Distributed Environmental Inversion for Multi-Static Sonar Tracking

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Abstract – This paper presents an approach for adapting a tracking algorithm to the acoustic propagation environment. This adaptation is performed by incorporating the expected target signal-to-noise ratio (SNR) into the data association step through the measured contact amplitude, as has been shown previously by [1]. In this work, expected SNR is provided via acoustic modeling; estimates of bottom loss and scattering strength, required by the acoustic model, are obtained via inversion of the acoustic model based on measured multi-static sonar reverberation data. This paper shows that the use of distributed sensors provides improved estimates of the environmental parameters, and hence better estimates of the expected SNR.

Keywords: Tracking, data association, environmental estimation, distributed sensors.

1 Introduction

Most target tracking algorithms are contact-based sequential state estimators [2]; however, target amplitude has been incorporated into tracking algorithms (e.g., [1]). Such methods involve the use of the probability density function (PDF) for the observed signal. In active sonar, this PDF is dependent on the expected SNR, which is a spatially and temporally varying quantity. The expected SNR is dependent on the source/receiver geometry, target state, and the acoustic propagation conditions, which in turn are dependent on environmental parameters such as bathymetry, sound speed, and bottom geo-acoustic parameters. While bathymetry and sound speed profile are often available, bottom geo-acoustics are typically unknown; however, they can have a significant impact on the accuracy of the expected SNR predicted by an acoustic model. To address this dependence, bottom loss and scattering strength functions are estimated via acoustic inversion using measured reverberation data. The estimated parameters are then used in the forward acoustic model to provide an improved estimate of the SNR, which can then be incorporated into the tracking algorithm via the target measurement PDF.

The paper addresses the problem of “environmentally-adaptive multi-static tracking.” Multi-static sonar measurements will be used to provide improved environmental parameter estimates as compared to those from a mono-static system. These improved parameter estimates will in turn lead to an improved estimate of the expected SNR, which can then be incorporated into the tracker signal processing. This paper will not address the question of which tracker architecture (e.g., extended Kalman filter, probabilistic multi-hypothesis tracker, etc.) is best for distributed environmentally-adaptive tracking. Rather, the focus is on the merging of the tracker signal processing with acoustic modeling and geo-acoustic inversion. The estimate of the expected SNR can be incorporated into the tracker via the approach of [1], which is reviewed next.

2 Motivation: Tracking with Target Amplitude

2.1 Background

In the standard approach to sequential estimation for target tracking, the target motion model is written as

$$x_{k+1} = Fx_k + Gv_k \quad (1)$$

where $x_k$ is the target state (e.g., position and velocity) at time $k$, and $v_k$ is the innovations process (residual). The measurement vector $z_k$ is related to the target state by

$$z_k = h(x_k) + w_k \quad (2)$$

where $h(\cdot)$ is the possibly nonlinear measurement function and $w_k$ is white noise. Tracks are formed and maintained on contacts that have exceeded some predefined detection threshold, set to achieve a desired $P_D$ and $P_{fa}$ based on an expected SNR and a priori data PDFs. If the expected SNR is erroneous, a degradation will be observed as either excessive false contacts or missed detections for weak targets. In [1], a method was presented for estimating the SNR from the target and false contact statistics and using the estimated SNR in the track association stage of the tracker. This method is known as Probabilistic Data Association Filter with Amplitude Information (PDAFAI) [1], and is described next.
2.2 Review: PDAFAI

This section reviews the PDAFAI as presented in [1]. Let $p^t_0(a)$ be the PDF of the measured data, where $a$ denotes the amplitude of the contact from the detector. Possible PDFs include the Rayleigh, mixture Rayleigh, and K distributions [3,4]. The PDF of the data when a target is present is denoted by $p^t_1(a)$; a common example is the Swerling I distribution, which was used in [1]. The reverberation is taken to be normalized so that the target PDF is parameterized by the SNR, denoted by $d$. So for the case of a Swerling I target and Rayleigh reverberation, the PDFs are given by:

$$p^t_0(a) = \frac{a}{1+d} \exp(-a^2/(2(1+d))) / P_d, a \geq \tau$$

and

$$p^t_1(a) = a \exp(-a^2/2)/ P_{FA}, a \geq \tau$$

where $d$ is the expected SNR of the target return, and the PDFs have been renormalized for the range of amplitudes that have exceeded the detection threshold $\tau$.

Given the $i$th contact at time $k$ with amplitude $a^i_k$, the “amplitude likelihood ratio” is defined as:

$$\lambda_i = \frac{p^t_1(a^i_k)}{p^t_0(a^i_k)}$$

This value is incorporated into the standard PDAF by modifying the association event probabilities $\{\beta_i\}$:

$$\beta_i = \frac{e_i \lambda_i}{b + \sum_{j=1}^{m_k} e_j \lambda_j}, \beta_0 = \frac{b}{b + \sum_{j=1}^{m_k} e_j \lambda_j}$$

where $e_i = N(v^i_k, 0, S_k)/P_0$, $m_k$ is the number of contacts within the track gate, $P_0$ is the probability that the measurement lies in the track gate, $v^i_k$ is the track residual, $S_k$ is the covariance of the residual, and $V_k$ is the volume of the track validation gate. The standard PDAF algorithm then proceeds as usual. See [1] for further details.

In [1], a predetermined SNR was used for track formation; for track maintenance, SNR was estimated from an observed track once enough target detections were available. In this work, the initial (expected) SNR is obtained from an acoustic model whose input parameters are updated based on measured reverberation, as described next. Note that the acoustic model could also be updated based on observed target SNR estimated from the data, as in [1]. That approach is not addressed here.

3 Distributed inversion of multi-static sonar data

The objective of the multi-static environmental inversion is to improve the estimate of the ocean bottom properties given additional sensors. The improved inversion in turn yields an improved estimate of the expected SNR, with the ultimate goal of improving the performance of the tracking algorithm. In this inversion, bottom loss (BL) and bottom scattering strength (SS) as functions of grazing angle, as well as their covariance and angular resolution, are estimated from reverberation time series. In this section, the inversion is demonstrated using both a simulated scenario with idealized geometry and real sonar data to clearly illustrate the improvement in BL and SS estimation that can be achieved with additional sensors. It is shown that with the addition of a second receiver, the variance of the bottom properties decreases and their resolution in grazing angle increases (corresponding to a decrease in the bias of the estimator). The inversion method used is the regularized iterative Gauss-Newton method, so an initial estimate of the solution is required from which to iterate. Once the solution is found, the standard deviation and resolution of the solution are computed and used to compare the three cases (receiver #1, receiver #2, and both receivers). The extension of the inversion to more than two receivers is straightforward.

3.1 Acoustic simulation definition

The simulation environment used for this demonstration is a ray-based ocean acoustic model called the Comprehensive Acoustic Simulation System (CASS) [5,6]. The "forward problem" in this work is the multi-static CASS simulation which computes the mean intensity of ocean acoustic reverberation as received at one or two receivers, from one source, over a sloped bottom that is otherwise geographically homogeneous. The forward problem input is a concatenated vector of the bottom loss (BL) and bottom scattering strength (SS) properties of the ocean bottom versus grazing angle. The output is a concatenated vector of the mean acoustic reverberation intensity versus time from one or two receivers. The geometry and other parameters such as frequency and sound speed profile are defined in Figure 1 and Table 1. The simulated data is a vector of noisy ocean acoustic reverberation in decibels from one or two receiver locations; the inverse problem is to estimate the concatenated vector of BL and SS, called the "model", from the reverberation data.

The BL and SS model is in fact a continuous function over angle, but this continuous function is parameterized by a finite number of parameters in order to compute the inverse solution. The choice of this finite parameterization must be done with with some thought to avoid imposing an arbitrary structural preference upon the model by the parameterization. In this demonstration the model is simply parameterized by discretization into one-degree angular intervals, a fine resolution which is chosen to sample all BL and SS features. These many angles overparameterize the model, an issue that is compensated via regularization, so that the actual resolution of the inverted model (which will be coarser than the one-degree discretization) will be figured as an inversion result rather than arbitrarily assumed beforehand. This process of parameterization and regularization is detailed in numerous inverse theory textbooks; the reader is referred to [7], for example.
concatenation of the time series from both receivers. For the two-reverberation case the vector of simulated data is the matrix is thus assumed to be diagonal. For the two-reverberation time series in decibels. The data covariance then by adding white Gaussian noise to the simulated compute the associated mean reverberation intensity, and model of BL and SS by using the forward problem to reverberation data is generated from the specified true (defined as the integral of the second derivative over angle), which discourages model features that are not required by the data.

In this demonstration the true model, from which the simulated data is computed, is associated with a bottom of "very fine sand". The iteration is initialized with a bottom model corresponding to "coarse sand". The inversion produces an estimate of the underlying geo-acoustic model. Measures of the quality of this estimate at each grazing angle are obtained from the standard deviation and resolution information of the inverted model solution.

The reverberation data are sampled at 20 samples per second, double the Nyquist rate of the reverberation as determined by the pulse length. The reverberation is defined here to be the fluctuations due to the scattering from only the sea bottom (as opposed to including surface or volume scattering), and is in the temporal range where the reverberation level is high enough to neglect the ambient noise. The reverberation is assumed to be Rayleigh distributed with constant variance in the log domain. The log of a smoothed Rayleigh random variable is approximately Gaussian. Accordingly, the simulated reverberation data is generated from the specified true model of BL and SS by using the forward problem to compute the associated mean reverberation intensity, and then by adding white Gaussian noise to the simulated reverberation time series in decibels. The data covariance matrix is thus assumed to be diagonal. For the two-receiver case the vector of simulated data is the concatenation of the time series from both receivers.

### Table 1. Acoustic simulation parameters. Only bottom reverberation is computed for this simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3500Hz</td>
</tr>
<tr>
<td>Pulse length</td>
<td>0.2s</td>
</tr>
<tr>
<td>Reverberation sampling rate</td>
<td>20 samples per second</td>
</tr>
<tr>
<td>Horizontal beamwidth of source and both receivers</td>
<td>1 deg centered in the vertical shared plane</td>
</tr>
</tbody>
</table>

In estimating the best model vector of BL and SS versus grazing angle associated with a given vector of measured reverberation data, it is important to note that one does not simply solve for the minimum data misfit. The measured data are noisy, and one does not want to perfectly match the noise. There are an infinite number of model vectors which equally fit the data to within the noise. In addition, the forward problem has a null space in it, so that an infinite number of model vectors equally fits the same data even without noise, as they can vary in the null space without constraint. For both of these reasons, one solves for an adequate misfit that is within the data’s noise and which also meets some additional criteria on the model. The additional criteria is the regularization and maximizes the smoothness of BL and SS over angle (defined as the integral of the second derivative over angle), which discourages model features that are not required by the data.

Inversion is performed using the regularized, iterated, Gauss-Newton method, as described in [7]. The inversion is performed based on received power rather than from the acoustic pressure signal itself, so that the forward problem is weakly nonlinear. As a result, the problem can be locally linearized given an initial estimate in a neighborhood of the solution. Steps are calculated toward the solution model using the Jacobian matrices of derivatives of the data with respect to the model at the points of linearization. Derivatives are approximated by finite differences. At the solution point the problem is linearized one more time, and the covariance and resolution matrices for the solution model are calculated. Since the forward problem is nonlinear and the PDF of the solution is non-Gaussian, the covariance matrix by itself only partially describes the statistics of the solution model. Monte Carlo methods can more fully approximate the non-Gaussian statistics of the solution model but would run prohibitively slowly with this problem.

The iterated Gauss-Newton method is regularized using higher order Tikhonov regularization according to the following formulation. The locally linear problem to be solved for a perturbation step is:

\[
\begin{bmatrix}
A(m_o) \\
\nu L
\end{bmatrix}
\delta m =
\begin{bmatrix}
\delta d \\
-\nu L m_o
\end{bmatrix}
\]

(6)

\(A(m_o)\) is the Jacobian matrix of partial derivatives of the reverberation in dB with respect to the BL and SS in dB evaluated at the model point \(m_o\) (and approximated by finite differences). The residuals at the model point \(m_o\) are \(\delta d = d - A(m_o)m_o\) where the noisy data are \(d\). For the inverse problem in this paper, \(L\) is a 2\(^{nd}\) order finite difference operator used in maximizing the smoothness of the solution model. The scalar regularization parameter \(\nu\) balances the data misfit and model smoothness norm in order to only fit the noisy data within the statistics of the noise. Its value is chosen by

![Figure 1. Geometry and sound speed profile (SSP) of the multi-static inversion simulation.](image-url)
the L-curve method [8] because the data noise level is unknown to the inversion code. Since the data noise is assumed Gaussian, we can measure the data misfit in terms of the $L^2$ norm of the residuals and use maximum likelihood and least squares to find the most likely solution to the problem defined in Equation (6). The perturbation solution is guaranteed to be unique due to the regularization, and is given by:

$$
\delta \hat{m} = (A^T A + \nu^2 L^T L)^{-1} (A^T \delta d - \nu^2 L^T L m_o) \quad (7)
$$

The 2nd order finite difference operator $L$ has boundary conditions of zero curvature on the boundary points. These are implemented by adding a row of zeros at each end of the BL and SS portions of the matrix – one row at the top, one row at the bottom, and two rows in the middle between the BL and SS. The two middle rows also serve to separate the smoothing on the BL and SS vectors so that the jump between the concatenated BL and SS vectors is not smoothed. The use of a 2nd order finite difference operator is to discourage features in the inverted model that are not required by the data.

If the covariance matrix of the noisy data is not available it must be estimated from the residuals at the solution point. The sample standard deviation $s$ of the data noise is calculated from the residuals, the number of data points $N$, and the number of model parameters $M$:

$$
s = \sqrt{\frac{(d - A(m_o) m_o)^T (d - A(m_o) m_o)}{N - M}} \quad (8)
$$

The data covariance matrix is thus assumed to be of the form $s^2 I$, which follows from the covariance of the reverberation data. The covariance matrix $C_m$ at the solution point under this formulation is based on $s$ and the generalized inverse $A^+$ of the linearized forward problem at the solution:

$$
A^+ = \left( A^T A + \nu^2 L^T L \right)^{-1} A^T \quad (9)
$$

$$
C_m = s^2 A^+ A^{+T} \quad (10)
$$

To avoid numerical issues when computing the matrix inverse within $A^+$, the inverse is done via generalized singular value decomposition of $A$ and $L$. The resolution matrix $R$ at the solution point is defined as:

$$
R = A^+ A \quad (11)
$$

This matrix is an MxM symmetric matrix, where $M$ is the number of model parameters. Note the generalized inverse in Equation (9) was also used in finding the model solution point in Equation (7), but the generalized inverse solution is unfortunately a biased estimator of the true solution. The resolution matrix gives information about that bias. If the resolution matrix equals the identity matrix, the resulting estimate is unbiased. But generally there are features of the true model that cannot be recovered in the inversion because the forward problem has smeared them out. Model parameters which are not recoverable at the full resolution of their parameterization have a wider diagonal at their location in the resolution matrix because they are weighted averages of other parameters. Often one plots the diagonal of this matrix to simplify interpretation of the resolution matrix. The axes of this plot are the “resolution index” on the vertical axis and grazing angle on the horizontal axis, and reflect the amount of bias in the model estimate caused by the regularization. The values will be one for model parameters that are perfectly resolved, close to one for well resolved parameters, and less with worsening resolution. The smaller the resolution index is at a given grazing angle, the more the inverted model estimate at that angle is a weighted average of model values at other grazing angles, and the more biased the estimate at that angle is. Due to this bias, the true model at that angle could even be far outside the confidence region around the model estimate.

### 3.3 Simulation results

The inverted model solutions, standard deviations, and resolutions are compared between the single- and two-receiver cases in Figures 2-4. The first point to make about the plots of the models in Figure 2 is that outside the roughly 3-17 degree angular range, the inverse solution has very poor fit to the true model (upon which the simulated data were based) in both BL and SS. This limited range in angles is due to the geometry of the problem: only certain grazing angles are sampled, so the data only contains information about the bottom at those angles. The model values outside that angular range are then constrained by the regularization, and the poor quality of the values at those angles is reflected in the covariance and resolution of the inverse solution.

**Figure 2.** Inverted solution models of bottom loss (BL) and bottom scattering strength (SS), for one and two receivers, with comparisons to the true model.
Figure 3. Standard deviations of the inverted solutions of bottom loss (BL) and bottom scattering strength (SS), for receiver #1 only, receiver #2 only, and both receivers.

Figure 4. Diagonals of the resolution matrices of bottom loss (BL) and bottom scattering strength (SS) for receiver #1 only, receiver #2 only, and both receivers.

The standard deviations of the inverted solution models are compared in Figure 3 and the resolutions are compared in Figure 4. For ease of comparison, only the standard deviations are shown from the covariance matrices of the solutions. The off-diagonal covariance terms widen the projection of the confidence region on the solution model parameters, but comparing the standard deviations is sufficient to demonstrate the improvements with the additional receiver. It is clear that all the cases have lower standard deviations in the 3-17 degree range, which matches the misfit seen in the model plots. This lower standard deviation is due to the additional acoustic path information at those angles. Note also that the main improvement in standard deviation for multiple compared to single receivers is in BL. A tradeoff angle can be seen in the BL standard deviations at about 10 degrees, below which receiver #1 has lower standard deviations and above which receiver #2 has lower standard deviations. The combined case takes advantage of the different grazing angles in each receiver to better cover both sides of that tradeoff angle. In general the resolution index is higher for the BL than for the SS results, and higher in the 3-17 degree range because that is where the acoustic paths interacted with the bottom. Analogous to the standard deviations, in the two-receiver case the resolution index is high at more angles compared to either individual receiver, reflecting a decrease in the null space of the problem due to the combination of acoustic paths with different grazing angles for the two receivers, and accordingly a decrease in bias.

3.4 Inversion results for multi-static sonar data

Figure 5 shows the measurement geometry and area of inversion of the DEMUS’04 multi-static active sonar experiment conducted by the NATO Undersea Research Centre. The location of the source and receivers are given by the white triangle and circles, respectively. The colormap denotes SNR estimates produced by an acoustic model, with dark red indicating higher SNR. The black line indicates the tow path of an echo repeater that represented a target for the experiment. Measured reverberation ensemble averages from receivers 1 and 2 for beams pointing approximately at the receiver were used in a joint, bistatic inversion. (Receiver 3 was not functioning during this exercise.) The parameters estimated were bottom loss and scattering vs. grazing angle over an approximate 1° - 26° angle range. Values outside that range were provided by the initial guess from a geoacoustic bottom interaction model.

Figure 6 shows two ensemble averages, together with the model fit to the measurements. The region of high reverberation at the beginning of each sequence (e.g., approximately 5-7 seconds for receiver 1, and 3-5 seconds for receiver 2) represents the direct blast signal. Neither the direct blast areas, nor areas where reverberation did not clearly dominate ambient noise, were inverted. The blue and green lines in figure 6 show the inverted solution.
Figure 7 shows the bottom loss and scattering solution. This inversion considers only grazing angles that contribute to the modeled reverberation within the inversion time window, resulting in a mixed over-determined and (slightly) under-determined problem. The model parameter regularization used in this inversion results in an identity model resolution matrix. Figure 8 shows the diagonal of the covariance matrices at the solution, and the clear improvement afforded by joint bistatic (multistatic) inversion.

4 Conclusions

This paper has presented the results of distributed geo-acoustic inversion motivated by environmentally-adaptive multi-static sonar tracking. The simulation demonstrates that inversion using multiple sensors, as available in a multi-static sensor network, can provide improved estimates of the bottom loss and scattering strength, as measured by the bias and variance of the parameter estimates. These improved estimates in turn can improve acoustic model predictions of target SNR, which can be incorporated into a tracking algorithms by methods such as that described in [1] and summarized in this paper.

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