Robust Processing of SAR Hologram Data to Mitigate Impulse Noise Impairments

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Abstract - The standard linear algorithm for synthetic aperture radar (SAR) image synthesis consists of fusing the real and imaginary components of the hologram data, and applying a matched filter to the radar returns from each point to generate the SAR image. This algorithm becomes inefficient when the SAR hologram data are corrupted by impulse noise due to data transmission and coding/decoding errors. The degradation of synthesised SAR images in the presence of impulse noise in the hologram data is considered and new nonlinear algorithms based on robust estimation are proposed. Experimental results for measured SAR data are presented. It is shown that the proposed algorithms efficiently reject impulse noise and substantially improve SAR image quality.

Keywords: Synthetic aperture radar, fusion, hologram, image synthesis, robust estimate.

1 Introduction

Synthetic aperture radar (SAR) sensors have been successfully used for remote sensing applications for several decades [1]. While the conventional side-looking radars use a “brute-force” approach to achieve the desired azimuth resolution by extending the physical antenna dimensions, SAR sensors use other operational principles and provide greater opportunities for ground-mapping and moving object detection applications [2]. SAR operates in coherent mode and use signal processing techniques to generate a long synthetic antenna instead of using a physically large actual antenna. The synthesis of the antenna aperture is provided by the forward motion of the SAR platform [2, 13]. The size of the synthetic aperture considerably exceeds that of a real antenna and the resolution of SAR sensors in the azimuth plane is commonly better in comparison with side-looking radar sensors.

The focused synthesis of SAR images is implemented in two stages. First, the radar hologram data are collected over some interval of time in the form of real and imaginary components. The complex hologram incorporates amplitudes and phases for the sum of signals reflected from the Earth’s surface within the particular range intervals and within the antenna azimuth pattern. At the next stage, the real and imaginary hologram components are fused together by processing in the azimuth direction and combining the returns from a number of spatial positions to generate a radar image. The processing adjusts the phases of raw signals, multiplying by weighting factors and taking the data vector sums.

The focused SAR aperture synthesis procedure is applied for each image element and can be efficiently implemented with a matched filter. The standard linear algorithm for SAR image generation [2,3] is optimal, assuming that both real and imaginary components of the complex raw signal are corrupted by additive Gaussian noise. This assumption does not hold for SAR holograms, if the data are coded and transmitted to a ground station for aperture synthesis and SAR image generation. Due to interferences, data coding/decoding errors and hardware failures the hologram data can be corrupted by impulse noise that considerably degrades the quality of SAR images, and for which the matched filter is no longer optimal.

Since the SAR hologram data are weakly correlated, it is difficult and inefficient to detect and eliminate impulse noise directly from the SAR hologram. The noise-like nature of the hologram is also a serious obstacle for elimination of impulse noise by direct application of nonlinear image filtering algorithms based on order statistics. However, order statistics filters are very efficient for mitigation of impulse noise for spatially correlated optical and radar images [4,5]. The standard aperture synthesis algorithm based on taking the data vector sums [2,3] also can not be considered an efficient algorithm in the presence of impulse noise. Even one impulse noise sample present in hologram data may substantially modify the linear amplitude estimate of the signal reflected from the Earth surface. So the problem is: how to provide estimates of complex parameters that are robust to impulse noise. Here “robust” [6] means that the estimate should be insensitive to deviations from the assumption of the Gaussian noise distribution due to the presence of data transmission and storage errors.
The algorithm for hologram components fusion and focused aperture synthesis applied for every image sample is based on estimation of the amplitude and phase of a complex signal reflected from a portion of the Earth’s surface of the same size as a resolution element. A similar task was considered by V. Katkovnik in [7], where a robust algorithm was developed for M-periodogram estimation of a signal contaminated by impulse noise errors having an unknown heavy-tailed distribution. To estimate amplitude and phase of the harmonic signal components use was made of an exponential error function $L(x) = |x|^p$ with parameter $p$ specifying the sensitivity to impulse noise errors. An iterative algorithm is presented in [7], which estimates the complex amplitude of a harmonic signal by minimising the exponential error function. This approach received a new development in [8], which proposes frequency-domain robust filtering of signals in the impulse noise environment. The filters are based on a discrete Fourier transform (DFT) estimate of the input signal. New forms of robust DFT were proposed in [8]. The operation of averaging was replaced by the operations of taking marginal median and L-estimates of the DFT complex amplitudes. It was also shown in [9] that a similar approach can be taken for different discrete transforms, such as Hadamard, Walsh, Haar, discrete cosine, etc.

We propose several new nonlinear algorithms for fusing SAR hologram components and generating SAR images. These algorithms are based on robust versions of M-, marginal median and L-estimates. The high performance of the proposed algorithms is demonstrated for measured SAR hologram data with simulated impulse noise impairments. Several quantitative performance measures are used to compare the quality of processed noisy images with that of the unimpaired original.

2 Robust algorithms for SAR aperture synthesis

The quality of the SAR image data is degraded by a number of factors, the most significant of which are:

- phase and amplitude fluctuations of the SAR signal caused by propagation through a turbulent troposphere [10];
- phase errors due to irregular motion and fluctuations of the SAR platform and instabilities in radar components;
- receiver thermal noise;
- errors in communication equipment.

The influence of the phase signal fluctuations on the SAR response is considered in [2, 10] and can be reduced by restricting the length of the synthetic aperture, while the effect of communication equipment errors is the subject for investigation in this paper. We assume that a digital communication system is used for transmitting the SAR hologram data to the ground station for signal processing and image generation. At this stage the data are corrupted by the impulse noise caused by the data bit errors and coding/decoding errors. The model for the hologram data impairments can be presented as:

$$h(r,c) = h_{Re}(r,c) + j \cdot h_{Im}(r,c),$$

$$h_m(r,c) = \begin{cases} s_m(r,c) + n_m(r,c), & \text{with prob. } 1 - P_{imp} \\ u_m(r,c), & \text{with prob. } P_{imp} \end{cases}, \quad (1)$$

where index $m=\{Re,Im\}$, $h(r,c)$ are complex samples of the SAR hologram corresponding to the $r$-th row and the $c$-th column. $h_{Re}(r,c)$ and $h_{Im}(r,c)$ are real and imaginary components of SAR hologram. $s_m(r,c)$ are the original hologram samples and $n_m(r,c)$ correspond to additive white Gaussian noise samples. Real and imaginary components of the original hologram are also corrupted by impulse noise $u_m(r,c)$ with probability $P_{imp}$. Since the impulse noise is caused by the bit errors in the communication system it can be characterised by the uniform distribution. The experimental hologram data are presented by 256 signal levels in [-128,127] interval which also defines the limit values of the impulse noise distribution.

The standard linear algorithm for focused SAR image synthesis employs multiplication of the complex hologram data with the complex conjugate of the point target response function, weighting the results and averaging [3]. Assuming that the hologram rows correspond to the azimuth direction, the algorithm can be expressed as follows:

$$J(r,c) = \frac{1}{N} \sum_{n=-N/2}^{N/2} h(r,c+n) \cdot f_w(r,n)^*,$$

$$I(r,c) = |J(r,c)|, \quad (2)$$

where $\cdot$ denotes the complex modulus, $r$ and $c$ are row and column indices of the SAR image, $N$ is the synthetic aperture length in hologram samples, $h(r,c)$ is the hologram complex sample, $f_w(r,n)^* = f(r,n)^* \cdot w(n)$ is the complex conjugate of the point target response function $f(r,n)$, multiplied by a weighting function $w(n)$. $J(r,c)$ corresponds to the complex amplitude estimate of the reflected signal and $I(r,c)$ are the SAR image samples generated by taking the absolute value. The weighting function $w(n)$ is applied in Eq. (2) to decrease the sidelobe level of the SAR response [2,3].

The linear estimate of complex amplitudes in Eq. (2) minimises the mean square error (MSE) between the complex hologram signal and the point target response function. It is optimal according to the maximum likelihood criterion if the hologram data are corrupted by Gaussian white noise. However, this assumption is not satisfied for the impairment model (1) where the hologram is corrupted by impulse noise. In the presence of communication equipment and coding/decoding errors the distribution of hologram noise becomes heavy-tailed and the linear estimate is not optimal.

As it has been shown in [7] the robustness of the signal complex amplitude estimate can be improved by applying the exponential error function $L(x) = |x|^p$, which provides
an optimal maximum likelihood solution for errors having a probability density function (PDF) of the form

$$f(x) = C \cdot \exp(-k \cdot |x|^p),$$  \hspace{1cm} (3)

where $C$ and $k$ are constants and $0<p<2$. For $p=2$ the distribution (3) corresponds to the Gaussian distribution and the optimal maximum likelihood estimate (M-estimate) can be obtained from the linear algorithm (2). Setting parameter $p=1$ for the exponential PDF (3) we obtain the Laplacian distribution with heavier tails. The M-estimate for this distribution minimises the absolute error function $L(x) = |x|$. For an arbitrary $p$ parameter in Eq. (3) the optimum amplitude estimate cannot be obtained directly and an iterative procedure must be used [7]. As applied to the task of robust SAR aperture synthesis the M-estimate algorithm should find the signal complex amplitude $J(r,c)$ by minimisation of the following error function

$$L(r,c) = \sum_{n=-N/2}^{N/2} |h(r,c+n) - J(r,c) \cdot f_w^*(r,n)|^p, \quad (4)$$

where parameter $p$ determines the robustness of the algorithm. The solution for the M-estimate (4) can be found by solving a nonlinear equation of the form $\partial L(r,c)/\partial J(r,c) = 0$. This equation can be solved iteratively [7] to obtain an approximate estimate $\hat{J}(r,c)$ of complex amplitude as

$$J_{(k)}(r,c) = \sum_{n=-N/2}^{N/2} d_{(k-1)}(n) \cdot h(r,c+n) \cdot f_w^*(r,n),$$

$$\gamma_{(k-1)}(n) = |h(r,c+n) - J_{(k-1)}(r,c) \cdot f_w^*(r,n)|^{p-2},$$

$$d_{(k-1)}(n) = \gamma_{(k-1)}(n) / \sum_{m=-N/2}^{N/2} \gamma_{(k-1)}(m), \quad k = 1...K, \quad (5)$$

where $K$ is the number of iterations. At the initial step of the iterative algorithm (5) we set $d(0)=1/N$. It follows from Eq. (5) that the M-estimate is obtained by weighted averaging operation with weights $d(0)(n)$ proportional to the distance between the hologram samples and the estimated signal. It was shown [7] that algorithm (5) is robust and substantially improves the accuracy of estimate in the presence of heavy-tailed noise, while for Gaussian noise the M-estimate performs only slightly worse than the standard algorithm (2). In addition, by changing the $p$ parameter in the interval $[0,2]$ it is possible to vary the robustness of the M-estimate. The algorithm has good convergence properties and usually requires not more than 3-5 iterations to find a solution [7].

An approach for a signal complex amplitude estimation that does not require an iterative procedure is based on the marginal median form introduced in [8,11]. For the task of robust SAR hologram synthesis we propose the following modification of the marginal median estimate algorithm:

$$\hat{J}_R(c,r) = \text{med}\{\text{Re}[h(c,r+n) \cdot f_w^*(r,n)]\}, \quad n = -N/2,...,N/2,$$

$$\hat{J}_I(c,r) = \text{med}\{\text{Im}[h(c,r+n) \cdot f_w^*(r,n)]\}, \quad n = -N/2,...,N/2,$$

where $\text{Re}[x]$ and $\text{Im}[x]$ are real and imaginary components of the complex variable $x$, and $\text{med}\{\}$ denotes the median estimate. The marginal median estimate of vector signals for the impulse noise environment provides a computational simplification with respect to the previous iterative and vector median [12] algorithms. It is also a “suboptimal” estimate for the hologram signals contaminated by Laplacian noise, which is the worst case for numerous forms of impulse noise. Consider the exponential error function (4). Assume that the point target response function is not weighted and can be presented in complex form as

$$f_w^*(r,n) = \exp[-j \cdot \psi(r,n)],$$

where $\psi(r,n)$ is the signal phase shift due to the forward motion of the SAR platform. Using this assumption and taking into account that phase shift does not change the amplitude of a complex variable, the error function (4) can be presented in the form

$$L(r,c) = \sum_{n=-N/2}^{N/2} |h(r,c+n) \cdot f_w^*(r,n) - J(r,c)|^p. \quad (7)$$

With the assumption that real and imaginary parts of the error function are statistically independent and impulse noise has Laplace distribution, the error function can be presented as

$$L(r,c) = \sum_{n=-N/2}^{N/2} \text{Re}[h(r,c+n) \cdot f_w^*(r,n) - J(r,c)]^p + \sum_{n=-N/2}^{N/2} \text{Im}[h(r,c+n) \cdot f_w^*(r,n) - J(r,c)]^p. \quad (8)$$

For the error function (8) the minimisation problem is reduced to the real-valued case. If we impose an additional restriction that the signal amplitude estimate $\hat{J}(r,c)$ should be chosen from the set of samples

$$\{h(r,c+n) \cdot f_w^*(r,n)\}, \quad n = -N/2,...,N/2,$$

the solution for (8) can be obtained by the marginal median algorithm (6).

The assumption that the data errors are independent for real and imaginary components of SAR hologram agrees with our model of hologram impairments (1). The vector median estimate can also be used for robust SAR hologram processing [9]. But, due to random phase shifts of the reflected signal the impairment-free hologram components can be considered as statistically independent and comparison of the performance of vector median and marginal median estimate algorithms requires additional investigation.
The marginal median estimate has excellent robust properties and is able to eliminate up to 50 percent of impulse noise samples. But algorithm (6) is less efficient in suppressing additive Gaussian noise components in comparison with the standard linear estimate (2). To provide a tradeoff between the robustness to the impulse noise influence and improved suppression of Gaussian noise, an L-estimate was considered in [9]. For the task of robust SAR image generation the marginal L-estimate algorithm can be defined as

\[
\hat{I}(r,c) = \left\lfloor \frac{N}{2} \right\rfloor \sum_{n=-N/2}^{N/2} a_n[r_n(n) + j \cdot i_n(n)] \right.,
\]

where \( a_n = 1 \), and \( r_n(n) \) and \( i_n(n) \) represent the values of the sets \( \{R_n(n), I_n(n), n=-N/2, \ldots, N/2\} \), sorted into ascending order: \( r_n(-N/2) \leq \ldots \leq r_n(N/2) \), \( i_n(-N/2) \leq \ldots \leq i_n(N/2) \). Coefficients \( R_n(n) \) and \( I_n(n) \) are defined as

\[
R_n(n) = \text{Re}[h(r, c + n) \cdot f_n^*(r, n)]
\]
\[
I_n(n) = \text{Im}[h(r, c + n) \cdot f_n^*(r, n)],
\]

where \( n = -N/2 \ldots N/2 \). For SAR aperture synthesis we will use the \( \alpha \)-trimmed mean form of L-estimate, where the coefficients \( a_n \) for odd \( N \) are calculated as

\[
a_n = 1/(N - 2\alpha), T = \left(1 - \alpha\right)N/2, \]

where \( n \in [T - \lfloor N/2 \rfloor, \lfloor N/2 \rfloor - T] \) and \( a_n = 0 \) otherwise. \( \lfloor x \rfloor \) denotes the largest integer less than or equal to \( x \). Parameter \( \alpha \) determines the number of samples of \( r_n(n) \) and \( i_n(n) \) used for the inner mean value estimation; the remaining smallest and largest samples are trimmed. The robustness and noise suppression properties of the \( \alpha \)-trimmed mean estimate can be changed by varying the \( \alpha \) parameter in interval \( 0 \leq \alpha \leq 1 \). It is easy to show that for \( \alpha = 0 \) the estimate (10, 11) is reduced to marginal median; and that for \( \alpha = 1 \) it becomes a mean estimate. The ability to eliminate impulse noise is defined by the number of trimmed samples and depends on the \( \alpha \) parameter. For the model of SAR hologram impairments considered in Eq. (1) the negative and positive impulse noise samples have equal probabilities \( P_{imp}/2 \) and the coefficients \( R_n(n) \) and \( I_n(n) \) in Eq. (10) are simultaneously corrupted by impulse noise appearing in both real and imaginary hologram components. The probability of coefficients \( R_n(n) \) and \( I_n(n) \) being degraded by only negative or only positive impulse noise samples is \( P_{imp} \). But the hologram components are corrupted by \( N \cdot P_{imp} \) impulse noise samples, the \( \alpha \)-trimmed mean algorithm excludes from the averaging at least \( \alpha \) of the smallest and largest samples in \( r_n(n) \) and \( i_n(n) \) to provide a robust estimate.

The proposed algorithms for SAR aperture synthesis based on M-, marginal median and \( \alpha \)-trimmed estimates are nonlinear and provide robustness properties in the presence of impulse noise. But this introduces nonlinearities and these also may become a source of nonlinear distortions of the generated SAR images. The aim of nonlinear processing is to minimise the image distortions due to additive Gaussian and impulse noise and to provide an acceptable level of distortions introduced by the algorithms’ nonlinearities. For \( \alpha \)-trimmed aperture synthesis the compromise can be achieved by proper selection of the \( \alpha \) parameter. The intensity and characteristics of nonlinear distortions depend on a number of factors, such as the intensity of the hologram noise, contrasts of image objects and their configuration, the characteristics of SAR sensor, etc. The distortions may change the form of the SAR response, increase the level of sidelobes and modify their form. Since it is not always possible to obtain analytical expressions taking into account various causes of SAR image degradation, an experimental validation of the proposed nonlinear algorithm efficiency is considered in the next section.

3 Analysis of robust estimators

To estimate the performance of the proposed algorithms, we considered a scenario consisting of image acquisition, transmission of the data to the ground station and impairment mitigation stages. The performance of the algorithms is estimated by comparing the synthesised image where noise is absent with the results of image generation for the hologram corrupted by impulse noise. The difference between the images was evaluated using the signal-to-noise ratio (SNR), the relative difference in intensity of the brightest object \( (D_{max}) \), and the correlation coefficient (CC) performance measures:

\[
\text{SNR} = 10 \log_{10} \left( \frac{\sum r \cdot c \cdot I(r, c)^2}{\sum r \cdot c \cdot (I(r, c) - \hat{I}(r, c))^2} \right),
\]
\[
D_{max} = \left| \frac{\sum r \cdot c \cdot I(r_m, c_m) - \hat{I}(r_m, c_m)}{\sum r \cdot c \cdot I(r_m, c_m)} \right|,
\]
\[
CC = \frac{\sum r \cdot c \cdot [I(r, c) - m(I)] \cdot [\hat{I}(r, c) - m(I)]}{\left( \sum r \cdot c \cdot [I(r, c) - m(I)]^2 \right) \cdot \left( \sum r \cdot c \cdot [\hat{I}(r, c) - m(I)]^2 \right)},
\]

where \( I(r, c) \) are the original image samples generated by the linear algorithm (2) when the impulse noise is absent and \( \hat{I}(r, c) \) is the image at the output of the proposed robust algorithm. \( (r_m, c_m) \) are the coordinates of the brightest object in the original image and \( m(\hat{I}) \) is the mean brightness estimate of image \( \hat{I} \).

Consider the measured SAR hologram obtained from SAR sensor under the following conditions:

- SAR sensor is installed onboard an airborne platform;
- wavelength \( \lambda = 1.8m \);
- signal azimuth sampling frequency \( f_{az} = 25Hz \) (downsampled from 1kHz frequency after linear filtering);
- carrier velocity \( V = 639km/h \).
- signal range sampling frequency \( f_s = 5 \text{MHz} \);
- the number of quantisation levels is 256.

The real and imaginary parts of the hologram are presented in Figs. 1(a) and (b).

![Fig.1. SAR hologram, wavelength \( \lambda = 1.8 \text{ m.} \): (a) real part; (b) imaginary part](image)

The hologram images are then corrupted by uncorrelated impulse noise with probability \( P_{imp} = 0.1 \) to simulate transmission and coding errors in the form of bit-error noise.

Synthesis of the image from the hologram was achieved using an aperture size of 261 samples, and a Hamming weighting window for synthesis of the response. For the simulation, tuning parameters for \( \alpha \)-trimmed estimate and M-estimate were selected for optimal performance depending on the intensity of impulse noise present. Studies were conducted to ascertain optimal values for parameter \( p \) in Eq. (4) and parameter \( \alpha \) in Eq. (11). \( \alpha \) was selected to maximise the SNR. The results are presented in Fig. 2. However, selection of the \( p \) parameter requires additional consideration. Quantitative results obtained from the simulation with \( P_{imp} = 0.1 \) indicated that \( p = 1.6 \) was optimal, in this particular case, but visual analysis of the synthesised image revealed a ‘streaking’ phenomena, where segments of several horizontal rows of the image appeared as dark stripes. Therefore it was proposed to select \( p = 1.7 \) which provided an acceptable compromise between image appearance and quantitative performance in this case.

![Fig.2. Optimal \( \alpha \) parameter selection for \( \alpha \)-trimmed estimate, impulse noise probability 0.1 < \( P_{imp} < 0.5 \)](image)

Initially, the estimation algorithms were applied to the noiseless SAR holograms, Figs. 1(a) and (b), to synthesise the radar image in the absence of impulse noise, as shown in Figs. 3(a)-(d). Visual analysis reveals that the average estimate, Fig. 3(a), preserves fine details for both the contours of line objects present mainly in the centre of the image, and bright point objects which are most notable in the upper-left and middle-right of the image, whilst achieving a high contrast overall. Conversely, the marginal median, Fig. 3(b), performs poorly in comparison with the average estimate, substantially reducing the intensity of bright point objects particularly in the upper-left and middle-right of the image, with faded contours and the lowest contrast for all estimators for this case. Both \( \alpha \)-trimmed estimate \((\alpha = 0.75)\) and M-estimate \((p = 1.7)\), Figs. 3(c) and (d) respectively, exhibit similar performance with regard to maintaining high contrast levels and high intensity of bright objects, comparable to that of the average estimate.

![Fig.3. Synthesised SAR image, impulse noise is absent: (a) average estimate; (b) marginal median estimate; (c) marginal \( \alpha \)-trimmed estimate \((\alpha = 0.75)\); (d) M-estimate \((p = 1.7)\)](image)

Of greater interest is algorithm performance in the presence of impulse noise and the ability to filter this noise from the signal while not destroying the image content. The hologram components shown in Figs. 1(a) and (b), were corrupted by impulse noise with probability \( P_{imp} = 0.1 \), and the synthesised images are shown in Figs. 4(a)-(d).

In the presence of impulse noise the average estimate fails to suppress noise sufficiently resulting in a poor quality image, Fig. 4(a), where only a few bright objects are left distinguishable in the top-left of the image, although the intensity of the object on the middle-right has been preserved well. The M-estimate offers adequate noise suppression but at the same time diminishes the intensity of bright objects and contours resulting in the loss of some bright objects from the image. Clearly, the \( \alpha \)-trimmed estimate produces the best image, Fig. 4(c), outperforming all estimators in this situation by efficiently eliminating noise and preserving the intensity of bright points and contours to fairly high degree. Careful visual analysis of the image produced by the M-estimate, Fig. 4(d), reveals evidence of the ‘streaking’ phenomena in the lower-left and centre of the image, although this less prominent than in the results obtained for \( p = 1.6 \).
The noise suppression properties of the M-estimate with $p=1.7$ demonstrate inefficiency with regards to eliminating impulse noise, resulting in loss of contours and some bright objects, although the intensities of bright objects in the top-left and middle-right have been retained.

In the case where noise is absent from the holograms the average estimate offers good performance in preserving details and intensity of bright objects, but in the presence of noise it is by far the worst performer. The marginal median performs almost equally poorly, producing remarkably similar results independent of whether noise is absent and present, although it offers poor contrast and reduced intensities of bright objects. Both the $\alpha$-trimmed estimate and the M-estimate offer comparable performance in the absence of noise, but when noise is present they exhibit remarkably different results, with the $\alpha$-trimmed estimate producing the better image while the M-estimate performs worse than the marginal median.

The robustness of the estimators with respect to impulse noise was investigated by subjecting the SAR holograms to varying probabilities of impulse noise where $0\leq P_{imp} \leq 0.5$, and performing analysis using the quantitative measures in Eq. (12). Appropriate tuning parameters for $\alpha$ were selected dependent on $P_{imp}$ and all other parameters were assigned typical values as in the previous experiment.

The simulation results obtained are shown in Figs. 5(a)-(c), illustrating the dependencies of the three performance measures on the variation of impulse noise probabilities. It follows from Fig. 5(a) that the marginal median estimate performs consistently for all noise probabilities maintaining a SNR of around 4dB. Visual analysis of the resulting images confirms that the marginal median mitigates noise effectively for all considered $P_{imp}$. Conversely, the average estimate shows intolerance towards any impulse noise offering the least desirable characteristics. For low probabilities of impulse noise where $P_{imp}<0.15$, the M-estimate performs better than the marginal median but is seriously affected by high noise probability. We note that this may be attributed to selection of a sub-optimal value for the tuning parameter $p$, and the fact that $p$ remains constant and does not vary to account for the intensity of noise. A detailed analysis is required to discover the reason for the ‘streaking’ phenomena when the optimal value of $p$ is assigned, but this is beyond the scope of this paper. Clearly, the $\alpha$-trimmed estimate offers superior performance in mitigating the impulse noise, even at high probabilities, attaining a gain of 10dB over the standard algorithm.

To illustrate the contrast-preserving properties of the proposed algorithms we measure the relative difference in intensity ($D_{max}$), Fig. 5(b), of the bright point object on the middle-right of the image after mitigation, compared to the unimpaired image.
The correlation between mitigated image and image in the absence of noise using the standard algorithm for synthesis, for varying probabilities of impulse noise is shown in Fig. 5(c). Obviously, when \( P_{imp} = 0 \) the average estimate produces perfect correlation because this is implemented in the standard algorithm, but note that it is the most sensitive to the presence of impulse noise resulting in serious degradation of the image when subjected to intense noise. Quantitative results indicate that although the marginal median is the worst performer in the absence of noise, it is the least sensitive to variations of impulse noise probability, performing consistently regardless of the intensity of noise present.

Visual analysis confirms the marginal median as having the lowest contrast properties but it offers comparable results for all noise probabilities, signifying robustness to impulse noise. The average estimate demonstrates complete intolerance to noise, with high \( P_{imp} \) resulting in extreme degradation in performance. However, the average estimate presents superior results in the absence of noise. Quantitative results indicate that the average estimate and the M-estimate fail to suppress noise well, but they possess the best contrast-preserving properties with respect to increasing probabilities. This is confirmed by visual analysis of the synthesised images, which reveals detectable bright point objects even in the presence of intensive impulse noise.

For low intensities of noise the \( \alpha \)-trimmed estimate and M-estimate offer comparable performance, although the M-estimate appears more sensitive to noise. High noise probabilities cause the M-estimate to behave much like the average estimate, while the behaviour of the \( \alpha \)-trimmed estimate approaches that of the marginal median for extreme intensities. Clearly, the \( \alpha \)-trimmed estimate is robust, providing desirable characteristics, efficiently suppressing noise while adequately preserving bright point objects and edges. The marginal median also demonstrates robustness, offering good noise suppression but at the cost of loss of contrast, resulting in lower intensities of bright objects. We observe that the M-estimate introduced distortions in the form of the ‘streaking’ effect, constraining us to use a sub-optimal choice for \( p \), and better performance should be attainable with the resolution of the ‘streaking’ issue and the selection of optimal value for \( p \).

The SAR normalised responses for each estimator are shown in Figs. 6(a)-(d). Impulse noise with probability \( P_{imp} = 0.1 \) and additive white Gaussian noise with \( SNR = 14dB \) were added independently to the real and imaginary components of the hologram. For simulation purposes an aperture size of 261 samples was selected, with an azimuth sampling frequency \( f_c = 100Hz \), a Hamming window for synthesis of the antenna pattern, and a point target response function amplitude of 10. Visual inspection of the normalised response for the average estimate, Fig. 6(a), reveals that the central lobe is barely distinguishable due to its inability to filter impulse noise effectively, having the mean sidelobe level around -8dB with the largest sidelobe being -3dB. The marginal median, Fig. 6(b), offers characteristics similar to the \( \alpha \)-trimmed estimate, Fig. 6(c), exhibiting a comparable response with distortions due to nonlinearities introduced by the algorithms. Finally, the SAR response for the M-estimate, Fig. 6(d), reveals more dominant sidelobes reaching a peak of -17dB and an average sidelobe height of -23dB. Again these distortions are a result of the nonlinearity of the algorithm.

![Fig. 6. SAR response: (a) average estimate; (b) marginal median estimate; (c) marginal \( \alpha \)-trimmed estimate (\( \alpha = 0.75 \)); d) M-estimate (\( p = 1.7 \))](image-url)
4 Conclusions

The marginal median, α-trimmed and M-estimate are proposed as alternatives to the standard linear algorithm for fusing hologram components and synthesising SAR images. They provide robust estimation in an impulse noise and Gaussian noise environment. We observe that the M-estimate introduced distortions in the form of the ‘streaking’ effect, constraining us to use a sub-optimal choice for $p$, therefore degrading the performance of the algorithm. We have shown the α-trimmed estimate to be the recommended algorithm, offering superior performance to that of the marginal median and the average estimates in suppressing even severe impulse noise. The robustness of the α-trimmed algorithm and the nonlinear distortions of the SAR response depend on the number of trimmed samples. An adaptive algorithm for optimal selection of $\alpha$ under impulse noise conditions could be applied to simultaneously minimise the SAR image degradation due to noise and nonlinear distortions. This is the topic for further study.

Acknowledgements

This research was supported by the Data Information Fusion Defence Technology Centre, United Kingdom, under DTC Project 4.7. The experimental SAR data were obtained by the Kalmykov Centre for Radiophysical Sensing of Earth, Kharkov, Ukraine and kindly made available to the authors for experimental study.

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