Assessment of Soil Parameter Estimation Errors for Fusion of Multichannel Radar Measurements

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Abstract – The application of multichannel radar measurement techniques for estimation of bare soil parameters is based on different principles of radiowave and soil surface interaction depending on radiowave frequency, polarisation and incidence angle. The accuracy of soil parameter estimation depends on the number of radar measurements and the choice of radiowave parameters. Random and systematic errors present in radar data may also have the impact on estimation results. To improve the accuracy of soil parameters estimation by fusion of multichannel radar data we propose a new method for assessment of estimation errors. It is based on local linear approximation of the radiowave scattering model and takes into account impairment characteristics, measurement conditions and radar parameters. This new method is applied to an example to illustrate how the estimation accuracy of soil moisture and roughness parameters can be improved by optimising the radar operating frequencies.

Keywords: Data fusion, errors, radar measurements, radar scattering model, soil roughness, soil moisture.

1 Introduction

Multichannel measurements are frequently used in radar remote sensing of the Earth’s surface to improve the information content of radar data, to increase the accuracy and reliability of measurement results and to estimate several parameters of the observed surface objects simultaneously. The high efficiency of the multichannel approach in retrieving the desired physical parameters of the Earth’s surface is provided by exploiting the different ways radiowaves interact with objects depending on frequency, polarisation and incidence angle of the scattered signal [1]. The intensity of the radar signal scattered from the Earth’s surface depends on surface geometric structure and dielectric characteristics. The form of this dependence is also a function of radar signal parameters, which can be modified to improve the information content of the scattered signal.

Radar measurements of the Earth’s surface at different radiowave frequencies, polarisations and incidence angles obtained simultaneously or within short time intervals provide detailed information about sensed objects and improve the accuracy of information retrieval. The efficiency of multichannel radar measurements depends on the choice of radar signal parameters, the number of measurements and the impact of measurement errors [2]. Hence, the multichannel technique can achieve high accuracy by optimisation of radar parameters and minimisation of radar measurement errors.

An important application of multichannel radar methods for soil remote sensing is the estimation of moisture and surface roughness parameters [3]. It has been shown in [2,4] that it is theoretically possible to estimate the roughness and moisture characteristics of soil with high accuracy by using multifrequency and dual-polarisation signals when radiowave frequencies and incidence angles of measurements are selected properly. The drawbacks of radar data fusion based on theoretical models are eliminated by application of empirical radar cross section (RCS) models based on experimental measurements [5]. The algorithm proposed by Oh et al. in [5] for simultaneous estimation of soil moisture and root mean square height parameters is based on fusion of three radar measurements at vertical (VV), horizontal (HH) orthogonal polarisations and cross-polarisation component (HV). The radar measurements at different polarisations are fused by inversion of the dependencies between the scattering signal and soil parameters. The dependencies are derived from the empirical soil scattering model developed by Oh et al. [5]. This approach has been extended in [6], where more than three radar measurements at several frequencies and incidence angles are fused to improve the accuracy and reliability of soil parameter estimates. The algorithm [6] is based on minimisation of the discrepancies between the measured radar signal scattered by soil surface and the signal predicted by an empirical or theoretical RCS model.

While the accuracy of soil parameter estimation is improved by increasing the number of independent radar measurements at different frequencies and incidence angles, the degree of improvement also depends on radar parameters. The standard approach for selecting optimal frequency and incidence angle radar parameters assumes the provision of high sensitivity of radar measurements with respect to variations of soil parameters [2]. For applications dealing with simultaneous estimation of several parameters the optimisation of radar data fusion performance requires joint analysis of different factors.
which have a complicated influence on the sensitivity of radar measurements. High sensitivity of measurements should be provided simultaneously for several estimated parameters and application of the standard approach for radar parameter optimisation is complicated.

The new method for assessment of soil parameter estimation errors proposed in this paper can be applied for any number of estimated soil parameters and for fusion of radar measurements with different characteristics. The method is based on linear approximation of the soil scattering model and uses the criterion of mean square error between the estimated and true soil parameters for evaluation of measurement errors. It takes into account both random and systematic measurement errors and can be used for optimisation of the efficiency of multichannel radar measurements in combination with any theoretical and empirical radar scattering models for bare soil surfaces. The application of the proposed method for optimisation of radar measurements is considered for the empirical RCS model of soil surface proposed by Oh et al. [5].

The paper is organised as follows. In section 2 we consider different factors that affect the multichannel radar data fusion results and soil parameter estimation results. The statistical characteristics of data fusion errors are derived for the ratios of RCS measurements at VV, HH and HV polarisations. The method for evaluating multichannel data fusion and soil parameter estimation errors is presented in Section 3. The experimental results of applying the proposed method to the assessment of two-frequency multipolarisation data fusion and to the optimisation of radar parameters are presented in section 4. Final conclusions are given in Section 5.

2 Soil Roughness and Moisture Content Estimation Errors

For multichannel radar remote sensing techniques the accuracy of soil parameter estimation depends on a number of factors. Some of them can be neglected while the influence of the others is significant and has to be taken into account. These factors are:

- RCS measurement errors at different frequencies and polarisations;
- the discrepancies between measured and predicted intensities of the scattered signal due to inaccuracies of the radar scattering model;
- the dependencies between scattered signal intensity and measured soil parameters (volumetric moisture content \( m_w \), root-mean-square (rms) height of surface roughness \( \sigma_s \), and soil composition);
- radar measurement parameters, such as the number of RCS measurements, operating frequencies and incidence angles;
- systematic errors introduced by the scattering model, radar sensor and data fusion algorithm.

RCS measurement errors usually contain both random and systematic components which can be considered as fluctuating and bias error signals respectively. The fluctuating component is caused by equipment internal noise and the interference nature of the scattered signal. Internal equipment noise has the properties of additive white Gaussian noise with zero mean and variance \( \sigma_w^2 \). The intensity of internal noise depends on the characteristics of the radar receiver and has fixed level selected at the stage of radar system design. The fluctuations in the radar signal due to interference are considered as statistically independent multiplicative noise with an intensity that is usually much higher than the intensity of internal equipment noise [1]. Radar signal fluctuations are suppressed by averaging of independent radar measurements that can be obtained by variation of operating frequencies and incidence angles [2].

The non-fluctuating component of errors represents a bias added to RCS measurements. It appears in radar data due to calibration errors and time variations of radar parameters [2]. Another possible source of bias errors in soil parameter measurements is the discrepancy between the predicted and measured intensities of the scattered radar signal. Errors of this type are observed for radar scattering models that are not accurate for the particular measurement conditions and soil characteristics, or for simplified empirical and theoretical scattering models. For example, the theoretical models of geometric optics, physical optics, small perturbation method and integral equation method, are only valid for highly restricted surface types and conditions [3] and their application for natural surfaces may produce bias errors. Considerably better agreement with experimental RCS measurements is achieved for empirical scattering models [7]. To establish the correspondence between soil properties and specific RCS values we used the empirical model developed by Oh at al. [5], that estimates the ratio of specific RCS \( p = \sigma_{HH}^0/\sigma_{VV}^0 \) and \( q = \sigma_{HV}^0/\sigma_{VV}^0 \) at HH, VV and HV radiowave polarisations. The coefficients \( p \) and \( q \) of Oh at al.’s model depend simultaneously on the rms height of surface roughness \( \sigma_s \) and soil dielectric constant \( \varepsilon \) [5] as follows:

\[
\begin{align*}
    p &= \left[ 1 - \left( \frac{2\theta_i}{\pi} \right)^{\frac{1}{3}} \argument 0 \exp(-k\sigma_s) \right]^2, \\
    q &= 0.23 \left( \frac{\lambda}{\sigma_s + 1} \right)^2 \left[ 1 - \exp(-k\sigma_s) \right]
\end{align*}
\]

where \( \Gamma_0 = \left| \frac{\varepsilon - 1}{\varepsilon - 1} \right|^2 \) is the Fresnel coefficient, \( k=2\pi/\lambda \) is a wavenumber, \( \theta_i \) is an incidence angle.

Consider the statistical characteristics of \( p \) and \( q \) ratios for Oh at al.’s empirical model. Assume that the following conditions are satisfied: the radar is calibrated with high accuracy; the errors of the empirical scattering model are small; the intensity of interference fluctuations is much higher than the intensity of equipment internal noise. In this case the bias component of RCS measurement errors and the equipment noise have considerably lower intensity than the interference fluctuations of the scattered signal and their impact is not significant.

Suppose that \( K \) independent measurements \( I_k, k=1...K \), of scattered signal intensity are available and the estimates of the surface specific RCS are formed by averaging of \( I_k \) radar measurements to suppress the interference
fluctuations and improve the accuracy. Then, the RCS estimate is

\[ I = \frac{1}{K} \sum_{i=1}^{K} I_i. \] (2)

The average value of the intensity \( I \) obeys a gamma distribution law with order parameter \( K \) [8] and has probability density function (pdf)

\[ p(I) = \frac{1}{\Gamma(K)} \left( \frac{K}{\sigma} \right)^K I^{K-1} e^{-K I / \sigma}, \quad I \geq 0 \] (3)

where \( \Gamma(\cdot) \) is the gamma function, and \( \sigma \) is a distribution parameter. The average intensity \( I \) is equal to \( \langle I \rangle = \sigma \) and its variance is \( \text{var}(I) = \sigma^2 / K \).

The specific RCS ratios \( p \) and \( q \) for the empirical model (1) are given by

\[ p' = \frac{I_{HH}}{I_{VV}}, \quad q' = \frac{I_{HV}}{I_{VV}} \] (4)

where \( I_{HH} \), \( I_{VV} \) and \( I_{HV} \) are signal intensities at HH, VV and HV polarizations. To find the statistical characteristics of \( p' \) and \( q' \) ratio estimates we assume that the parameter \( \sigma \) of gamma distribution in (3) is equal to 2 and the average estimate \( I \) is scaled by \( K \) and \( I' = I \cdot K \). Then, random variables \( I' \) have a chi-squared distribution \( \chi^2_n \) with \( n=2K \) degrees of freedom. It can be shown that the ratio of two variables with chi-squared distribution \( \chi^2_n \) obeys the F-distribution law [9] with the pdf given by

\[ p_r(x) = \frac{\Gamma(n)}{\Gamma(n/2)^2} \frac{x^{n/2-1}}{(1+x)^n}. \] (5)

F-distribution (4) has mean value \( m_F = n/(n-2) \) and variance \( \sigma^2_F = \frac{4n^2(n-1)}{n(n-2)^2(n-4)} \). Obviously, the statistics \( p' \) and \( q' \) in (4) also have F-distributions, but their mean and variance values are different. The mean values for \( p' \) and \( q' \) are calculated as

\[ m_p = p_i \frac{K}{K-1}, \quad m_q = q_i \frac{K}{K-1} \] (6)

where \( p_i \) and \( q_i \) are the true values of the intensity ratios. It follows from (6) that the estimates (4) are biased and their magnitudes depend on the number of averaged measurements \( K \). The variances of \( p' \) and \( q' \) statistics are

\[ \sigma^2_p = \frac{p_i^2 K(2K-1)}{(K-1)^2(K-2)}, \quad \sigma^2_q = \frac{q_i^2 K(2K-1)}{(K-1)^2(K-2)}. \] (7)

According to (7) the variances of \( p_i \) and \( q_i \) ratio estimates depend on the number of averaged measurements and the true values of estimated parameters. These results confirm the fact that the interference noise is multiplicative. When the number of averaged measurements \( K<3 \) the variance of signal intensity ratio is not defined, and at least three independent measurements are required to produce a reliable estimate. The pdfs for the F-distribution and for the number of measurements \( K=3 \) and \( K=200 \) are shown in Figure 1.

![Figure 1: Pdfs of \( p \) and \( q \) statistics for averaged measurements: (a) \( K=3 \); (b) \( K=200 \)](image)

The Gaussian pdf with equal mean and variance parameters is depicted in Figure 1 by a dashed line. For \( K=3 \) the form of the F-distribution is significantly different from the Gaussian, but for \( K=200 \) there is no significant difference between two distributions. For relatively large numbers of averaged samples the distributions of \( p' \) and \( q' \) statistics are approximately Gaussian and the estimates of soil parameters based on mean square error criterion are quasi-optimal, as for Gaussian distribution the least mean square estimate and maximum likelihood estimate are identical.

### 3 Assessment of Data Fusion Errors

For assessment of soil parameter estimation errors we have to make assumptions about the algorithm used for multichannel radar data fusion and the model used to describe the radar signal scattering from the bare soil surface. We assume that an algorithm based on mean square error criterion [6] is used for radar data fusion and there are no restrictions imposed on the choice of soil scattering model. But it is expedient to choose a particular scattering model for consideration. For predicting the radar signal intensities we used Oh at al.’s empirical model [5]. This provides a good match between the estimated and true soil parameters for a wide range of soil
types and characteristics [6]. Oh at al.’s model establishes the dependencies between \( p \) and \( q \) RCS ratios at different radiowave polarizations and the parameters of bare soil, such as rms height of surface roughness \( \sigma_0 \) and soil dielectric constant \( \varepsilon \). The dielectric properties of soil are functionally dependent on the volumetric moisture content \( m_v \), percentage of sand \( S \) and clay \( C \) in soil and other factors.

To calculate the dielectric constant \( \varepsilon \) we used the water-soil mixture model presented in [3], which establishes the dependence between the dielectric constant \( \varepsilon \) and moisture content \( m_v \), as \( \varepsilon(m_v) \) for the particular soil type and composition. Then, the ratios of RCS at different polarizations are considered as functions of soil parameters \( p(\sigma_0, m_v) \) and \( q(\sigma_0, m_v) \).

To assess the soil parameter measurement accuracy we used an approach based on local linear approximation of \( p \) and \( q \) ratios as functions of soil roughness \( \sigma_0 \) and moisture content \( m_v \) parameters. We assume that the errors in \( p \) and \( q \) measurements are relatively small and that near to the true \( p_t \) and \( q_t \) values the dependencies \( p(\sigma_0, m_v) \) and \( q(\sigma_0, m_v) \) are linear:

\[
p(\sigma_0, m_v) \equiv p(\sigma_0, m_v, 0) + \frac{\partial p}{\partial \sigma_0}(\sigma_0 - \sigma_0, 0) + \frac{\partial p}{\partial m_v}(m_v - m_v, 0)
\]

\[
q(\sigma_0, m_v) \equiv q(\sigma_0, m_v, 0) + \frac{\partial q}{\partial \sigma_0}(\sigma_0 - \sigma_0, 0) + \frac{\partial q}{\partial m_v}(m_v - m_v, 0)
\]

where \( \sigma_0, 0 \) and \( m_v, 0 \) are true parameter values. The analytical expressions (8) are obtained by Taylor expansion of \( p(\sigma_0, \varepsilon(m_v)) \) and \( q(\sigma_0, \varepsilon(m_v)) \) equations for Oh at al.’s model, where the dependence \( \varepsilon(m_v) \) is given by the water-soil mixture model [3]. Soil parameters \( \sigma_0 \) and \( m_v \) are estimated by standard least-squares method assuming that the errors of \( p \) and \( q \) ratio measurements are small and the mean square error criterion can be used for parameter estimation [6].

Consider the vector \( H' = [p_1, p_2, \ldots, p_J, q_1, q_2, \ldots, q_J] \) of radar measurements for \( p \) and \( q \) RCS ratios at different operating frequencies and incidence angles. In general, the number of measurements for \( p \) and \( q \) ratios can be different. The measurement vector \( H' \) is presented as

\[
H' = H + S = Z + \delta H + S
\]

where \( Z \) is a vector of true values of \( p_t \) and \( q_t \) ratios, \( \delta H \) is a vector of random measurement errors due to the signal interference fluctuations and equipment internal noise. As it has been shown in section 2, the distributions of \( \delta H \) vector elements are close to Gaussian. \( S \) is a vector of systematic errors, such as radar calibration errors and soil scattering model errors. The elements of vector \( S \) are split into groups of similar errors, which correspond to different radar sensors and measurement conditions. The systematic error vector is \( S = [s_1, s_1, \ldots, s_2, s_2, \ldots, s_M, s_M]' \), where \( M \) is the number of groups.

Now, there is a need to assess the impact of random and systematic errors on the accuracy of soil parameters estimation, and consider the opportunity to detect and compensate the bias error component. We have assumed in (8) that the functions \( p(\sigma_0, m_v) \) and \( q(\sigma_0, m_v) \), are linear close to the true parameters \( \sigma_0 \) and \( m_v \). The vector of RCS ratios \( p \) and \( q \) can be expressed as \( Z = A_{\sigma_0} X_{\sigma} \), where \( X_{\sigma} \) is a vector of soil parameters of length \( n \) (the vector \( X_{\sigma} \) is equal to \( \sigma_0, m_v, 1 \) as we estimate the roughness and moisture content parameters). \( A_{\sigma_0} \) is an array of dimension \( N \times n \). The elements of \( A_{\sigma_0} \) array are calculated as partial derivatives of \( Z = [z_1, z_2, \ldots, z_N]' \) functions with respect to \( x_1, x_2, \ldots, x_n \) variables as

\[
A_{\sigma_0} = \frac{\partial Z(X_{\sigma})}{\partial X} = \begin{bmatrix}
\partial z_1(X_{\sigma}) / \partial x_1 & \partial z_1(X_{\sigma}) / \partial x_2 & \ldots \\
\partial z_N(X_{\sigma}) / \partial x_1 & \partial z_N(X_{\sigma}) / \partial x_2 & \ldots
\end{bmatrix}
\]

(10)

where \( X_{\sigma} \) is a vector of true soil parameters. For the radar scattering model in (1) and water-soil mixture model in [3] the elements of \( A_{\sigma_0} \) array in (10) are calculated by Ridder’s numerical differentiation method [10].

### 3.1 Random errors

Consider radar measurements without systematic errors, when \( S = 0 \). Then, application of a least-squares method for estimation of soil parameters gives the estimate of \( X_{\sigma} \) vector as

\[
\hat{X}_n = (A_{\sigma_0}^T W A_{\sigma_0})^{-1} A_{\sigma_0}^T W H
\]

(11)

where \( W \) is an array of weighting coefficients that weight the measurements in proportion to their accuracy.

The covariance matrix of least-squares estimate of \( \hat{X}_n \) vector is then given by the expression [11]

\[
R_{\sigma} = D R_{H} D^T
\]

(12)

where \( D = B^{-1} A_{\sigma_0} W \) and \( B = A_{\sigma_0} W A_{\sigma_0} \cdot R_{H} \) is the covariance matrix of random measurement errors \( \delta H \) for \( p \) and \( q \) ratios.

### 3.2 Bias Errors

The non-fluctuating component of errors \( S \neq 0 \) presented in the vector of measurements \( H' \) has an impact on both the estimates of soil parameters \( X_n \) in (11) and their statistical characteristics. When \( S \neq 0 \) the mean of the estimated vector of parameters \( \hat{X}_n \) is given by

\[
m(X_n) = X_n + DS
\]

(13)

It follows from (13) that non-fluctuating error component adds a bias to soil parameter estimates. Since the errors of soil parameter measurements are evaluated as the difference between the estimated and true parameters \( X_n \),
we can assume that true parameters are equal to \( X_n = 0 \). This assumption simplifies the mathematical expressions for the assessment of estimation errors and is valid for linear dependencies between measurements \( H' \) and estimated parameters \( X_n \). Then, the measurement errors can be characterised by the matrix of second moments \( K_{X_n} \).

Consider the matrix of second moments for vector \( \hat{X}_n \) in (11). It can be shown that for the bias of magnitude \( S \) in the measurements the matrix of second moments for the least-square estimates is given by

\[
K_{\hat{X}_n} = DR_{H'}D^T + DSS^T D^T. \tag{14}
\]

Hence, the matrix \( K_{\hat{X}_n} \) in (14) differs from the covariance matrix \( R_X \) in (12) by the additional component \( DSS^T D^T \), which is proportional to the magnitudes of the systematic errors and can be found from (13) as the square of bias \( DS \). The matrix of second moments \( K_{\hat{X}_n} \) also depends on the covariance matrix \( R_H \) of random measurement errors.

### 3.3 Correction of Bias Errors

The accuracy of soil parameter measurements can be significantly improved by minimising the influence of systematic errors. This is achieved by application of efficient radar calibration techniques and accurate radar scattering models based on experimental measurements. But in many practical situations the bias component of the residual errors may have a significant impact on the accuracy of soil parameter estimation. In the presence of radar calibration and scattering model errors the accuracy of radar data fusion algorithm can be improved by estimating the systematic error of vector \( S \) together with the unknown soil parameters and subsequent correction of estimated errors. The elements of vector \( S \) can be considered as additional estimated parameters and added as new elements of vector \( \hat{X}_n \). Then, the influence of bias errors is eliminated and the second component \( DSS^T D^T \) in equation (14) becomes equal to zero. By adding new estimated parameters we also increase the impact of random errors and decrease the accuracy of estimates. The influence of random errors is characterised by \( DR_{H'}D^T \) component in equation (14). As the impact of random errors is increased, the improvement of the accuracy of estimates must be questioned. To evaluate the improvement achieved by estimation of \( S \) error vector we used the matrix \( K_{\hat{X}_n} \) of second moments of least-squares estimates. The diagonal elements of \( K_{\hat{X}_n} \) in (14) indicate changes of soil parameter mean square errors when the additional parameters are estimated.

Consider the vector of estimated parameters \( \hat{X}_{n+k} \) extended by a \( k \)-element vector \( S \) of systematic errors. The least-squares method gives the estimate of the \( \hat{X}_{n+k} \) vector as

\[
\hat{X}_{n+k} = (A_{N,n+k}^T W A_{N,n+k})^{-1} A_{N,n+k}^T W H'. \tag{15}
\]

The vector of estimates can be presented as \( \hat{X}_{n+k} = [x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_{n+k}]^T \), where \( x_n \) is a vector of soil parameters and \( \hat{X}_k \) is a vector of systematic errors. The elements of the extended array \( A_{N,n+k} \) are found as partial derivatives of functions \( Z' = Z + S \) with respect to the extended set of variables \( X_{n+k} \)

\[
A_{N,n+k} = \frac{\partial Z'(X_0')}{\partial X}. 
\]

where \( X_0' \) is a vector of true values of parameters \( X_{n+k} \). It can be shown that the estimates \( \hat{X}_{k+n} \) in (15) obtained by the least-squares method are unbiased [11]. Then, the mean of \( \hat{X}_{k+n} \) is given by

\[
m(X'_{n+k}) = X_{n+k} = [X_n X_k]^T.
\]

The covariance matrix for the least-squares estimate is given by

\[
R_{\hat{X}_{n+k}} = D_{n+k} R_H D_{n+k}^T \tag{17}
\]

where

\[
D_{n+k} = B_{n+k}^{-1} A_{N,n+k}^T W \quad \text{and} \quad B_{n+k} = A_{N,n+k}^T W A_{N,n+k}.
\]

As mentioned above, we can assume that the true values of estimated parameters are equal to zero, and \( X_{n+k} = 0 \). Taking into account that the estimates \( \hat{X}_{k+n} \) are not biased, it can be shown that the covariance matrix \( R_{\hat{X}_{n+k}} \) is equal to the matrix of second moments \( K_{\hat{X}_{n+k}} \).

Hence, the matrix of second moments is given by

\[
K_{\hat{X}_{n+k}} = R_{\hat{X}_{n+k}} \tag{16}
\]

Consider \( K_{\hat{X}_{n+k}} \) matrix as a block matrix

\[
\begin{bmatrix}
K_{\hat{X}_{n,n}} & K_{\hat{X}_{n+1,\hat{k}}} \\
K_{\hat{X}_{n+1,\hat{n}}} & K_{\hat{X}_{\hat{k},\hat{k}}}
\end{bmatrix}
\]

The block matrix element \( K_{\hat{X}_{n,n}} \) of size \( n \times n \) is the covariance matrix for soil parameter measurements \( \hat{X}_{n} \). Its analytical expression is obtained by substitution of the \( A_{N,n+k} \) array in equation (17) into a form of block array \( [A_{N,n} A_{N,k}] \). It can be shown that \( K_{\hat{X}_{n,n}} \) matrix of second moments is presented as the sum of two components [12]
where the component $R_{R_{n,n}}$ is equal to the covariance matrix $R_{\hat{X}}$ in (12) for the least-squares estimates when the regular component of errors is absent. The component $Q$ in (18) indicates the deterioration of the estimation accuracy when the number of estimated parameters is extended by the $X_k$ vector of additional parameters.

Hence, the accuracy of soil parameter estimation is restricted by the $Q$ component in (18) when the bias component of errors is estimated and corrected, and by the $DSS^T D^T$ component in (14) when the $S$ vector is not estimated simultaneously with soil parameters. To evaluate the efficacy of systematic error correction we used the criterion based on comparison of the error second moments $K_{X_k}$ and $K_{R_{n,n}}$ given by the expressions (14) and (17). After eliminating the equal elements $R_{R_{n,n}}$ and $R_{\hat{X}}$ in both expressions we need to compare only the $DSS^T D^T$ and $Q$ components. For calculation of the $DSS^T D^T$ matrix we have to know the true values of bias errors defined by vector $S$. But for the majority of practical applications the vector of bias errors $S$ is not known a priori and for evaluation of the impact of systematic errors we use the approach based on approximation. If the dependencies between the measured data and estimated parameters are linear, for calculation of $DSS^T D^T$ matrix we can use the estimates of systematic errors $\hat{S}$ given by equation (15). When Oh at al.’s radar scattering model [5] is applied in combination with the water-soil mixture model [3] for soil parameter estimation, the dependencies between the estimated parameters are nonlinear and the equation (15) based on linear least-squares method can not be applied. To estimate the vector of bias errors $\hat{S}$ for Oh at al.’s scattering model we use the nonlinear least-squares method based on numerical algorithms for objective function minimisation [7]. After $\hat{S}$ vector is found the decision is made about the importance of systematic error correction by comparison of the $DSS^T D^T$ and $Q$ matrices. The criterion we use to make a decision is

$$\text{tr}(Q) > \text{tr}(DSS^T D^T)$$

where $\text{tr}(\cdot)$ denotes trace of the matrix. If condition (19) is satisfied we assume that the influence of $S$ component in (9) is significant and bias errors should be estimated as additional parameters. Otherwise, only the soil parameters are estimated.

### 4 Experimental Results

The proposed method establishes a relationship between the soil parameter estimation errors, radar measurement errors and radar parameters such as frequency, polarisation and incidence angle. This relationship can be used to improve the accuracy of measurements by correction of radar calibration errors and optimisation of radar parameters. The approach for mitigation of systematic errors has been presented in Section 3.3. Now, we consider how the proposed method can be used to minimise soil parameter estimation errors by proper selection of radar operating frequencies.

For estimating volumetric moisture content $m$, and rms height of surface roughness $\sigma_h$ parameters we fuse multichannel radar measurements of a specific RCS taken at HH, VV and HV polarizations, two frequencies $f_1$ and $f_2$ selected in the range 1-30GHz and incidence angle 0-30°. Assume that radar measurements are taken for a uniform region of bare soil with the following characteristics:

- the percentage of sand in soil $S=0\%$;
- the percentage of clay in soil $C=29.7\%$;
- packed soil density $\rho_v=1311$ kg/m$^3$;
- volumetric moisture content $m_v=0.3$ g/cm$^3$;
- rms height of surface roughness $\sigma_h=0.4$ cm.

To establish the dependencies between the considered soil parameters, radar operating parameters and the intensities of soil scattered signal we use Oh at al.’s empirical scattering model and water-soil mixture model [3]. To estimate soil parameters we applied the nonlinear least-squares algorithm [6] that minimises the discrepancy between the measured and Oh at al.’s model predicted ratios $p=\sigma_{HH}/\sigma_{VV}$, $q=\sigma_{HV}/\sigma_{VV}$ of specific RCS. The algorithm assumes weighting of $p$ and $q$ radar measurements according to their accuracy. The weighting coefficients for $W$ matrix (11) are selected in two different ways:

- the covariance array $R_H$ of radar measurement errors $\delta H$ of RCS ratios $p$ and $q$ is known a priori and the weighting array is selected as $W=R_H^{-1}$;
- the accuracy of radar measurements is not known in advance and the measurements are weighted equally as $W=I$, where $I$ is a unit matrix.

The measurements of $p$ and $q$ RCS ratios are considered to be statistically independent and the covariance matrix $R_H$ to be diagonal. Statistical independence of $p$ and $q$ measurements is provided by using different radar operating frequencies and by varying the radar position and operating time intervals for the particular case when the operating frequencies $f_1$ and $f_2$ are identical. The elements of matrix $R_H$ are calculated from the expression (7) for the variance of $p$ and $q$ parameters.

The experimental results presented in Figure 2 and Figure 3 illustrate the dependencies of second moments for measurement errors $m^2(m)$, and $m^2(\sigma_h)$ for soil moisture content and surface roughness parameters estimated by the proposed method. For calculation of measurement errors we used equation (14) assuming that the components of $S$ array are equal to zero when the regular component of errors is absent. The radar operating frequencies are selected in the interval 1-30GHz and the number of averaged radar measurements is equal to $K=200$ for every frequency. As it follows from expression (7) for the number of averaged data $K=200$ the standard deviation of estimated RCS ratios is equal to $\sigma_p=0.1\, p$ and...
Figure 2: second moments of soil parameter estimation errors for $W = R_H^{-1}$ and $S=0$: (a) soil roughness, $S=0$; (b) soil moisture, $S=0$; (c) soil roughness, $S=0.02$; (d) soil moisture, $S=0.02$.

Figure 3: second moments of soil parameter estimation errors for $W = I$ and $S=0$: (a) soil roughness; (b) soil moisture

$\sigma_q = 0.1q$ for $p$ and $q$ parameters respectively. Hence, the assumption about linear dependencies of RCS with respect to soil moisture $m_v$ and roughness $\sigma_h$ parameters is satisfied in the vicinity of true $p$ and $q$ ratios.

For the dependencies in Figure 2 the measurements are weighted as $W = R_H^{-1}$ and the covariance matrix is known a priori. Figures 2(a)-(b) show second moments of measurement errors for $m_v$ and $\sigma_h$ soil parameter when the bias error is absent. The dependencies are monotonic and the accuracy of measurements is improved when both operating frequencies $f_1$ and $f_2$ are decreased simultaneously. The minimum value of measurement error for the considered interval of operating frequencies is achieved for $f_1=f_2=1$. The second moments of errors are equal to $m_v^2(m_v)=0.0009$ and $m_h^2(\sigma_h)=0.001$ for soil moisture and roughness parameters respectively.

When all the elements of $S$ array are equal to 0.01, the bias error of the same order as random errors is added to all measurements of $p$ and $q$ ratios. Then, the estimation errors of $m_v$ and $\sigma_h$ parameters are increased for the whole range of operating frequencies. This is illustrated in Figures 2(c)-(d). The form of the dependencies is not changed significantly. However, for the biased measurements the function $m_v^2(\sigma_h)$ in Figure 2(c) is increased for small operating frequencies and reaches its minimum at frequencies of about 4 to 6 GHz.

It follows from Figure 3 that the dependencies for the second moments of errors $m_v^2(m_v)$ and $m_h^2(\sigma_h)$ have more complex forms when the measurements are not weighted.
and $W=I$. When the systematic errors are absent from radar measurements the best estimation accuracy of soil roughness parameter $\sigma_h$ is achieved for radar frequencies in the range 10-15GHz and for similar frequencies of lower magnitude (see Figure 3(a)). For the $m_v$ soil moisture parameter the best accuracy is provided in Figure 3(b) for similar frequencies and for the frequencies selected in the range 20-30GHz. The bias error of magnitude $S=0.01$ added to the measurements of RCS ratios does not change the form of the functions in Figure 3 significantly. However, a more complicated dependency between the accuracy of estimated parameters and radar operating frequencies is observed.

The obtained results agree with the experimental data given in [5]. It is shown in [5] that the sensitivity of $p$ and $q$ ratios with respect to $m_v$ and $\sigma_h$ soil parameters is decreased when radar operating frequencies are increased. Hence, the accuracy of soil parameter measurements can be improved by application of lower operating frequencies. It also follows from the results presented in Figure 3 that the estimation accuracy can be improved by selecting similar operating frequencies if the covariance matrix is not known a priori and measurements are not weighted. These recommendations agree with the conclusions given in [2] concerning the enhancement of specific RCS ratio sensitivity with respect to soil moisture parameter when similar radar operating frequencies are used for multichannel measurements at different frequencies and incidence angles.

Hence, the efficacy of the proposed method for the assessment of soil parameter estimation errors is illustrated by the experimental results for two-frequency radar measurements.

5 Conclusions

A method based on local linear approximation of specific RCS dependencies has been proposed in this paper for assessment of soil parameter estimation errors, for fusion of multichannel radar measurements at different operating frequencies, polarisations and incidence angles. The proposed method does not require any specific assumption about the type of radar scattering model and can be used for radar data fusion based on both theoretical and empirical models. The method evaluates the impact of random and systematic errors on the results of radar data fusion and the accuracy of parameter estimation. Its application has been considered for the assessment of soil moisture and roughness parameter estimation errors for two-frequency multipolarisation radar measurements. It has been shown that for Oh at al.’s radar scattering model the choice of radar operating frequencies has a significant impact on radar data fusion and parameter estimation accuracy.

In future work our method will be used for remote sensing applications to assess the Earth surface parameter estimation errors and to optimise radar remote sensing system characteristics.

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References