Set Based Tracking in Clutter as a Percolation Process

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Abstract – In this paper, we investigate the performance of a set based tracker in clutter with no target present. If there is a positive probability of clutter-only tracks lasting an infinite time in such an environment, then a tracker cannot be expected to perform well under these clutter conditions when a target is present. In order to analyse the problem, a continuous space model is simplified and discretised. It is shown that this discretised model is a percolation process, which undergoes a phase transition. This means that there exists a critical false alarm probability for the system, below which clutter tracks die out in finite time, and above which clutter tracks have a positive probability of surviving indefinitely. A theoretical lower bound for this critical clutter probability is derived, which is then verified by Monte Carlo simulation.

Keywords: Tracking, filtering, set based estimation, clutter, percolation.

1 Introduction

In most realistic tracking systems, spurious measurements that do not originate from a target of interest are obtained; such measurements are typically referred to as clutter. In active sonar, clutter may originate from fish, sea bottom features or other random scatterers. The tracking system must in some way either elucidate the target originated measurement (if present), or use all of these measurements to update its estimate of target state, preferably with very limited adverse effect on the tracker performance.

It is intuitive that the performance of a tracking algorithm in clutter is dependent on the amount of clutter present. If there are only one or two clutter measurements every now and then, they should have little effect on the tracker performance. However, if there are consistently a large number of clutter measurements at each time step, then it is difficult to distinguish the target track from other sets of associations over time, and the tracker would be expected to perform poorly.

The tracker performance is also dependent on the type of tracker that is used, including the way that the clutter measurements are treated and the estimation criteria. Many association algorithms have been developed to track clutter; some such as ‘Nearest Neighbour’ (NN) and ‘Strongest Neighbour’ (SN) use a single association of measurement to track, while others use multiple measurements to update the track. Of these, some may be probabilistically weighted to form a single updated estimate, such as used in Probabilistic Data Association (PDA), or others, such as Multiple Hypothesis Tracker (MHT), maintain all or most of the associations separately, but are more computationally intensive.

It is difficult to characterise the performance of a tracking algorithm in clutter. Simulation based approaches such as Monte Carlo analysis may be employed using known target states to quantitatively assess the performance of an algorithm in a given scenario, e.g. [1, 2]. However, this will not give an overall characterisation of the performance of the tracking algorithm, particularly in real situations where the true target state progression is unknown.

There have been some attempts to theoretically analyse the performance of specific tracking algorithms in clutter under a minimum mean square error estimation criterion. Rogers [3, 4] considers nearest neighbour association for a Kalman filter with clutter measurements. He models the tracking error as a diffusion process to obtain estimates for the mean track life and track half-life as a function of the clutter density. Li [5] has theoretically analysed the Strongest Neighbour version of this filter. Fortmann et al [6] and Li and Bar-Shalom [7] have considered the performance of the PDA filter for tracking in clutter. One of the problems introduced with the PDA filter is that the covariance update becomes dependent on the measurements that are obtained. Since these are random variables, the covariance cannot be evaluated, so that this covariance must be approximated in some way [6–8].

In this paper, we investigate the performance of a set based tracker in clutter. The aim of a set based tracker is to obtain an estimate comprising a set that is guaranteed to contain the true target state at any time, based on the measurements observed and a model for the target behaviour [9, 10]. Set based tracking in clutter was examined in [11] and [12] where it was recognised that any measurement may have originated from a target, and must be used to update the tracker. An advantage of working with a set based tracker is that it provides a bounded set in which to search for a measurement, thus limiting the number of measurements used to update a track.\(^{1}\)

\(^{1}\)Compare with the Kalman filter, which uses (infinite support) Gaussian distributions for target state, meaning that any measurement to track association is feasible. Gating is often used in this
Very little has currently been done to characterise the performance of a set based tracker in clutter. In this paper, we use percolation theory to analyse a simple set based tracker in clutter. We assume that there is no target present, the justification being that if tracks may be maintained in such an environment, then they would also be maintained with a target present. The models used are simplistic, but provide a first step towards developing theory on set based tracker performance in clutter.

Section 2 describes set based tracking techniques and their application in cluttered environments. In Section 3, the simple discretised model used for our analysis is described, including a description of the discretised clutter model. Section 4 introduces percolation theory, particularly that of oriented site percolation. In Section 5, it is shown that the simple discretised model actually represents an oriented site percolation problem. In this section, it is also shown that this model undergoes a phase transition. Theoretical lower and upper bounds for the critical false alarm probability are derived, and compared with a Monte Carlo simulation to gain a numerical estimate for this value. Extensions to the model are also described. Finally, concluding remarks are made in Section 6.

2 Set Based Tracking

A typical target tracking problem uses a linear model to describe target state transitions (1), and a linear relation between target state and observed measurements (2),

\[ x_{k+1} = F_k x_k + v_k, \quad z_k = H_k x_k + w_k. \]  

The target state at any time \( k \) is denoted \( x_k \) and the matrix \( F_k \) describes the state transitions. An additive process noise, \( v_k \), is used to include small random perturbations of the target state. Measurements, \( z_k \), of the target are linearly related to the target state through the matrix \( H_k \). An additive measurement noise, \( w_k \), is used to model inaccuracies in the measurement process.

The assumptions made by a set based tracker are that the process and measurement noises are bounded, with the bound known,

\[ v_k \in \Omega_Q(k), \quad w_k \in \Omega_R(k), \]  

where \( \Omega_Q(k) \) is a bound on the process noise at time step \( k \), and \( \Omega_R(k) \) is a bound on the measurement noise at time step \( k \). This is in contrast to many tracking systems, which often assume an explicit distribution for the noises, for example, Kalman filter based trackers assume a Gaussian distribution for the noises. In addition, the set based tracker makes the assumption that an initial bound on target state is known; this becomes the initial estimate of target state.

2.1 Set Based Estimation

The aim of a set based estimator is to maintain a set of feasible target states over time, based on the measurements received and the assumed target dynamics. Provided these are modelled correctly, the estimate at each time is this set of feasible target states and is guaranteed to contain the true target state. The process works in two stages: prediction, where the set of possible target states are projected forward to the next time; and update, where the set of feasible target states is updated based on measurements of the target. Further detail may be found in many references including [9, 10].

Assume that the updated estimate at time \( k - 1 \) is the set \( \Omega(k - 1|k - 1) \). The prediction stage projects the estimate from time \( k - 1 \) through the target dynamics (1) to obtain a set that is guaranteed to contain the target at time \( k \). Firstly, the set \( \Omega_P(k|k - 1) \), which is the projection of the estimate at time \( k - 1 \) through the linear equation (1) assuming no process noise \( (v_{k-1} = 0) \), is calculated,

\[ \Omega_P(k|k - 1) = \{ y : y = F_{k-1} x, x \in \Omega(k - 1|k - 1) \}. \]

The Minkowski summation (denoted \( \oplus \)) of this set with the set bounding the process noise combines any possible target position with any possible process noise [9, 10],

\[ \Omega(k|k - 1) = \Omega_P(k|k - 1) \oplus \Omega_Q(k - 1). \]  

Thus the prediction, \( \Omega(k|k - 1) \), is a set guaranteed to contain the true target state at time \( k \).

The update stage occurs after a measurement has been received. The set of all target states consistent with the measurement is intersected with the prediction set to obtain an updated estimate at time \( k \) [9, 10],

\[ \Omega(k|k) = \Omega(k|k - 1) \cap \Omega_z(k). \]  

The set \( \Omega_z(k) \) contains all target states consistent with the measurement \( z_k \) at time \( k \),

\[ \Omega_z(k) = \{ y : H_k y = z_k - w_k, w_k \in \Omega_R(k) \}. \]

2.2 Set Based Tracking in Clutter

As described in Section 1, it is very important for a tracking algorithm to be able to track in cluttered environments. A set based tracker must thus maintain a guaranteed bound on target state under these conditions. Whenever multiple feasible measurements are obtained, they must all be used to update the tracker to ensure that the premise of a guaranteed bound on target state is maintained. We use a track split approach similar to that described in [13] and used in [11, 12].

The approach proceeds as for the set based tracker described in Section 2.1, but updates using each of the feasible measurements. Assume that a set encompassing all possible target states at time \( k - 1 \) is known and denoted \( \Omega(k - 1|k - 1) \). This set is predicted forward as in (5) to obtain \( \Omega(k|k - 1) \), the set of all possible target states at time \( k \).
A set of $N$ measurements, $\{z_i^k, i = 1, \ldots, N\}$ is obtained at time $k$. For each of these measurements, the set of possible target states based solely on the measurement is calculated,

$$\Omega_i^t(k) = \{y : H_k y - w_k = z_i^k, w_k \in \Omega_R(k)\}.$$  

For each of these sets, the intersection with the prediction provides the set of feasible target states based on that measurement,

$$\Omega_i^t(k|k) = \Omega(k|k - 1) \cap \Omega_i^t(k).$$

The union of all of these sets provides the set of all feasible target states at time $k$ and becomes the updated estimate

$$\Omega(k|k) = \bigcup_{i=1}^N \Omega_i^t(k|k). \quad (7)$$

### 3 Clutter Only Tracking Model

As introduced in Section 1, we are investigating the process of tracking in clutter only, where there is no target present. This provides a technique for assessing the performance of a tracker. If there is a positive probability of a track being sustained for an infinite time in an environment when it is known that there is no target, this tracker could not be expected to perform well when a target is present.

#### 3.1 Target Model

A typical target state transition and measurement model was described in Section 2. We consider a simplified version of this model in order to make an initial investigation into the performance of the set-based tracker in clutter. Investigations into more complicated models and higher dimensional models are beyond the scope of this paper, but are recommended as future work.

Our model assumes that target state is one-dimensional and is discretised, with a labelling convention used such that the target state, $x_k$, is taken from the integers, $x_k \in \mathbb{Z}$. The model assumes that the target is almost stationary, and changes of position occur only through process noise. The target state is perturbed by process noise, which is also discrete and taken from a bounded set, $\Omega_Q$; we assume that this set is given by $\Omega_Q = \{-1, 0, 1\}$.

$$x_{k+1} = x_k + v_k, \quad v_k \in \Omega_Q, \quad (8)$$

where it is assumed that $x_0 \in \mathbb{Z}$, so that the target state is always an integer.

This model allows the target to stay in its current state, or traverse to the neighbouring state on either side of the current state. The target state transitions are illustrated in Figure 1.

Measurements of the target are assumed perfect,

$$z_k = x_k. \quad (9)$$

This is an unrealistic assumption, but allows us to investigate the existence of clutter tracks. The idea of the process is that after a measurement is received, the prediction step opens a window in which to look for the target at the next time. If a measurement is found in this window, the track continues, and the process is repeated.

#### 3.1.1 Discussion

This may be thought of as a coarse discretisation, into cells of size 1 unit, of the continuous space model

$$x_{k+1} = x_k + v_k, \quad v_k \in \left[\frac{-3}{2}, \frac{3}{2}\right] \quad (10)$$

$$z_k = x_k. \quad (11)$$

A finer discretisation would be achieved by allowing more transitions from each state, for example, allowing a target to transition to its current state, or to any of the two neighbouring states on either side of the current state.

This provides a model for expected target behaviour, and is required whether a target is present or not. Often in realistic tracking applications, we are unsure if a target is present. The tracking algorithm is required to look for measurements exhibiting target-like behaviour. If such measurements occur, there is no way to distinguish them from a real target, and the tracker would continue to track as if it is a target.

#### 3.2 Clutter Model

The measurements that are obtained are caused by clutter only, there is no target present for this model. Since the model used assumes that the measurements are of target state, the measurements must take integer values, $z_k \in \mathbb{Z}$. There are two types of clutter: random clutter, where measurements occur randomly anywhere in the measurement space and persistent clutter, which is generated by something in the environment, such as sea floor formations in active sonar. The clutter model we use assumes that this clutter is simply random clutter, and that persistent clutter may be filtered out of the measured data.

The model for this random clutter assumes that clutter is distributed among the discrete set of possible measurements according to a binomial distribution. Each possible target state is considered independently, and receives a clutter measurement, or false alarm, with probability $p$.

$$P\{z = x\} = p, \quad \forall x \in \mathbb{Z}. \quad (12)$$

Fig. 1: Feasible target state transitions over time. This defines the graph $E_{alt}^2$. 
This model matches that used in [2], in which the measurement space was discretised into error cells related to the sensor resolution, and the probability of false alarm in each cell is \( p \). This is the model from which the typical continuous space clutter distribution was defined; in the limit as the size of discretisations tends to zero, the number of clutter measurements tends to a Poisson distribution, with these measurements uniformly distributed over space.

We claim that this target and clutter model is equivalent to an oriented site percolation process. This will be demonstrated in Section 5, but firstly, oriented site percolation processes are described in Section 4.

### 4 Oriented Site Percolation Processes

Percolation theory studies the flow of "fluid" through a medium. The medium is usually described by a graph or lattice through which the fluid may flow. In the site percolation process, sites of the lattice are open with probability \( p \) and closed otherwise. Fluid may only flow through open sites. Of interest in such systems is whether fluid may freely flow through the medium and visit an infinite number of sites (note that this does not mean that the fluid visits all sites). If the fluid is able to reach an infinite number of sites, then it is said that percolation has occurred. Percolation processes differ from diffusion processes in that they assign randomness to the medium through which the fluid flows, where diffusion processes assign the randomness to the fluid itself [14].

A classic example of a percolation process is that described by Frisch and Hammersley [14] of a porous stone being immersed in water. It is of interest whether the water is able to penetrate the stone; if so, then it is said that percolation occurs. When modelling this phenomenon, it is usually assumed that the stone is made up of a set of sites that are joined together by bonds. For the bond percolation process, it is assumed that the bonds are open with probability \( p \), in which case water may pass through them, and are closed otherwise. For the site percolation process, the sites are open with probability \( p \), and closed otherwise. For either of these processes, if the penetrating water may reach an infinite number of sites, then percolation has occurred. References describing percolation processes include [15, 16] and others.

A percolation process is defined on a graph, \( G = (S, B) \), which consists of a set of vertices or sites \( S \) that are joined by edges or bonds \( B \). Site percolation processes assume each site \( s \in S \) is open with probability \( p \), and closed otherwise, with all bonds assumed to be open\(^2\). The bonds may have an orientation associated with them, in which case the flow through these bonds may only occur in the direction of their orientation, resulting in an oriented percolation process. The orientations of bonds may be used to model effects of outside forces such as gravity, or the passage of time. If fluid may flow through an infinite number of sites through these oriented bonds, then percolation has occurred. A more formal description of oriented site percolation follows, further information may be found for example in [17].

A site \( i \) may be reached from a site \( s \), written \( s \rightarrow i \), if a path exists from \( s \) to \( i \) through only open sites and in the direction of any bonds. Denote the set of open sites that may be reached from some site \( s \) by

\[
C_s = \{ i \in S : s \rightarrow i \}. \tag{13}
\]

This is often referred to as a cluster. If there are an infinite number of sites in a cluster, \( |C_s| = \infty \), then percolation has occurred. Generally, it is assumed that the graph is translationally invariant, so that all sites are statistically equivalent. Then we need only consider an arbitrary site; label this site \( 0 \) and denote the set of open sites that may be reached from \( 0 \) by \( C \). The probability of percolation, denoted \( \theta(p) \), may thus be defined as

\[
\theta(p) = \mathbb{P}\{|C| = \infty\}. \tag{14}
\]

This is a non-decreasing function of \( p \) [14–16]. An illustration of the probability of percolation, \( \theta(p) \), as a function of \( p \) is shown in Figure 2. Many such systems undergo a phase transition. For values of \( p \) below a threshold, there is zero probability of percolation occurring, while for values of \( p \) above the threshold, there is positive probability of percolation occurring. This threshold, denoted \( p_c \), and shown in Figure 2, is known as the critical probability, and is defined by

\[
p_c = \sup\{ p : \theta(p) = 0 \}. \tag{15}
\]

![Fig. 2: The probability of percolation as a function of the probability of a site being open, \( p \). The critical probability \( p_c \) is the point above which there is a positive probability of percolation occurring.](image)

One of the important problems when analysing percolation processes is to determine whether phase transition occurs, and if it does, to calculate the critical probability at which this phase transition occurs. Unfortunately, in general this is extremely difficult to calculate analytically, and has been ascertained for very few percolation processes.

### 5 Clutter Only Set Based Tracking as an Oriented Site Percolation Process

In Section 3, we claimed that the clutter only tracking problem described in that section is, in fact, an oriented site per-
alation process. This section will justify that claim.

Recall the process described in Section 3. The target state space was discretised so that the target could take states only from the integers. The target state evolves dynamically over time, so that to entirely specify a target state sequence, both the time and the target state must be known. Thus the vertices of the two-dimensional graph are defined by $S = \{(k, x) : k \in \{0\} \cup \mathbb{Z}^+, x \in \mathbb{Z}\}$, where $k$ represents time, and $x$ is the target state.

The bonds between these sites are defined by the allowed target state transitions over time. The target trajectory model dictates that a target in state $x$ at time $k$ may be in states $x - 1$, $x$ or $x + 1$ at time $k + 1$. Thus, the only feasible target transitions starting at site $(k, x)$ are to sites $(k + 1, x - 1)$ or $(k + 1, x)$ or $(k + 1, x + 1)$. Since these transitions represent time progressions, the bonds are directed from site $(k, x)$ to the $(k + 1, x - 1)$, $(k + 1, x)$ and $(k + 1, x + 1)$ as illustrated in Figure 1. This graph is the graph $\mathbb{L}_{alt}^2$ described in [18]. This notation will be used to refer to our graph, illustrated in Figure 1, from this point on.

So we have specified the graph structure, denoted $\mathbb{L}_{alt}^2$, which consists of a set of sites $S$ in two dimensions representing the target state at specific times, and a set of bonds $B$ dictated by the target state transition model.

Consider the measurement process. Measurements are generated through a random clutter model, allowing measurements in any target state $x \in \mathbb{Z}$ with probability $p$ at any time $k \in \{0\} \cup \mathbb{Z}$. This is equivalent to having any site in the graph $\mathbb{L}_{alt}^2$ open with probability $p$ and closed otherwise, where an open site corresponds to one with a measurement.

A clutter track is any path through the clutter measurements using only the allowed target state transitions. The graph $\mathbb{L}_{alt}^2$, illustrated in Figure 1, dictates the allowed transitions, and open sites are distributed according to the measurement distribution. Thus a flow through this graph is equivalent to a path through clutter measurements, a clutter track. Thus, if $p$ is chosen so that the percolation process has a positive probability of percolation, $\theta(p)$, then clutter tracks may exist that last indefinitely long.

### 5.1 Phase Transition

To demonstrate that a phase transition exists for this system, it suffices to show that the critical probability, is bounded such that $0 < p_c < 1$. We derive an upper and a lower bound for $p_c$ in this section.

#### 5.1.1 Upper Bound for Critical Probability

An upper bound on the critical probability may be derived by considering the graph formed by deleting all sites $(k, x)$ such that $x + k$ is odd, and all bonds connected to these sites. The graph for this model is often referred to as $\mathbb{L}^2$ and is illustrated in Figure 3. This reduces the number of allowed transitions between target states, and the target is forced to change to a neighbouring state at each time transition. Consider a site percolation process on this graph, and denote the probability of site percolation on this graph by $\alpha(p)$. Because this process is a subset of the site percolation process on the original graph, $\mathbb{L}_{alt}^2$, then the probability of percolation on this graph will be less than or equal to the probability of percolation on $\mathbb{L}_{alt}^2$, $\alpha(p) \leq \theta(p)$. (16)

Thus, the critical probability for these graphs is such that

$$p_c^0 \geq p_c,$$ (17)

where $p_c^0$ is the critical probability for site percolation on the graph $\mathbb{L}^2$. It has been shown by Liggett [19] that an upper bound for $p_c^0$ is given by $\frac{3}{4}$. This also provides an upper bound for $p_c$,

$$p_c \leq p_c^0 \leq \frac{3}{4}. \quad (18)$$

#### 5.1.2 Lower Bound for Critical Probability

This section considers a lower bound for the critical probability of site percolation on our graph. Bishir [20] derived a lower bound for the critical probability for the site percolation process on the graph $\mathbb{L}^2$ shown in Figure 3 above. The lower bound derived by Bishir does not apply to our graph $\mathbb{L}_{alt}^2$. In this section, we use a similar technique to that used by Bishir to derive a lower bound for the critical probability of site percolation on the graph $\mathbb{L}_{alt}^2$, illustrated in Figure 1.

A lower bound for the critical probability may be found by considering a dominating process for the site percolation process on $\mathbb{L}_{alt}^2$. This dominating process is the same as the percolation process, with a modification so that if any two sites are wetted at time $t$, then it is assumed that all sites between them at time $t$ are also wetted. This process also has a probability of percolation associated with it, denote this by $\beta(p)$. Clearly,

$$\theta(p) \leq \beta(p);$$

This model cannot be physically interpreted in terms of our target in clutter problem, but does provide a theoretical dominating process for which the critical probability can be theoretically calculated.
because this modified process comprises all sites that are wetted in the original percolation process, as well as additional sites introduced by the modification to the original process. Thus the critical probabilities for each of these processes satisfies

\[ p^c_\beta \leq p_c, \]

where \( p^c_\beta \) denotes the critical probability of the modified process. It may be shown that the critical probability for the modified process is given by \( p^c_\beta = \frac{1}{2} \). For the sake of brevity, the proof is not included, instead it may be found in [11] or will be included in a paper that is currently being prepared.

Thus, we have shown that the set based tracking in clutter only problem described in Section 3 exhibits a phase transition, and that

\[ \frac{1}{2} \leq p_c \leq \frac{3}{4}. \]

Of particular interest is the lower bound on this critical probability. Percolation does not occur for the system for \( p < p_c \), so it will certainly not occur for \( p < \frac{1}{2} \). Returning to the original model, this means that false alarm probabilities less than \( \frac{1}{2} \) for this discretised model will guarantee that clutter only tracks die out in finite time.

5.2 Monte Carlo Simulation Results

It is generally very difficult to theoretically calculate the critical probability for any percolation system. Often Monte Carlo simulations are performed in order to calculate a numerical estimate for this critical probability [21, 22]. Monte Carlo simulation was used in order to obtain a numerical estimate of the critical probability for the oriented site percolation process on the graph \( \mathbb{Z}^2_{alt} \) described in Section 5. The false alarm probability was varied from 0.53 to 0.54 in steps of 0.0001. The simulation consisted of 1000 runs for each of these false alarm probabilities. Each of these runs was allowed to continue for 20000 time steps. The proportion of clutter only tracks still alive after this time are illustrated in Figure 4. This gives an indication of the proportion of tracks alive after an infinite time, or the probability of percolation. This will be slightly inaccurate due to the fact that tracks that last 20000 steps may still die out before an infinite number of steps have been reached. The value of \( p \) at which this becomes non-zero is the critical false alarm probability, \( p_c \). Observation of Figure 4 suggests that it may be approximated by \( p_c \approx 0.535 \) for the oriented site percolation process on \( \mathbb{Z}^2_{alt} \).

5.3 Extensions to the Model

The current description focuses on a very simple and coarse discretisation of the target state space and thus requires extension to become more realistic. Firstly, the size of the discretisations should be reduced, in order to obtain a better approximation of the continuous system. However, this leads to more allowed transitions between states, and more complicated calculations to find lower bounds. It seems intuitive that a phase transition also occurs for a continuous space system and that a critical clutter density threshold exists, similar to the critical false alarm probability, below which clutter only tracks die out. If it were possible to find the critical probability for the discretised systems, then this could be obtained in the limit as finer discretisations are used. This will require much more research into solving these discretised percolation systems for their critical probabilities.

In addition, models using higher dimensional target states and more complicated motion models should be considered. Such models should still result in percolation processes, but these will be defined on more complicated graph structures and would be more difficult to study. The effect of measurement uncertainty, which has not been considered in the current system, should also be added to the model.

This analysis may also be extended to other tracking processes such as Hidden Markov Models, by including the transition probabilities for the allowed transitions. This leads to a mixed bond-site percolation process.

6 Conclusions

A simple discretised model for set based tracking in clutter only, with no target present, has been described and analysed. For a tracker to be expected to perform well, these tracks should die out in finite time, so that when a target is present, it will provide the only persistent track. We have shown that based tracking using this simple discretised model is an oriented site percolation process, for which a phase transition occurs. This means that there is a critical probability of false alarm, below which clutter only tracks are guaranteed to die out in finite time, whilst above which, there is a positive probability of clutter tracks lasting forever. This critical false alarm probability is difficult to calculate analytically, but lower and upper bounds have been calculated, suggesting that \( \frac{1}{2} \leq p_c \leq \frac{3}{4} \). A Monte Carlo simulation verified these bounds, and provided a numerical approximation of \( p_c \approx 0.535 \).

This presents a potential new framework under which to study the problem of tracking in clutter with bounded errors. Many extensions to the simple model considered here should be investigated further in order to allow analysis of a more realistic model.
References