Abstract - The local position measurement system LPM is a novel system for fast and accurate measurements of object positions with active transponders. With an accuracy of few centimeters and measurement rates of 1000 measurements per second LPM is by now one of the most advanced systems in this field. For sensor fusion purposes each transponder is equipped with a telemetry channel. In this contribution we discuss the sensor fusion of the position measurements of the LPM system with accelerometers and gyroscopes. The focus of this work is to investigate the impact of additional sensor information on the accuracy of the estimated states based on different state transition matrices of a Kalman filter. The results based on synthetic data are verified by measurements of a test car on a race track.

Keywords: Position estimation, tracking, sensor fusion, Kalman filtering.

1 Introduction

Precise local position measurement is still a technical challenge. The local position measurement system LPM as introduced in [1] is a novel system for fast and accurate measurements of object positions based on active transponders. Applications reach from sports e.g. tracking of race cars, ski racers or soccer teams to industrial or research applications. In [2] an integrated telemetry channel for the LPM system was introduced. The telemetry channel enables the ability to take into account measurements of additional sensors like accelerometers and gyroscopes to improve the accuracy of the estimated position among other states of the target like velocity or acceleration based on Kalman filtering. A key element of sensor fusion is a proper description of the basic maneuverability of a certain target by the state transition matrix of the Kalman filter.

The paper is organized as follows: In section 2 we briefly repeat the basic principle of the LPM system, Section 3 discusses the basic concepts of target tracking based on Kalman filtering, followed by discussing several 2-D tracking models in section 4. The basic properties on selected models are demonstrated in section V, which are verified on measured data in section 5. A conclusion is given in section 6.

2 LPM Measurement System

The LPM concept is, in some aspects, inverse to GPS ([3]), as there are active transponders operating around 5.8 GHz, whose positions are measured, and as there are fixed passive base stations around the covered field of view. The basic arrangement of the LPM system is sketched in Fig. 1.

Figure 1 : Arrangement of LPM components and signal flow.
The LPM measurement equation for the differential measurement length $c \cdot t_{(BS,k)}^{(BS,k)}$ system for BS $k$ for a measurement transponder (MT) at position $z^{(MT)}$ is given by

$$
\rho_k^{(k)} = c \cdot t_{(BS,k)}^{(BS,k)} - \left\| z^{(MT)} - z^{(BS,k)} \right\| + w^{(off)}
$$

(1)

with $\left\| \cdot \right\|$ denoting the Euclidian norm, $c$ is the velocity of the electromagnetic wave in air. The coordinates of the reference transponder (RT) $z^{(RT)}$ and the base station (BS) $z^{(BS,k)}$ are assumed to be known. The unknown time offset between the signal of the RT and the MT is given by $w^{(off)} c^{-1}$. A detailed discussion on the LPM measurement equation is given in [4]. The error propagation of the LPM system is discussed in the next chapter.

3 Target Tracking

3.1 Kalman Filter

Generally, a Kalman filter estimates the target trajectory state vector $X_n$ based on the random system dynamic model while it evolves over time

$$
X_{n+1} = \Phi_n X_n + U_n,
$$

(2)

where $U_n$ is the dynamic model driving noise vector. The state transition matrix describes the system dynamic model. For non-uniform sampled signals, the state transition matrix depends on the time index $n$. The predictor equation (3) estimates the state of the object $X_{n+1,n}$ at time $n+1$ based on measurements up to time $n$,

$$
X_{n+1,n} = \Phi_n X_{n,n}.
$$

(3)

A detailed discussion on mathematical models for modeling the dynamic of a target is given in the next chapter. The filtering equation (4) provides an update of the predicted state

$$
X_{n,n} = \Phi_n X_{n+1,n} + H_n (Y_n - M X_{n+1,n}).
$$

(4)

The basic structure of a Kalman tracking filter consists always of two parts: predict and update (Fig. 2).

![Figure 2: Basic structure of a Kalman tracking filter.](image)

The observation matrix $M$ is the link between the state vector $X_{n,n}$ and the measurement vector $Y_n$. The Kalman matrix gain $H_n$ for time $n$ is calculated by the weight equation

$$
H_n = S_{n,n-1} M^T (R_n + M S_{n,n-1} M^T)^{-1}.
$$

(5)

The covariance matrix $R_n$ of the observed measurements $Y_n$ describes the accuracy of the measurement, with $R_n$ defined by

$$
R_n = \text{cov}(Y_n).
$$

(6)

The covariance matrix $S_{n,n-1}$ of the estimated state vector $X_{n,n-1}$, calculated by the predictor covariance matrix equation

$$
S_{n,n-1} = \Phi_n S_{n-1,n} \Phi_n^T + Q_n
$$

(7)

is an estimate of the accuracy of the predicted state $X_{n,n-1}$. The covariance matrix $S_{n,n-1,k}$ of the state vector is an estimate of the accuracy of the updated state $X_{n,n}$. $Q_n$ denotes the covariance of the dynamic model driving noise vector $U_n$ defined by

$$
Q_n = \text{cov}(U_n).
$$

(8)

The filtered estimated covariance matrix $S_{n,n-1}$ is in turn obtained from the previous predicted covariance matrix $S_{n-1,n-2}$ by the corrector equation

$$
S_{n,n-1} = (I - H_n M) S_{n-1,n-2} + \tilde{w}^{(off)}
$$

(9)

where $I$ represents the identity matrix. A detailed essay on the Kalman filter and its potential use for tracking applications can be found in [5-7].

3.2 Position Data

Due to the independence of the measurements of each BS to one another the covariance matrix of measurements for the BSs is a diagonal matrix given by

$$
R_{(BS)}^{(BS)} = \text{diag}(R_{BS,1}^2, \ldots)
$$

(10)

with $k$ as the ID of the BS. According to the signal to noise ratio in each base station, it is possible to calculate the specific variance $\sigma_{BS,k}$ of each measurement signal. In [4] a discussion on the calculation of the object position in the LPM system is given. According to a specific base station setup concerning the actual position of the measurement transponder, the covariance matrix of the position as well as the measurement offset $w^{(off)}$ is given by

$$
R_{(pos)} = (G^T G)^{-1} G^T R_{(BS)} G (G^T G)^{-1},
$$

(11)

with

$$
G = \begin{bmatrix}
\frac{\partial \rho^{(1)}}{\partial z^{(MT)}} & \frac{\partial \rho^{(1)}}{\partial z^{(MT)}} & \cdots \frac{\partial \rho^{(1)}}{\partial w^{(off)}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \rho^{(N)}}{\partial z^{(MT)}} & \frac{\partial \rho^{(N)}}{\partial z^{(MT)}} & \cdots \frac{\partial \rho^{(N)}}{\partial w^{(off)}}
\end{bmatrix}
$$

(12)

The covariance matrix of measurement errors describes the coordinate coupling according to a specific BS setup. The coupling terms are especially of meaning for an uneven distributed setup.

3.3 Sensor Data

The telemetry channel of the LPM system offers the potential to take into account additional information from sensors like gyroscopes or accelerometers. For tracking in Cartesian Coordinates the normal and tangential sensor
information of the accelerometers have to be rotated according to the heading of the object $\phi^{(head)}$

$$
\begin{bmatrix}
    a^{(a)} \\
    a^{(v)}
\end{bmatrix} =
\begin{bmatrix}
    \cos(\phi^{(head)}) & -\sin(\phi^{(head)}) \\
    \sin(\phi^{(head)}) & \cos(\phi^{(head)})
\end{bmatrix}
\begin{bmatrix}
    a^{(norm)}
\end{bmatrix}
$$

(13)

to gain the sensor information in the tracking coordinates.

For a non-sliding object it can be assumed that the heading is equal to the velocity heading $\phi^{(vel)}$. Otherwise, the heading of the object has to be estimated by taking into account a second transponder. Alternatively it would be possible to use an electronic compass, but such a device is extremely vulnerable to magnetic distortions. A sketch of the circumstances of a sliding object is shown in Fig. 3. A detailed discussion on accelerometers and gyroscopes for tracking applications is given in [8].

Figure 3 : Sketch of object heading $\phi^{(head)}$ vs. velocity heading $\phi^{(vel)}$ of a sliding object.

4 2-D Motion Models

Most target maneuvers are coupled across different coordinates. For simplicity many maneuver models assume that these coordinate couplings are weak and can be neglected. Furthermore the calculation complexity of the Kalman filter is $o^3$ with $o$ as the number of states. Hence a decoupled Kalman filter reduces the calculation complexity significantly.

Many 2-D target tracking models are naturally turn motion models [9]. These curvilinear motion models are mostly established and rely directly on the target kinematics. Coordinate-coupled target models are highly dependent on the choice of the state variables. The choice of the state vector in combination with the kinematics model is a challenging task, where target dynamics, sensor coordinates, and accuracies of approximations among others must be taken into account [10]. The LPM measures directly Cartesian coordinates, therefore Cartesian coordinates were also chosen for the state vector.

4.1 Constant velocity

The state vector for a moving point target in Cartesian coordinates for estimating the position as well as the velocity is given by

$$
\mathbf{X}^{(CV)} = \begin{bmatrix} x_n & \dot{x}_n & y_n & \dot{y}_n \end{bmatrix}^T
$$

(14)

with $x_n$ and $y_n$ as the positions in the x- and y-coordinates. The simplest motion model for a moving target assumes a (nearly) constant velocity (CV) in each coordinate, acceleration terms are not taken into account. Its state transition matrix $\Phi^{(CV)}$ is given by

$$
\Phi^{(CV)} = \text{diag}(\Phi^{(CV),a}, \Phi^{(CV),v})
$$

(15)

with the state transition matrix for a single axes given by

$$
\Phi^{(CV),a} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(16)

The covariance of the dynamic driving noise vector is self-evident also block diagonal

$$
Q^{(CV),a} = \sigma^2_a \begin{bmatrix} \frac{1}{2}T^2 & \frac{1}{2}T & 0 \\ \frac{1}{2}T & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

(17)

with $\sigma^2_a$ as the variance of the random part $U_a$. The observation matrix for the CV model for each axes is given by

$$
M^{(CV)} = \begin{bmatrix} 1 & 0 \end{bmatrix}
$$

(19)

The CV model is inappropriate for highly maneuvering targets because any maneuver can only be modeled by the variance $\sigma^2_a$ of the random part $U_a$. Furthermore this model is not suitable to take into account measurements of accelerometers or gyroscopes.

4.2 CV circular turn model

As discussed above, many 2-D target tracking models are naturally turn motion models. The CV circular turn (CT) model also known as CV model with known angular velocity assumes that the absolute value of the velocity is constant. Furthermore, this model takes into account the angular velocity $\omega$ of the target. Its state transition matrix is coupled among the two axes and given by

$$
\Phi^{(CV),\omega}(\omega) = \begin{bmatrix} 1 & \sin(\omega T_n) & 0 & 0 \\ 0 & \cos(\omega T_n) & 0 & 0 \\ 0 & -\sin(\omega T_n) & 1 & 0 \\ \sin(\omega T_n) & 0 & 0 & \cos(\omega T_n) \end{bmatrix}
$$

(20)

For a known $\omega$ the state transition is still linear. It should be noted that the angular velocity is not modeled as a state. The covariance matrix of the noise term $U_a$ is given by

$$
Q^{(CV),\omega}(\omega) = \text{diag}(Q^{(CV),a}, Q^{(CV),\omega})
$$

(21)

The observation matrix for this model is given by

$$
M^{(CV)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
$$

(22)

Unlike the CV model this model is capable of taking into account the angular velocity of an object measured for
instance by a gyroscope, but the model is not suitable for estimating the angular velocity out of the estimated states. Still acceleration terms can not be taken into account.

### 4.3 CV-CT model with tangential acc. \( a_l \)

In [11] the CV-CT model was extended to take into account tangential acceleration terms as an input but not as a state. The predictor equation for this model is given by

\[
X_{n+1} = X_n + G \left( \varphi_{n}, \omega_{n} \right) \Delta_{n}^{(\text{tan})}
\]

with

\[
G \left( \varphi_{n}, \omega_{n} \right) = \begin{bmatrix}
\cos(\varphi_{n+1}) & \cos(\varphi_{n+1}) \cdot T_n \sin(\varphi_{n}) \\
\sin(\varphi_{n+1}) & \sin(\varphi_{n+1}) \cdot T_n \cos(\varphi_{n}) \\
\sin(\varphi_{n}) & \cos(\varphi_{n}) \cdot T_n \cos(\varphi_{n}) \\
\cos(\varphi_{n}) & -\sin(\varphi_{n}) \cdot T_n \sin(\varphi_{n}) \\
\end{bmatrix}
\]

and

\[
\varphi_{n+1} = \varphi_{n} + \omega_{n} \Delta_{n}^{(\text{tan})}.
\]

The covariance matrix of the noise term \( \Sigma_{n} \) as well as the measurement matrix equal are identical compared to the CV-CT model. For highly maneuvering object, the tangential acceleration is significantly smaller compared to the normal acceleration of the target e.g. for a constant turn maneuver the tangential acceleration is zero.

### 4.4 Constant acceleration

The state vector for a moving point target in Cartesian coordinates for estimating the position, the velocity, and the acceleration is given by

\[
X_{n}^{(\text{CA})} = [x_n, \dot{x}_n, \ddot{x}_n, y_n, \dot{y}_n, \ddot{y}_n]^T.
\]

The state transition matrix for an object moving in 2-D with (nearly) constant acceleration (CA) is given by

\[
\Phi_{n}^{(\text{CA})} = \begin{bmatrix}
1 & T_n & \frac{T_n^2}{2} \\
0 & 1 & T_n \\
0 & 0 & 1
\end{bmatrix}
\]

with the state transition matrix for each axes given by

\[
\Phi_{n}^{(\text{CA},i)} = \begin{bmatrix}
1 & \sin(\omega_{n} T_n) & \frac{\sin(\omega_{n} T_n)^2}{2} \\
\omega_{n} & 1 & \frac{T_n}{2} \\
\omega_{n}^2 & \omega_{n} T_n & \frac{T_n^2}{2} \\
\end{bmatrix}
\]

The covariance of the dynamic driving noise vector is self-evident also block diagonal

\[
Q_{n}^{(\text{CA})} = \begin{bmatrix}
\sigma^2_{\dot{x}} & \sigma^2_{\ddot{x}} & \sigma^2_{\dot{y}} & \sigma^2_{\ddot{y}} \\
\sigma^2_{\dot{x}} & \sigma^2_{\ddot{x}} & \sigma^2_{\dot{y}} & \sigma^2_{\ddot{y}} \\
\sigma^2_{\dot{x}} & \sigma^2_{\ddot{x}} & \sigma^2_{\dot{y}} & \sigma^2_{\ddot{y}} \\
\sigma^2_{\dot{x}} & \sigma^2_{\ddot{x}} & \sigma^2_{\dot{y}} & \sigma^2_{\ddot{y}} \\
\end{bmatrix}
\]

\[
Q_{n}^{(\text{CA},i)} = 2 \alpha \sigma^2_{\omega} \begin{bmatrix}
\frac{\omega_{n}}{T_n} & \frac{\omega_{n}^2}{2 T_n} & \frac{\omega_{n}^2}{2} \\
\frac{\omega_{n}}{T_n} & \frac{\omega_{n}^2}{2 T_n} & \frac{\omega_{n}^2}{2} \\
\frac{\omega_{n}}{T_n} & \frac{\omega_{n}^2}{2 T_n} & \frac{\omega_{n}^2}{2} \\
\frac{\omega_{n}}{T_n} & \frac{\omega_{n}^2}{2 T_n} & \frac{\omega_{n}^2}{2} \\
\end{bmatrix}
\]

If only the position is measured the observation matrix for the CA model for each axes is given by

\[
M_{n}^{(\text{CA},i)} = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\]

and if the position as well as the acceleration are measured the observation matrix becomes

\[
M_{n}^{(\text{CA})} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The two axes are not coupled with one another. This model is very universally applicable but it has the drawback that measurements of gyroscopes can not be taken into account.

### 4.5 CA with angular velocity

The state transition matrix for the CA model can be extended ([12]) to take into account the angular velocity of the target by

\[
\Phi_{n}^{(\text{CA},\omega)} = \text{diag}(\Phi_{n}^{(\text{CA},i)}(\omega_{n}), \Phi_{n}^{(\text{CA},i)}(\omega_{n}))
\]

with

\[
\Phi_{n}^{(\text{CA},\omega)}(\omega_{n}) = \begin{bmatrix}
1 & \sin(\omega_{n} T_n) & 1 - \cos(\omega_{n} T_n) \\
\omega_{n} & 1 & \frac{T_n}{2} \\
\omega_{n}^2 & \omega_{n} T_n & T_n^2
\end{bmatrix}
\]

The coupling between the two axes is done via the angular velocity, hence for each axes a separate Kalman filter can be implemented. The angular velocity itself is not modeled as a state of the filter, hence the state transition matrix is still linear. Unlike (20) the sign of \( \omega \) in the state transition matrix is of no meaning. The angular velocity can be both, measured or estimated. For a planar circular turn the turn rate can be estimated by

\[
\omega_{n} = \alpha_{n}^{(\text{turn})}.
\]

The normal acceleration is given by the projection of the acceleration vector onto the velocity vector. The covariance matrix of the noise term \( \Sigma_{n} \) as well as the measurement matrix are equal as for the CA model. Furthermore, for \( \omega = 0 \), the CA model with angular velocity (34) is identical to the CA model. The CA model with angular velocity is a very universal model suitable for many different tracking applications.

### 5 Simulations

In this chapter the impact of additional sensor information for improved state estimation depending on the added noise of the measured parameters and the chosen state transition model is discussed.
Figure 4: Simulated tracking of a car moving with constant velocity, then driving a 180° left turn followed by a 180° right turn with state transition model 1. CA model with measured position (CA|p), 2. CA model with measured position and acceleration (CA|pa), 3. CA model with \( \omega \) with measured position (CA\( \omega \)|p), 4. CA model with \( \omega \) with measured position and acceleration (CA\( \omega \)|pa), 5. CA model with \( \omega \) with measured position and \( \omega \) (CA\( \omega \)|p\( \omega \)), and 6. CA model with \( \omega \) with measured position, acceleration, and \( \omega \) (CA\( \omega \)|pa\( \omega \)); (a) zoomed position of the first turn, (b) velocity in the x-coordinate, (c) acceleration in the x-coordinate, as well as (d) angular velocity. Measurement noise of the acceleration sensor and the gyroscope is modeled by zero mean white Gaussian noise with a standard deviation of (1) \( \sigma_x = 3 \text{ m} \), \( \sigma_a = 3 \text{ m/s}^2 \), (2) \( \sigma_x = 3 \text{ m} \), \( \sigma_a = 8 \text{ m/s}^2 \) and (3) \( \sigma_x = 3 \text{ m} \), \( \sigma_a = 8 \text{ m/s}^2 \). The dynamic of each filter is chosen identical.
5.1 Test scenario

The test scenario simulates a car moving with constant velocity, then driving a 180° left turn followed by a 180° right turn. For a better comparability of the simulation results, only the performance of the CA model (28) and the CA model with angular velocity \( \omega \) (34) are compared to one another depending on the available measurements, resulting in 6 test cases:

1. CA model with measured position (CA\(_{lp}\)), 2. CA model with measured position and acceleration (CA\(_{lpa}\)).
3. CA model with angular velocity with measured position (CA\(_{olp}\)), 4. CA model with angular velocity with measured position and acceleration (CA\(_{olpa}\)), 5. CA model with angular velocity with measured position and angular velocity (CA\(_{olp\omega}\)), and 6. CA model with angular velocity with measured position, acceleration, and angular velocity (CA\(_{olpa\omega}\)).

The simulation results in Fig. 4 show (a) the zoomed position of the first turn, (b) velocity in the x-coordinate, (c) acceleration in the x-coordinate, as well as (d) angular velocity. Measurement noise of position estimation and acceleration sensor is modeled by zero mean white Gaussian noise with a standard deviation of \( \sigma_x = 8 \text{ m/s}^2 \), \( \sigma_x = 3 \text{ m} \), \( \sigma_x = 8 \text{ m/s}^2 \) and \( \sigma_x = 8 \text{ m} \), \( \sigma_x = 8 \text{ m/s}^2 \). No noise was added on the angular velocity.

Calculating the acceleration of the object (c.2), CA\(_{lp}\) compared to CA\(_{olp}\) lags more behind but has less undershooting; CA\(_{lpa}\) compared to CA\(_{olpa}\) lags more behind, but has less undershoot, both improve the performance significantly compared to CA\(_{lp}\). CA\(_{lpa}\) lags a little more than CA\(_{olpa}\), but undershoot of both are similar. For the angular velocity (d.1) CA\(_{olp}\) lags extremely behind with a significant undershooting. CA\(_{olpa}\) lags moderate behind with a slight undershooting. For a large deviation in both (.3 in Fig. 4), position measurements and acceleration data, the position estimation results of (a.3) are similar to (a.2), but the total lag is increased significantly. Velocity estimation results of (b.3) have a similar characteristic compared to (b.2), but the total lag and the overshooting are increased significantly. The results of estimating acceleration (c.3) and the angular velocity (d.3) are very similar to (c.2).

6 Measurement Results

The motor sport center at the Wachau race track, Austria, is equipped with an LPM prototype system. The LPM master processing unit is installed at the paddock club above the pit lane. The basic test scenario is a car driving a slalom. Tracking of an object driving a slalom is one of the most challenging test scenarios. To demonstrate the impact of additional sensor data to improve the state estimation of the LPM system the test car was equipped with accelerometers, gyroscopes and two transponders. Two MTs are necessary to estimate the heading of the car. The 32-bit telemetry channel of the LPM system was used to transmit sensor data of two accelerometers, each coded with 11 bits, and of one gyroscope coded with 10 bits. The results of estimated position state, velocity state, acceleration state, as well as measured angular velocity and heading are shown in Fig. 5. Due to the non-sliding behavior of the object the velocity heading is identical to the object heading. For simplicity only the results for CA\(_{lp}\), CA\(_{lpa}\), CA\(_{olp}\), and CA\(_{olpa}\) are displayed.

The adding of acceleration data shows a significant raise in the accuracies in all states as expected from the simulation data. Due to the high sample rate and the high accuracy position measurement results of the LPM system the angular velocity measurement has only a slight influence on the accuracy of estimation position, velocity or acceleration state.

7 Conclusions

Data fusion is a very powerful method to increase the accuracy as well as the dynamic of object tracking systems. By taking only measurements of the object position into account for tracking a highly maneuverable target, a Kalman tracking filter will always lag behind the true state, or, if the filter is tuned to have an extreme dynamic, the states will be estimated extremely inaccurate. Additional sensor information of higher derivatives of the state like acceleration reduces this lag significantly. In this contribution a detailed discussion is given on discrete 2-D tracking model with the focus on
the applicability of these models for sensor fusion purposes. Simulation results demonstrated the influence of sensor noise. Additional sensor information tends always to increase the accuracy of state estimation, but as it was shown, these increases can be, depending on the specific circumstances, negligible. The simulation results where verified on real measurement data of a car driving a slalom, equipped with an LPM transponder for position estimation as well as a sensor unit consisting of two accelerometers and one gyroscope.

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References


