multiple dimensions.

As a state vector we consider:

$$s_k = [x_k, y_k, \dot{x}_k, \dot{y}_k, L_k]^T. \quad (4)$$

Here $x_k, y_k$ is the position of the center of the target and $\dot{x}_k, \dot{y}_k$ is the velocity of the center of the target. $L_k$ is the extension or length of the target.

The systems dynamics are:

$$s_{k+1} = f(s_k, m_k) + g(s_k, m_k, w_k) \quad (5)$$

with

$$f(s_k, m_k) = \begin{pmatrix} 1 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} s_k. \quad (6)$$

where $T$ is the update time. The process noise input model is given by

$$g(s_k, m_k, w_k) = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ \sigma_{\dot{x}} & 0 & 0 \\ 0 & \sigma_{\dot{y}} & 0 \\ 0 & 0 & \sigma_l \end{pmatrix} w_k. \quad (7)$$

where $w_k = [w_{k1}, w_{k2}, w_{k3}]^T$, and $\{w_{ka}\}_{a=1}^{3} \sim \mathcal{N}(0, 1)$. The modal state of the system evolves according to a transitional probability matrix,

$$m_{k+1} = \Pi_{ij}(m_k) \quad (8)$$

where

$$\Pi_{ij} = \begin{pmatrix} 1 - P_b & P_b \\ P_d & 1 - P_d \end{pmatrix}. \quad (9)$$

This transition model represents transitions from target absence to presence and vice versa.

2.2 Measurement Model

First we briefly describe the measurement model under the assumption of a point target. This description has been taken from [5].

As already mentioned, the measurements consist of the power levels in $N_r \times N_d \times N_b$ sensor cells, where $N_r, N_d,$ and $N_b$ are the number of range, Doppler, and bearing cells respectively.

In figure 2 an illustration of part of a measurement is given. The power is plotted as a function of range and Doppler cells for one fixed bearing angle. The target is clearly visible in the measurement data. In this example the Signal to Noise Ratio (SNR) is 10dB.

Thus one measurement consists of a frame of reflected target power levels over a three dimensional array.

The measurement model describes how these measurements are related to the target state.

Figure 2: Illustration of power measurements as a function of range and Doppler cells for one fixed bearing angle, $SNR = 10dB$.

$$z_k = v_k, \quad m_k = 0, \quad (10a)$$

$$z_k = h(s_k, v_k), \quad m_k = 1. \quad (10b)$$

The power measurements per range-Doppler-bearing cell are defined by

$$z_{ijl}^k = |z_{Aijl}^k|^2, \quad k \in \mathbb{N} \quad (11)$$
where \( z_{ijk} \) is the complex amplitude data of the target which is

\[
z_{A,k} = A_k h_A(s_k) + n_k, \quad k \in \mathbb{N}
\]

(12)

where

\[
A_k = \tilde{A}_k e^{i \phi_k}, \quad \phi_k \in (0, 2\pi)
\]

(13)

is the complex amplitude of the target and \( h_A(s_k) \) is the reflection form that is defined for every range-Doppler-bearing cell by

\[
h_{ijl}^A(s_k) = e^{-\frac{(r_i - r_k)^2}{2r^2}} L_r \frac{i}{2} \frac{(r_i - r_k)^2}{2r} L_d - \frac{(y_i - y_k)^2}{2y^2} L_o,
\]

(14)

\( i = 1, \ldots, N_r, \quad j = 1, \ldots, N_d, \quad l = 1, \ldots, N_b \) and \( k \in \mathbb{N} \).

The relation between the Cartesian coordinates in which the system state is given and the radar coordinates is given through the nonlinear transformation,

\[
r_k = \sqrt{x_k^2 + y_k^2}, \quad d_k = \frac{x_k}{x_k}, \quad b_k = \arctan\left(\frac{y_k}{x_k}\right)
\]

(15)

(16)

(17)

which are the range, Doppler, and bearing respectively of the target. \( R, D \) and \( B \) are constants, related to the size of a range, a Doppler and a bearing cell. \( L_r \), \( L_d \) and \( L_o \) represent constants of losses.

The noise is defined by

\[
n_k = n_{lk} + i n_{qk}
\]

(18)

which is complex Gaussian, where \( n_{lk} \) and \( n_{qk} \) are independent, zero-mean white Gaussian with variance \( \sigma_n^2 \). In this way the power measurements in a range-Doppler-bearing cell are defined by

\[
z_{ijl}^2 = |z_{ijl}^A|^2 = |A_k h_{ijl}^A(s_k) + n_{lk} + i n_{qk}|^2
\]

(19)

These measurements conditioned on the system state \( \{ s_k, m_k \} \) are now assumed to be exponentially distributed, and the likelihood function is given as

\[
p(z_{ijl}^A|s_k, m_k) = \frac{1}{H_{ijl}^0} e^{-\frac{\|z_{ijl}^A\|^2}{H_{ijl}^0}},
\]

(20)

where

\[
H_{ijl}^0 = E[z_{ijl}^A|s_k, m_k].
\]

(21)

where

\[
H_{ijl}^0 = P h_{ijl}^A(s_k, t_k) + 2\sigma_n^2
\]

(22)

with

\[
h_{ijl}^A(s_k, t_k) = \left(h_{ijl}^A(s_k, t_k) \right)^2 =
\]

(23)

Observe that the likelihood under the noise only assumption is readily obtained from the above formulations as well, i.e. for \( P = 0 \).

More detail on the above model can be found in [5].

2.3 Extended Target Model

In this subsection we will introduce the extended target model. We start out by partially following the discussion in section 2.3. of [7]. In that paper a so called spatial distribution model for extended objects is assumed. Furthermore, we will point out how this model can be used in the filter, i.e. through modification of the likelihood.

The spatial extension is modelled by the probability density function (pdf) \( p(\tilde{s}|s) \). This pdf can be interpreted as a generator of a point source \( \tilde{s} \) from an extended target with its center given by the state vector \( s \).

Receiving a measurement from a source \( \tilde{s} \) somewhere on the target leads to a likelihood \( p(z|\tilde{s}) \). Using this model the total likelihood is obtained as the convolution between \( p(z|s) \) and \( p(\tilde{s}|s) \), i.e.

\[
p(z|s) = \int p(z|\tilde{s}) p(\tilde{s}|s) d\tilde{s}
\]

(24)

Note that we have dropped the dependence on the modal variable.

Furthermore, the model of equation (24) also covers the point target case. This case is obtained or retrieved for:

\[
p(\tilde{s}|s) = \delta(\tilde{s} - s)
\]

(25)

Where \( \delta \) denotes the delta function.

Also the case of a finite number of point target sources over the target extent is covered by the above model. For example if there are \( n \) sources at location \( s'\), \( i = 1, \ldots, n \), the pdf \( p(\tilde{s}|s) \) is defined as:

\[
p(\tilde{s}|s) = \sum_{i=1}^{n} w(s') \delta(\tilde{s} - s')
\]

(26)

In the application under consideration here, we assume a target that is extended in one dimension. Given the state vector:

\[
s_k = [x_k, y_k, \hat{x}_k, \hat{y}_k, L_k]^T
\]

(27)

The center of the target is \([x_k, y_k]^T\) and the orientation of the target is assumed to be along the velocity vector \([\hat{x}_k, \hat{y}_k]^T\) and its length is \(L_k\). This length, in practice fixed, is added to the state vector and treated as an unknown. The 'dynamics', assumed for this truly static parameter is almost trivial and equals:

\[
L_{k+1} = L_k + w_{L_k}
\]

(28)

This is in accordance with the model introduced in section 2.1. So, the (particle) filter will also estimate the length of the target.

The general expression for the likelihood of an extended target, as represented by equation (24), might not be available analytically. In this case this expression might always be approximated numerically.
through an importance sampling approach. This approximation is readily calculated by:

\[
p(z|s) \approx \frac{1}{M} \sum_{r=1}^{M} p(z|\tilde{s}^r) \quad (29)
\]

where \(\tilde{s}^r, r = 1, \ldots, M\) independently drawn according to \(p(\tilde{s}|s)\).

### 3 Simulations

We present results of an extended target filter and also compare the extended target filter with a point target filter. This comparison clearly shows that it pays off to account for the target extent in the filter.

A particle filter (PF) will be used on simulated measurements from a target with a physical spatial extent, such that the target occupies multiple resolution cells. The performance of a PF with a point target model will be compared to a PF with an extended target model, in the example referred to as ETPF.

#### 3.1 Comparison of different filters

It has been observed that if the 'plain' point target model is used on extended target data the filter often diverges. The reason for this divergence does not lie in type of filter used, i.e. a particle filter, but due to a model mismatch: the target extent induces virtual accelerations. These virtual accelerations, \(a_v\), could 'worst case' amount to

\[
a_v = \frac{2L}{T^2} \quad (30)
\]

In the proceeding we will compare three types of filters. These are the point target filter and two versions of the extended target particle filter. In the point target filter we have increased the amount of process noise to avoid divergence. In fact the amount of process noise has been increased by a factor of \(\frac{1}{2}a_v\), this proved to be enough to ensure convergence of the filter. Furthermore we will test the extended target particle filter with a level of process noise, accounting only for true target accelerations this filter is referred to as ETPF\(_1\). Thirdly we will also test a version of the extended target particle filter, for which the process noise has been matched to same level as the one used in the point target filter. This filter is referred to as ETPF\(_2\). This third model allows us to isolate the performance gain due only to the adaption of the measurement model.

In the simulation the target is assumed to be extended in one dimension, its extent is assumed to be 20m. As can be seen from table 1, this means that the target may, depending on its orientation, occupy as much as 10 range cells.

![Figure 3: Trajectory and particle filter output for the position at time step 10](image)

Furthermore, in the example the target appears at time \(k = 6\) at \([9.65, 0]\) km, and is initially moving with a constant velocity of \([-10, 0]\) m/s towards the sensor. The dynamics of the target are captured by a constant velocity model.

The standard deviations for the process noise inputs are listed in table 2. The maximum target accelerations, i.e. without the virtual acceleration term, are assumed to be \(a_{x,max} = 4\) m/s\(^2\) and \(a_{y,max} = 4\) m/s\(^2\).

Furthermore, the transition probability matrix is assumed to be

\[
\Pi_{ij} = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix} \quad (31)
\]

The average target SNR has been set to 10 dB and is assumed to be known to the filter. We emphasize that this assumption can be relaxed on straightforwardly
by including the average SNR or equivalently target Radar Cross Section (RCS) into the filter, see e.g. [1]. Measurement data for both a straight moving ob-
ject and a turning object have been generated. For the turning object the lateral acceleration was 1m/s². This is well inside the specified maximum acceleration bounds.

All filters (all of the ‘particle filter’ type!) run with 2000 particles. This has been proven (on a simulation basis) to be enough for this application.

Illustrative results of the scenario and the filter behavior are shown in figures 3, 4, 5, 6, 7 and 8. Note that these figures are from a single scenario. It is interesting to see that in the beginning of the scenario, both the length and orientation of the object are not estimated very well, see especially figure 4. This improves considerably later on in the scenario.

Furthermore, figures 9, 10, 11, 12, 13, and 14 show the performance of the different filters over 100 Monte Carlo runs. It is clear from these performance figures that the extended target model leads to an improved performance in terms of accuracy. This holds even for case where the process noise level of the extended target filter (ETPF₂) has been increased to match the level of the process noise in the point target filter.
4 Conclusions

In this paper we have shown how to use a track before detect approach for extended targets. It has been shown that a processing, that can deal with such an extension is fairly easily implemented through a particle filter. Furthermore, it has been shown that using an extended target model in the filter greatly improves upon the performance of the filter in terms of track accuracy.

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References