Adaptiv

e Target Tracking in Slowly Changing Clutter∗

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Abstract - False track discrimination performance of a target tracking algorithm in a heavy clutter environment depends on the track confirmation and the track termination thresholds. The optimum value of these thresholds depends on the environment, in particular on the given probability of detection and on the existing clutter density. When tracking ground targets the probability of target detection is nominally constant, whereas the clutter measurement density varies significantly. Previously it was shown that, for a wide range of target signal to noise (+clutter) ratio in a uniform clutter density environment, and given the opportunity to set signal detection thresholds, the optimum value of clutter measurement density is almost constant (and the probability of detection will vary). We propose a scheme where the feedback from the target tracking system corrects the detection thresholds for each sensor resolution cell to obtain the constant and optimal clutter measurement density in each cell, when the clutter statistics changes slowly. This results in better false track discrimination capabilities of the tracker and also replaces the CFAR block in the signal processing unit.

Keywords: Optimal signal detection probability, tracking, IPDA, track existence, false track discrimination, probability of detection feedback, CFAR.

1 Introduction

Fast clutter estimation based on an unbiased spatial estimator enables an adaptive feedback to control the measurement generation process. Previously we have experimentally determined the optimal detection probability for a range of signal to noise ratios [1]. This showed a fairly low and constant optimal clutter density which we utilize in this paper to derive an adaptive Integrated Probabilistic Data Association (IPDA) algorithm that estimates clutter density and the probability of detection [2]. This leads to two significant performance improvements: faster true track confirmation given a constant confirmed false track rate and even more impressive at least a magnitude of order faster processing.

Data association is the most crucial part in tracking, as it greatly determines the performance of a tracking system. Measurements of uncertain origin have to be associated with tracks of real targets, as well as false tracks which do not follow targets. The task for the tracking system is now to use the available measurements and associate them with tracks of targets whose existence can only be known in a probabilistic sense. Algorithms like Generalized Pseudo-Bayesian Probabilistic Data Association (GPB1-PDA, [3]) or the Interacting Multiple Model Probabilistic Data Association (IMM-PDA, [4]) treat false track discrimination in clutter and indirectly calculate the probability that that target is detectable (probability of true track) by including a zero detection target model. The probability of the zero detection model is the complement of the probability that the target is detectable. Integrated Probabilistic Data Association (IPDA) recursively estimates the probability of target existence [2]. A further extension is the IPDA-MAP algorithm which uses a spatial clutter density map [6]. These algorithms and the observation that the optimal probability of detection varies while the optimal clutter density stays roughly constant (for details see [1]) are combined and extended to estimate the locally optimal probability of detection, see section 2.3. In each sensor resolution cell detection thresholds are modified to achieve the optimal clutter measurement density, which affects the probability of target detection. Measured clutter measurement density in each cell is part of the feedback to the signal detection thresholds. The feedback loop aims to deliver the optimal clutter density. In this submission an ideal feedback loop is assumed. This results both in improved false track discrimination capabilities of target tracker when compared to the fixed detection threshold case, and also replaces the CFAR block in the signal processing unit. Simulations show the false track discrimination performance benefits of this approach when tracking a single target with constant radar cross section in an environment of heavy, non-uniform and slowly changing clutter.

In IPDA the probability of existence is recursively estimated based on new measurements and their likelihoods of belonging to the track under investigation. Based on the Bayes-Chapman-Kolmogorov equation,
posterior probabilities are calculated from the prior probabilities and the new measurements. IPDA nicely treats also the possibility of not having a target as well. This is done in conjunction with a Markov model for target existence propagation. There are two models: Markov chain One and Markov chain Two. The first one is a special case of the second where target existence is equal to the target being detectable, whereas in the second model they are distinct.

The clutter map extension is based on [7] and as shown there the spatial estimator is able to give reasonable clutter density estimates with only 10 scans. This is due to the unbiased estimate of the inverse clutter density which the IPDA algorithm relies on to calculate the data association probabilities.

In the design of tracking systems for radar or sonar, the probability of detection is a central parameter. The probability of detection should be high enough to capture true targets but low enough to prevent overwhelming numbers of false detection. While there has been some investigation in the optimal probability of detection, there has been little research in adaptive feedback of the probability of detection for improved measurement generation, which is the process of turning the analog (amplitude) signal into discrete measurements of position and is further described in section 2.1.

Li and Bar-Shalom have developed a hybrid conditional average algorithm [8] to determine a temporal and spatially constant probability of detection. Similarly, Hong and Evans have developed a theoretical approximation framework to estimate the probability of detection [9], assumed to be constant over space and time. Willett et al. [10] has introduced some feedback from the PDA filter to the detector to allow for a temporary and spatially varying threshold, integrating Bayesian detection based on prior probabilities of expected location and covariance of target measurements. While the first two publications are dealing with no a priori knowledge (all measurements are treated equally), the latter one is changing the tracking setup by incorporating knowledge based on measurements. This may be beneficial in some applications but in general varying clutter distributions might introduce some bias.

Furthermore, two applications on adaptive detection threshold optimization for tracking in clutter are reported by Fortmann [11] and Gelfand in [12], where the second paper is an improvement of the earlier work by Fortmann, with regard to the optimization and modeling. In [12] a modified Riccati equation is used to minimize the mean-square state estimation error over detection thresholds. Two optimization algorithms, prior and posterior to the reception of the set of measurements $Y_k$ at time $k$ are proposed. However, there is no direct feedback based on the clutter density involved, rather number of measurements $m_k(\gamma)$ are modeled as sum of the number of false and target measurements by $m_k^F(Pf(\gamma)) + m_k^T(Pd(\gamma))$, where $\gamma$ is the detection threshold.

In this paper, Monte Carlo simulations based on the IPDA-MAP algorithm [2], are used to determine the true and false track discrimination performance. To compare the true tracks statistics the confirmation threshold is chosen such that the rate of confirmed false tracks scans is about one in 50, which seems reasonable for practical applications.

Four experiments have been designed to compare results and a fifth experiment is outlined and only some preliminary results are reported thereof. The first two experiments use uniform high and low clutter densities with different probability of detections. The third and forth experiment use non-uniform clutter density with low and heavy clutter areas with ideally adapted $P_d$ in experiment three and spatially constant non-ideal $P_d$ for experiment four. This is to investigate a proof of concept and maximal possible gains. This idea is currently developed in a fifth experiment for an extended publication later with spatial clutter-map estimator in IPDA-MAP algorithm and probability of detection map (Pd-MAP). The first four experiments use ideal feedback, meaning the clutter density and therefore the optimal probability of detections are known, whereas the fifth experiment uses the clutter density estimated by the spatial inverse density estimator. In all experiments the signal to noise ratio is assumed to be known spatially. In practice this would have to be estimated as well and one method how to do this can be found in [13].

2 Data flow in an IPDA-MAP tracking system

2.1 Data preprocessing and detection thresholding

Fig. 2.1 shows the signal flow in the experiment. The IPDA-MAP algorithm performs the tracking part, delivers probabilities of target existence which are then used in true track detection module to obtain confirmed tracks and estimates the spatial clutter density $\hat{p}(x, y)$.

The probability of detection $P_d$ is a consequence of signal processing which generates measurements when the signal energy is higher than the threshold in a hard limiter. Thus, the probability of target detection is correlated with the probability of false alarm or clutter measurement generation.

Fig. 2.1 shows the classification made in the threshold detection unit into noise (=’negatives’) and measurements (=’positives’) with respect to the amplitude threshold $X_t$ corresponding to a probability of detection $P_d$. Based on the threshold and the two
pdfs, four probabilities can be calculated. The two according to the ‘positive’ classification, which generates measurements for the tracking stage, are of particular interest. These are the probability of true accepted track measurements, i.e. the probability of detection, \( Pr(\text{TP}) = P_d \) (TP=true positives), and the probability noise signals regarded as track measurements, i.e. the probability of false alarm, \( Pr(\text{FP}) = P_{fa} \) (FP=false positives).

A receiver operating curve (ROC) with the confirmation threshold as threshold parameter can be used to determine the performance. Fig. 2.2 shows the ROC with two amplitude threshold values \( X_{t1} \) and \( X_{t2} \) with corresponding detection probabilities \( P_{d1} \) and \( P_{d2} \). The area under the bell curve is the figure of merit and the larger it is the better the performance. Ideally the probability of confirmed false tracks, \( Pr(CFT) \), should be zero and the probability of confirmed true tracks, \( Pr(CTT) \), should be one. The figure of merit is then 1, whereas the diagonal line corresponds to a figure of merit of 0.5, indicating a track confirmed as a false track would be as likely as a track confirmed as true track, and hence a random classification, as obtained by throwing a fair coin, would take place.

### 2.2 Tracking

The measurements will then be validated if they are inside some gating window determined by the IPDA algorithm on a track by track basis. These validated measurements are then associated with corresponding tracks. Based on a Markov chain model and Bayesian inference the probability of target existence is calculated for these tracks which are then classified as confirmed or terminated tracks, depending on whether the probability of target existence exceeds a confirmation threshold \( T_c \) or falls below a termination threshold \( T_t \), respectively. In an ideal situation the remaining tracks stay unconfirmed and over time their probability of existence will wander towards one or zero depending on whether it is a true or false track. In practice some of the confirmed tracks are false tracks that got misclassified due to the overlapping probability distributions, see Fig. 2.2.

### 2.3 Mathematical Notation for IPDA-MAP

A similar notation is used as in [1] but extended for IPDA-MAP and because \( P_d \) in the general case is not constant anymore but rather a function of target position where the gating window \( V_k \) is located:

\[
P_d = \int_{V_k} p_d(z) dz,
\]

modeled as Gaussian likelihood function centered at the predicted observation \( \hat{z}_{k|k-1} \). The integral is approximated as in (25) for fast approximate evaluation. To distinguish the approximated spatial varying probability of detection from the traditional constant \( P_d \) a bar-symbol is used: \( \bar{P}_d \). \( P_d(\hat{z}_{k|k-1}) \) denotes the estimated probability of detection by the Pd-MAP. If there are no measurements \( \bar{P}_d \) is approximated by \( P_d(\hat{z}_{k|k-1}) \).

The following notation is used:

- \( SNR_{dB} \) signal to noise ratio in dB
- \( SNR \) signal to noise ratio
- \( P_d \) probability of detection
- \( P_{fa} \) probability of false measurement
- \( \rho_{fa} \) clutter density
- \( V_{rc} \) resolution cell volume
This gives the relations:
\[
SNR = 10 \log_{10} SNR_{db} \quad (1)
\]
\[
P_{fa} = P_{d}^{1+SNR} \quad (2)
\]
\[
\rho_{fa} = P_{fa}/V_{rc} \quad (3)
\]
\[
V_{rc} = \Delta x \Delta y \quad (4)
\]
\[
\Delta x = 2\sqrt{3} \sigma_{x} \quad (5)
\]
\[
\sigma_{x} = \sqrt{\frac{1}{5} \sum_{i=0}^{50} \beta_{i}(k)z_{i}^{2}|_{k}} \quad (6)
\]
\[
\Delta y = 2\sqrt{3} \sigma_{y} \quad (7)
\]
where the relation (2) is given in [14, 15]. \(\Delta x\) and \(\Delta y\) are approximate side lengths of the resolution cell area, assuming equal distributions \(\frac{1}{2\Delta x}\) and \(\frac{1}{2\Delta y}\) within the cell area, yielding a variance of \(\Delta x^2\) and \(\Delta y^2\), respectively. Equating these expressions with the zero-mean Gaussian observation noise variances \(\sigma_{x}^2\) and \(\sigma_{y}^2\), yields expressions (5) and (6).

Fig. 2.2 shows how tracks are confirmed based on the Monte Carlo simulation. The number of confirmed false tracks is evaluated and the confirmation threshold \(T_{c}\) is varied until the sum of confirmed false track scans divided by the number of scans and runs is roughly one out of 50.

The target motion model is:
\[
x_{k+1} = Fx_{k} + v_{k} \quad (8)
\]
\[
x_{k} = [x, \tilde{x}, y, \tilde{y}]^T \quad (9)
\]
\[
F = \begin{bmatrix} F_{T} & 0 \\ 0 & F_{T} \end{bmatrix}, \quad FT = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (10)
\]
where \(T\) is the sampling time, \((x, y)\) and \((\tilde{x}, \tilde{y})\) are the position and velocity coordinates, respectively. \(v_{k}\) is zero-mean white Gaussian noise with known variance
\[
E[v_{k}v_{l}^T] = Q_{k} \delta(k,l) \quad (11)
\]
\[
Q = \begin{bmatrix} Q_{T} & 0 \\ 0 & Q_{T} \end{bmatrix}, \quad Q_{T} = \begin{bmatrix} T_{x}^2 & T_{x}T_{y} \\ T_{x}T_{y} & T_{y}^2 \end{bmatrix} \quad (12)
\]
with \(q = 0.75\).

The observation model is given by:
\[
z_{k} = Hz_{k} + w_{k} \quad (13)
\]
\[
H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (14)
\]
w\(_{k}\) is zero-mean white Gaussian noise with known variance:
\[
E[w_{k}w_{l}^T] = R_{k} \delta(k,l) \quad (15)
\]
\[
R = \begin{bmatrix} \sigma_{x}^2 & 0 \\ 0 & \sigma_{y}^2 \end{bmatrix} \quad (16)
\]
The Markov chain model one for the probability of target existence conditioned on the set of observations \(Z_{k-1}\) of all observations \(z_{i}^{k}, i = 1, ..., m_{k}\) made at previous times \(k = 0, ..., l - 1\) is as follow:
\[
\begin{bmatrix} P\{x_{k}|Z_{k-1}\} \\ 1 - P\{x_{k}|Z_{k-1}\} \end{bmatrix} = \begin{bmatrix} 0.98 & 0 \\ 0.02 & 1 \end{bmatrix} \begin{bmatrix} P\{x_{k-1}|Z_{k-1}\} \\ 1 - P\{x_{k-1}|Z_{k-1}\} \end{bmatrix} \quad (17)
\]

with initial probability of \(P\{x_{0}|Z_{0}\} = P\{x_{0}\} = 0.12\) used in the two-stage track initialization run at each scan.

State and covariance estimates \(\hat{x}_{k|k}\) and \(\hat{P}_{k|k}\) are given by the standard PDA algorithm:
\[
\hat{x}_{k|k} = \sum_{i=0}^{m_{k}} \beta_{i}(k)z_{i}^{k|k} \quad (18)
\]
\[
\hat{P}_{k|k} = \sum_{i=0}^{m_{k}} \beta_{i}(k)(\hat{P}_{k|k}^{i} + (\hat{x}_{k|k} - \hat{x}_{k|k})(\hat{x}_{k|k} - \hat{x}_{k|k})^{T}) \quad (19)
\]
where \(z_{i}^{k|k} = \hat{x}_{k|k-1}\), \(\hat{P}_{0|k}^{k} = \hat{P}_{k|k-1}\) and data association probabilities are given by:
\[
\beta_{0}(k) = \frac{1 - P_{G}\hat{P}_{d}}{1 - \delta_{k}} \quad (20)
\]
\[
\beta_{i}(k) = \frac{P_{G}\hat{P}_{d}(z_{i}^{k})}{1 - \delta_{k}} \frac{P_{d}(z_{i}^{k})}{\hat{P}_{k}^{i}} \quad , \quad i = 1, ..., m_{k} \quad (21)
\]
\[
\delta_{k} = \begin{cases} P_{G}\hat{P}_{d} - P_{G} \sum_{i=1}^{m_{k}} P_{d}(z_{i}^{k})/\hat{P}_{k}^{i}, & m_{k} = 0 \\
\sum_{i=1}^{m_{k}} P_{d}(z_{i}^{k})/\hat{P}_{k}^{i}, & m_{k} \neq 0 \end{cases} \quad (22)
\]
\[
p_{d}(z_{i}^{k}) = \Lambda_{k} = \frac{1}{P_{G}} N(z_{i}^{k}; \hat{x}_{k|k-1}, \hat{S}_{k}), \quad i = 0, ..., m_{k} \quad (23)
\]
\[
m_{k} = \begin{cases} 0, & m_{k} - \hat{P}_{d} P_{G} P\{x_{k}|Z_{k-1}\} = 0 \quad (24) \\
\sum_{i=0}^{m_{k}} \Lambda_{k}, & m_{k} \neq 0 \end{cases}
\]
where \(m_{k}\) is the number of validated observations \(z_{i}^{k}\); \(P_{G}(\cdot)\) and \(P_{G}\) are the probabilities of detection as a function of space and gating validation, respectively. \(p_{d}(\cdot)\) is the measurement likelihood in the gating window, assumed to be Gaussian. The approximation in (25) may be seen as a crude but fast quadrature integral-like evaluation. In IPDA-MAP \(\hat{P}_{k}^{i}\) is the estimated clutter density at the \(i\)-th observation \(z_{i}^{k}\) at time \(k\) and simplifies in the standard IPDA algorithm to \(\hat{P}_{k}^{i} = \frac{m_{k}}{m_{i}}\), \(i = 1, ..., m_{k}\).

The probability of target existence update is determined by:
\[
P\{x_{k}|Z^{k}\} = \frac{1 - \delta_{k}}{1 - \delta_{k} P\{x_{k}|Z^{k-1}\}} P\{x_{k}|Z^{k-1}\} \quad (26)
\]
in conjunction with the Markov update (17).

Estimation and prediction are obtained by the standard Kalman filter:
\[
\begin{align*}
\hat{S}_{k} &= H\hat{P}_{k|k-1} HT + R \\
K_{k} &= \hat{P}_{k|k-1}HT\hat{S}_{k}^{-1} \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_{k}(z_{k} - H\hat{x}_{k|k-1}) \\
\hat{P}_{k|k-1} &= (I - K_{k}H)\hat{P}_{k|k-1} \\
\hat{z}_{k+1|k} &= F\hat{z}_{k|k} \\
\hat{P}_{k+1|k} &= F\hat{P}_{k|k} FT + Q
\end{align*} \quad (27) - (32)
\]

### 2.3.1 Spatial Clutter-Map Estimation

In [7] it was shown that an efficient non-biased estimator of the inverse clutter measurement density is
the averaged distance $N_{k-1}(c)$ from resolution cell center to the nearest Poisson measurement point. This is a direct property of the Poisson distributed measurements, as then the mathematical distance $\xi$ between two Poisson measurements has exponential distribution (33) and expectation (35), [16, 17, 18]. The averaging process $N_k(c)$ can be done either according to a moving average (36) or by autoregressive filter (37), where $\mu_k(c)$ is the inverse clutter density measurement at cell $c$ at time $k$.

$\rho_c(\xi) = \rho e^{-\xi} U(\xi), \quad \text{with} \quad (33)$

$U(\xi) = \begin{cases} 1 : \xi \geq 0, \\ 0 : \xi < 0. \end{cases} \quad (34)$

$\mathcal{E}(\xi) = \frac{1}{\rho} \quad (35)$

$N_k(c) = \frac{1}{L} \sum_{j=k-L+1}^{k} \mu_j(c), \quad \text{moving average, or} \quad (36)$

$= \alpha_k N_{k-1}(c) + (1 - \alpha_k) \mu_k(c), \quad \text{auto regressive with} \quad (37)$

$\alpha_k = \left\{ \begin{array}{ll} \frac{k-1}{L} : k < L, \\ \frac{L}{L-1} : k \geq L. \end{array} \right. \quad (38)$

The averaged measurements have zero bias because the sum $y = \sum_{i=1}^{L} \xi_i$ of $L$ independent, exponentially distributed random variables $\xi_i$ have a Gamma distribution (38) with mean (39) and variance (41). A crucial property is that the relative variance of the estimator is given by (42), which means it converges asymptotically with $\frac{1}{L}$ to the true value of $y$ and simulations have shown that estimates become reasonable for $L$ as small as 10, in other words after only 10 scans a useable clutter density estimate is available. However, as the analysis is done for Poisson distributed clutter, real clutter densities with discontinuities, e.g. different areas with different mean values of poisson distributed clutters, again will have bias and a transition between these regions. For further details see [7].

$P_y(y) = \frac{\rho^L}{(L-1)!} y^{L-1} e^{-\rho y} U(y) \quad (39)$

$E_y = \frac{L}{\rho} \quad (40)$

$E_y^2 = \frac{L(L+1)}{\rho^2} \quad (41)$

$\sigma^2(y) = E_y^2 - E^2_y = \frac{L}{\rho^2} \quad (42)$

For the spatial inverse clutter map estimator, $N_k(c)$, $\xi_k(c)$ is the mathematical distance and is determined by the size $\mu_k(c) = \min_{i=1, \ldots, m_k} A_k^i(c)$ of a fixed 2-dimensional shape (e.g. circle or square), centered at resolution cell $c$, just touching the nearest observed clutter measurement $z_k^i$ and is clipped outside the surveillance area, see figure 2.3.1.

As the measurement generation process has measurements from targets as well as of clutter, the following size measure is used:

$\mu_k(c) = \sum_{i=1}^{m_k} A_k^i(c) P_i(i) \rho c(z_k^i), \quad \text{where} \quad (43)$

$P_i(i) = \begin{cases} 1, \quad & i = 1, \\ P_i(i-1) \left(1 - P_c(z_k^i)\right), \quad & i = 2, \ldots, m_k \end{cases} \quad (44)$

Where $I(.)$ is an index (sorting) function such that $A_k^j(i) \leq A_k^j(i+1)$, $\forall i = 1, \ldots, m_k - 1$. $P_c(z_k^i)$ is the probability of $z_k^i$ to be a clutter measurement and $P_c(i) > 1$ is the probability that all $i - 1$ previous measurements $z_k^j$, $j = 1, \ldots, i - 1$, were from targets and not clutter. Let $\chi_k$ and $\chi_k,\beta_k,i$ be the events that the target exists, and, that of target existence and measurement $z_k^i$ is a target measurement, respectively. Then $P\{\chi_k,\chi_k,\beta_k,i|Z^k\}$ denotes the probability of target existence and measurement $z_k^i$ is from the target given all previous and current observations. Therefore

$P_c(z_k^i) = 1 - P\{\chi_k,\chi_k,\beta_k,i|Z^k\} \quad (45)$

$P\{\chi_k,\chi_k,\beta_k,i|Z^k\} = \frac{P_c P_d(z_k^i)}{1 - \delta_k P\{\chi_k|Z^{k-1}\}} \frac{P\{\chi_k|Z^{k-1}\}}{P\{\chi_k|Z^k\}} \quad (46)$

where $i = 1, \ldots, m_k$.

Remark: The data association probabilities given before are related as $\beta_k,i \equiv P\{\chi_k,i|\chi_k,\beta_k,i\} = \frac{P\{\chi_k,\chi_k,\beta_k,i|Z^k\}}{P\{\chi_k|Z^k\}}$. Equation (45) is valid for single target tracking. For multi-target tracking, $P_c$ is the product of (45) across all tracks $\tau = 1, \ldots, T(z_k^i)$ which select $z_k^i$, i.e. $P_c(z_k^i) = \prod_{\tau=1}^{T(z_k^i)} P_c^\tau(z_k^i)$.

2.3.2 Updates of $P_d$-MAP

For a real system the detection thresholds are needed which are determined based on $P_{fa}$, $P_d$ and $SNR$, all of which may vary spatially and even slowly temporarily. The actual threshold $\gamma$ used in the thresholding unit to generate measurements is determined by $P_{fa}$ and $P_d$, see figure 2.1. $P_{fa} = V_{ref} P_{fa}$ is estimated by the spatial clutter density estimator. For a chosen spatially varying $P_d$, the signal to noise ratio could be estimated and based on (47) and the optimal values for a given $SNR$ can be obtained from [1]. However, this may lead to instabilities and an autoregressive filter for $P_d$ with a
longer time constant may be a better way. This is currently under investigation and will be published later. Another way is to use an independent estimate of the SNR, as given in [13]. Therefore, for the experiments 1 to 4, a known SNR is assumed.

3 Experiments

The objective of is to experimentally confirm an improvement in the tracking performance when spatial clutter density estimation by the IPDA-MAP algorithm is used to build a spatial probability of detection map (Pd-MAP). As mentioned before five experiments are designed. In the first and second experiment the clutter densities are high \( (5.41 \times 10^{-4} m^{-2}) \) and low \( (2 \times 10^{-5} m^{-2}) \), respectively, and uniform over the whole surveillance area. The probabilities of detection are selected as \( P_d = 0.9 \) and \( P_d = P_{d2} = 0.74 \), respectively. The signal to noise ratio is chosen to be \( 12 dB \), such that the second experiment has roughly optimal probability of detection and optimal clutter density (taken from \cite{1}), where as the first one only matches the Swerling model I relation

\[ \rho_{fa} = \rho_{fa} \left( \frac{P_d}{P_d} \right)^{1+SNR} \]  \hfill (47)

In the third and forth experiment an optimal and a non-uniform clutter with low and high-clutter density areas are used, respectively, as shown in figure 3. For the third experiment, ideal feedback, that is the optimal probabilities of detection are used, where as in the forth experiment a constant probability of detection of 0.9 for the whole surveillance area is assumed. In the fifth experiment, which will be done as part of further research, the same parameters as in experiment four are used initially but then the clutter density is estimated, using the spatial clutter map estimator (36,43). As the IPDA algorithm with two-point differencing initialization needs roughly \( P_d \geq 0.7 \) to reliably initialize tracks the estimated Pd-MAP is threshold and kept at 0.7 for lower estimates. Table 1 gives the values used in experiments three, four and the initial values for experiment five.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Exp. 3</th>
<th>Exp. 4 and 5 (init.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_1 )</td>
<td>( 2 \times 10^{-5} m^{-2} )</td>
<td>( 0.965 )</td>
</tr>
<tr>
<td>( P_{d1} )</td>
<td></td>
<td>0.9</td>
</tr>
<tr>
<td>( SNR_1 )</td>
<td>( 21dB )</td>
<td>( 21dB )</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>( 2 \times 10^{-6} m^{-2} )</td>
<td>( 5.41 \times 10^{-4} m^{-2} )</td>
</tr>
<tr>
<td>( P_{d2} )</td>
<td>0.74</td>
<td>0.9</td>
</tr>
<tr>
<td>( SNR_2 )</td>
<td>( 12dB )</td>
<td>( 12dB )</td>
</tr>
</tbody>
</table>

Table 1: Parameters for experiments three, four and five (initially).

To determine the performance confirmed true tracks over 50 scans are measured while keeping the confirmed false track statistics constant at a rate of 1 in 50. An automatic initialization of tracks is used that is based on two successive scans (two point differencing). At every scan new tracks are initialized for all possible measurement combinations of any two measurements from different scans, if the absolute measurement differences of such a combination fall below a certain threshold (maximum target speed). All of these tracks are assigned an initial probability of target existence. During a simulation run this probability is normally increased for true tracks by the validation of later measurements, while for false tracks it will decrease on average. If it drops below a termination threshold the track will be terminated. If the probability of target existence exceeds a confirmation threshold, the track becomes a confirmed track.

Depending on whether a track follows a target or not, they are classified as true or false tracks, respectively. This leaves four possibilities, confirmed true and false tracks (CTT and CFT) and terminated true and false tracks (TTT and TTF). Naturally, the goal is to have as many confirmed true tracks with a low limited number (ideally zero) of confirmed false tracks and no terminated true tracks. Because of the stochastic nature of the measurements, the associated pdfs on the numbers of true and false tracks have some variance, are overlapping and are functions of underlying parameters.

The initial target existence probability, \( P\{\chi_0\} = 0.12 \), and the termination threshold, \( T_t = 0.02 \), were chosen to be of reasonable value from experience based on experimental optimization [6] but held constant here and only the confirmation threshold \( T_c \) is adapted to keep the confirmed false track rate roughly constant.

4 Results

The results of experiments 1 and 2, given by figure 4, show that feedback for an adaptive probability of detection can improve performance drastically by raising the detection threshold to have an optimal clutter
density. Figure 4 shows the maximal performance gain in terms of the temporal behavior of the true track statistics. While experiment 1 has a high, non-optimal $P_d = 0.9$ and a resulting high clutter density, the performance lacks far behind the one of experiment 2, which has optimal probability of detection and clutter density for the given signal to noise ratio. The comparison between the IPDA-MAP which uses a clutter map and the standard non-parametric IPDA algorithm shows a clear advantage for the former with the biggest gain stemming from the optimal signal detection.

Fig. 4 shows the results of experiment 3 and 4 with the setting given in figure 3 and table 1. Clearly, using the optimal probability of detection for ideal thresholding increases the performance significantly. The vertical dashed lines show the area boundaries in x-direction in which the target is moving. Interestingly, the difference in performance is increased when the clutter density of the areas are swapped, i.e. $\rho_1 \leftrightarrow \rho_2$. Then the track starts in a high clutter area, followed by low, then high and finally low clutter again. In this case (red lines), a non-optimal probability of detection corresponding to a higher clutter density, yields significantly worse results for non-optimal selection. Also, in the second high clutter area the non-optimal selection starts loosing tracks, as seen in the enlarged plot area in figure 4. This shows that the optimal probability of detection and corresponding clutter density are crucial to initialize new tracks. A second observation is that the IPDA algorithm maintains tracks very well, once they are confirmed, even in the case of non-optimal selection of the detection probability. Nevertheless, true/false-track discrimination performance can still be improved by the use of the optimal probability of detection.

Fig. 4 shows the estimated clutter density of the spatial estimator after only 50 scans and a time constant of 10 scans for the autoregressive filter. This would be the initial clutter density for experiment 5 before feedback is turned on.

Fig. 4 shows the estimated clutter density when feedback is used to bring the clutter density to the optimal density of $2 \times 10^{-5}m^{-2}$. Further investigations on necessary time constants in the feedback loop will be done in future research.

Furthermore, a crucial side effect of performing spatially adaptive detection thresholding is that the algorithm’s running time can be reduced drastically. This is due to the fact the number of false tracks is proportional to the square of the number of clutter measurements received, which influences the total run time. Running times are given in table 2.

## Conclusions

In this paper the effect of spatially optimal probability of detection via feedback has been investigated. Experiments 1 to 4 looked at ideal feedback with exactly known clutter densities and probabilities of detection. The results are very promising and show a large potential of performance improvements in terms of the number of confirmed true tracks and in the actual runtime performance. The use of the spatial clutter density estimator with its low time constant is crucial for tracking slowly varying clutter, without loosing a lot of accuracy. Thus, it is very reasonable to expect good performance of a completely closed loop with adaptive probability of detection and clutter density estimation. From a radar operators point of view, this could remove the CFAR unit for the thresholding unit and thus remove manual interaction of setting threshold parameters.

## References


Figure 8: Results of experiments 3(non-optimal) and 4(ideal feedback) in non-homogenous clutter. If the high and low clutter density areas are swapped ($\rho_1 \leftrightarrow \rho_2$), the differences between experiments 3swap and 4swap are even more evident. The lower subplot is a magnification of the above one and the vertical dashed lines indicate the clutter area boundaries in the horizontal direction, compare figure 3.

Figure 9: The spatial clutter density estimator with ARMA filter is already fairly accurate after only 50 scans and a time constant of 10 scans.

Figure 10: Preliminary experiment 5 shows that it is possible to drive the clutter density towards the optimal density around $2 \times 10^{-5} \text{m}^{-2}$. However, some oscillations may occur but performance will be far better than in experiment 4, much closer to experiment 3.


