A General Algebraic Structure for Situation Analysis

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Abstract—The aim of this paper is to present recent works made in the study of distributed systems and knowledge-based programs and show how these results can contribute to the formalization of the Situation Analysis (SA) problem. Precisely, we propose to use the algebraic concepts detailed in a recent book of Fagin, Halpern, Moses and Vardi as a blueprint for SA system design. In this paper we show how the formal model in question can be used to handle and distinguish numerical evaluations of probabilities and belief as well as means to represent and reason on knowledge. After a presentation of key models and concepts of Situation Awareness (SAW) and SA we proceed with a brief review of formal models recently used and associated published work. Building upon Fagin and Halpern’s work but also on Bundy’s which extend the probability structure proposed by Nilsson this paper shows how to translate the basic concepts of functional SA models in the proposed formal algebraic framework. The algebraic concepts exposed and studied herein are those of agents and environment, local and global states, temporal sequences of global states called runs, systems or sets of runs, actions, protocols and finally contexts.

I. INTRODUCTION

Fusion is often defined as the process of combining information in order to estimate or predict entity states. Several models either functional [1], process [2] or formal [3] have already been proposed as guides for the design and implementation of information fusion systems. The usual functions considered for information fusion in the Joint Directors of Laboratories (JDL) model [1] are conveniently segregated into so-called levels of processing, even though practice has shown a less clearcut specialization. In [4], the JDL model is extended to include a fifth level of data fusion addressing the issues associated with the human interface.

Referring to the JDL functional model terminology the present paper focuses specifically on level 2 (Situation assessment) and indirectly on levels 3 (Impact assessment) and 4 (Process refinement). Compared to the global process of information fusion few models have been proposed for the detailed formalization of higher levels invoked in the JDL model. We believe that such formalization is essential for a better understanding of the objects and processes to be described, recognized and acted upon in practice. Furthermore and according to guidelines proposed in [1] regarding current problems of High-Level Data Fusion and Distributed Data Fusion the present paper focuses on the development of solutions regarding formal knowledge representation, multitype uncertainty management, heterogeneous schemes of reasoning as well as multiagent formalisms.

In Section II a brief review of Situation Analysis and Situation Awareness definition and models is made before a more detailed discussion on SA concepts, presented in Section III. Building on the conclusions developed in the previous section these basic concepts are then further formalized in Section IV. The presentation of the self-contained formalism proposed for handling distributed SA is presented in Section V where we show how this general framework is extended to deal with uncertainty and epistemic notions of knowledge and belief via the algebraic definition of an interpreted system.

II. SITUATION ANALYSIS

Situation Analysis (SA) can be defined in terms of contextual analysis of objects and processes in order to provide a certain form of knowledge to an agent or collection of agents. The particular form of knowledge we refer to herein is known in the data fusion community under the designation of Situation Awareness (SAW), where the contextual notion of situation relates to an idea of a spatial, temporal, functional or structural information partition [5]. Moreover, depending on the level of detail and time scales, objects and processes or both, can be considered as long as their attributes and properties remain stable for a sufficient period of time to allow perception and analysis. The agents are participants of the situation, playing a role in a certain number of processes. Agents can determine the initiation of a process or its termination, can play an active role in a process without being responsible for its initiation or termination or on the contrary be present but have no active role in the process. Finally an agent can be the product of a given process. Apart from agents roles, SA is interested in the determination and monitoring of the essence or identity of agents, their resources, plans and goals in a given context.

A. Epistemic concepts

Since SAW is conceived as a state of knowledge, explicit reasoning on knowledge and the problems linked to its representation are distinctive features of SA. In fact for cognitive scientists, SAW is a matter of mental representation used to study the interface between the external world and mind. It is a view particularly popular in Artificial Intelligence, although opposition is well argumented for example in [6].
For the Representational Theory of Mind (RTM), mental states are relations between agents and mental representations. Formally, and following Pitt’s formulation [7], for an agent to be in a psychological state $\Psi$ with semantic property $\Phi$ is for that agent to be in a $\Psi$-appropriate relation to a mental representation of an appropriate kind with semantic property $\Phi$. The Computational Theory of Mind (CTM), an updated version of RTM, claims that mental processes are computations on mental representations. It is the standpoint we adopt here.

The SA process relies on the integration of internal and external representations of the situation. Internal representations model the awareness of the agent about itself, while external representations cope with awareness about the environment. The situation model represents the elements of the situation and their relationships in order to help a given system in a problem-solving situation to grasp the situation at hand.

Our epistemic position towards Situation Analysis is to consider not only the sources of knowledge but also the justification processes involved as well and the truth functionality issues. Instead of focusing on Perception and Reasoning, our standpoint is to, following [8], consider also the categories of Memory, Testimony and Consciousness as basic sources of Situation Awareness. The formal representation and determination of computing means of each of these five sources are open problems of Artificial Intelligence. Formalization efforts for the joint handling of these sources of knowledge for Situation Assessment purposes are only mumbling at the time being.

B. Situation Awareness and Situation Analysis models

In this section we present briefly two models of SAW and detail roughly their main components in order to relate them to the formal structures presented below in Sections IV and V. The first model is the functional model proposed by Endsley and Garland [9] and the second is a formal ontology for SAW proposed by Matheus, Kokar and Baclawski [10].

For Endsley and Garland [9], Situation Awareness is a knowledge state involving the perception, the comprehension and the projection of the situation elements. They define SAW as “the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning and the projection of their status in the near future”. SAW is also defined as “the active mental representation of the status of current cognitive functions activated in the cognitive system in the context of achieving the goals of a specific task”. This model of SAW for decision making is represented on Figure 1.

Even though Endsley’s model is quite complete and covers much of the knowledge sources discussed in the introduction of the present section, as well as processing and decision steps involved in the exploitation of SAW, it is still a very high-level description and leaves untouched formalization and precise implementation issues.

A model such as Matheus, Kokar and Baclawski’s [10] is much more precise concerning the mathematical relations between situation elements. SAW is defined herein as the knowledge of a specification of a goal $T_g$, an ontology $T_o$ referring to the world at hand, a stream of measurements $W_1, W_2, \ldots$ related to time instances $t_1, t_2, \ldots$, fused theories $T^t = \nabla^T(T_1^t, T_2^t, \ldots, T_n^t)$ relevant to goal $T_g$ as well as fused theories $T^{t+1}$ relevant to $T_g$ at $t+1$, the fused model $M^t = \nabla^M(M_1^t, M_2^t, M_3^t, M_4^t)$ combining models relevant to $T_g$ as well as $M^{t+1}$ at some time instance in the future, and finally relations among objects $R^{t} \subset O^{t} \times O^{t}$ relevant at time $t$ as well as $R^{t+1} \subset O^{t+1} \times O^{t+1}$. The principal classes of entities related in the Core SAW Ontology are those of Situation, Situation object, Goal, Relation, Event, Attribute and Value for the purpose of “maintaining information in an organized fashion to make the answering of the queries possible and efficient” (see Figure 2).

Although useful for human factors studies [9] and database management problem solving [10], these models are not aimed at formal representations of epistemic states nor at the representation of uncertainty. The environment and the hostile non-cooperative entities it may contain are not explicitly formalized. Furthermore the difficult problem of situation-
bounded multiagent instance representation and reasoning is left untouched. Although in [10] the authors mention explicitly Situation and Situation objects, they do not have any special status and are just objects of the model and the notion of context itself is not formalized.

III. TOWARDS FURTHER FORMALIZATION OF SA CONCEPTS

A. Ontological aspects

Defining situation analysis, Roy [11] takes a more intuitive and practical approach and considers five basic situation elements: Environment, which is left undefined; Entity, an existing thing (as contrasted with its attributes), something that has independent, separate, self-contained, and/or distinct existence and objective or conceptual reality; Event, something that happens (especially a noteworthy happening); Group, a number of individuals (entities and/or events) assembled together or having some unifying relationship, i.e. an assemblage of objects/events regarded as a unit; Activity, embedding the ideas of action, movement and motion. The term activity is appropriate when something has the quality or state of being active, i.e. when something is characterized by action or expressing action as distinct from mere existence or state. These five situation elements, their links, inputs and outputs are represented on Figure 3.

![Fig. 3. Roy’s five situation elements [11].](image)

Roy’s model omits to explicitly mention two basic notions that are those of agents and processes. We think that processes are central notions around which SA can be articulated. Indeed a description of SA taking the notion of process as a basic building block seems particularly suitable and efficient since natural language descriptions of the world are very often process-based. Yet it seems that few formalizations have been proposed to link the notions of processes and situations. An example of this kind of work is given by Mourelatos and Passonneau [12] in the field of linguistics. Mourelatos’ typology of situations is shown in Figure 4 where situations are either seen as states or occurrences. In [13], Lambert parallels the JDL model with the Endsley’s work on Situation Awareness in dynamic systems and proposes that situations be understood as processes i.e. “collections of related spatio-temporal facts”.

Another argument for process-based SA resides simply in the fact that system and network level descriptions and operations in Computer Science are commonly stated in terms of discrete processes which are conveniently modeled as algebraic structures relating states and events, counters who relate the ticks of a clock, discrete processes and the set of integers or threads corresponding to subprocesses running in parallel.

As we recalled earlier, the distinction between objects and processes is only a matter of detail and time scale. Processes being a larger class of entities embedding the ideas of initiation, continuation and cessation when referring to continuous processes, while discrete processes can be distinguished into events and states according to Sowa [14]. For now on, objects will be identified to states while changing objects will be designated as events.

In SA we are interested in the causal relations between processes, on the prediction of future processes and most of all in the roles of the different entities. We believe that as participant of the situation (see Section II), the notion of agent must be explicitly included in the SA model and following Sowa [14] we define an action to be an event caused by an agent, and an activity to be a process caused by an agent or a group of agents.

Hence, redrawing Roy’s model, entity and event can be regrouped under the single notion of process, activities and actions being only particular kinds of processes. The environment is nothing else than a special agent with the particularity of being only partially controllable (see Section V). The notion of group does not need to be explicitly mentioned since grouping is the action of organizing objects into classes whose members share similar characteristics. For agents the most classical way of grouping objects is by defining equivalence relations. This operation is central to SA.

B. Formal aspects

We believe that the minimal baseline formalism for Situation Analysis must have at least the following features:

- A formal ontology;
- A structure for representing and reasoning about psychological states and semantic properties such as belief and knowledge;
- A way for representing and dealing with different aspects of uncertainty [15];
- Rules for combining information from different sources in order to estimate or predict states;
- Being generalizable to the multiagent case;
- An explicit definition of context.
This notion of context is twofold. On one hand there is a idea of partition of information, built most of the time on constraints of time, space, structural or functional features in order to reflect for example the limitations of intelligent agents. It is a low-level notion of context that can be formalized by appropriate, and yet simple, equivalence relations or more of less elaborate definitions of threshold. Low-level contexts are present in most existing SA models and systems, even if only implied. But there is also a higher-level notion of context that deserves more attention from the SA community as far as formalization issues are concerned. It is the setting in which a given situation is observed, its elements measured, the behavior and mindset of the agents evolving in it, identified and studied, psychological and physical states predicted. Thus, according to a given high-level context actions and activities will be planned, programed and executed differently.

Among the available mathematical and logical approaches, modal logics are one of the most popular tools to represent and reason about knowledge, and this for many agents. Moreover, many-dimensional logics [16] have been recently introduced to extend modal logics to combine different kinds of modalities. However, these logics cannot directly account for measures and numerical valuations that is the strength of numerical approaches. For example the theory of evidence is a good tool for representing uncertainty and ignorance and combining different pieces of information. However, it lacks clarity when it comes to reasoning.

Based on the possible worlds semantics, the formal system-based model proposed by J. Halpern and Y. Moses in 1990 that will be detailed in Section V, besides giving clear ontological definitions, combines in a general structure the basis of modal logics as well as those of evidence theory. This tool seems to be an interesting candidate to cope with the above mentioned SA requirements. In the following section, we review some basic structures that be used to fulfill the above requirements.

IV. Basic structures

In [5], [17], we showed how possible worlds semantics can serve as a basis for SA reasoning. In particular, we outlined the fact that Dempster-Shafer’s structures [18], [19] for belief functions which are extensions of Nilsson's structures [20] and probability structures [18] for probabilities conciliate many requirements for a SA mathematical structure. Indeed, based on the possible worlds semantics, DS structures are general enough to measure and reason about uncertainty, combine pieces of information and manage conflict. However, we did not highlight the multiagent aspect, the integration of time, nor reasoning about knowledge. These aspects will be detailed in the present paper. Before presenting the general model of Fagin et al. (Section V), we recall some basic properties of Kripke, Nilsson and Dempster-Shafer’s structures. Let \( \Phi \) be a set of formulas describing ground facts about the situation. \( \Phi \) can contain propositions carrying the following facts “The target we track is accelerating”, “It is raining”, “The ESM reports Emitter n° 12 for Target n° 3”. Elements of \( \Phi \) are denoted by \( \phi \) or \( \psi \) and called atomic propositions or simply atoms.

A. Knowledge and awareness

A knowledge structure defined in [21] is a tuple \( S_K = (S, \pi, R) \) where \( S \) is the set of the possible states of the world, \( \pi : S \rightarrow \Phi \rightarrow \{True, False\} \) is a truth assignment to the atoms per possible world (i.e. an interpretation for the propositions in \( \Phi \) over \( S \)) and \( R \) is an accessibility relation over \( S \). \( S_K \) is identical to the interpreted system \( I \) defined bellow (Section V-B) and is nothing else than a Kripke structure [22].

Given this semantics, we can introduce the knowledge operator \( K \) used independently of any formal definition of systems of modal logic. For a Kripke model of structure \( S_K \) and given \( s \in S \), the following clause \( S_K, s \models K\phi \) iff \( \forall tR(s, t) \) it holds that \( S_K, s \models \phi \) states that an agent is said to know proposition \( \phi \) if this proposition is True in every state considered possible by the agent being in the world (or state) \( s \). Paraphrasing [23], the agent may have doubts about the true nature of the world since it may consider more than one epistemic alternative as possible, but it has no doubts about the truth of \( \phi \).

Such a structure is the basis for epistemic logic models for reasoning on knowledge and belief. Indeed, the accessibility relation \( R \) defines axioms for the system which model the agent’s properties. Logics of knowledge and belief lie on five main axioms (K, T, 4, 5, D), plus (PC) which represents all instances of tautologies of propositional calculus. The axiom (K) is the axiom of distribution axiom and tells us that if an agent \( i \) knows that “\( \psi \) follows \( \phi \)” then it follows that the agent \( i \) knowing \( \psi \) follows from agent \( i \)'s knowledge of \( \phi \). The axiom (T) is the axiom of true knowledge and tells us that if an agent \( i \) knows that \( \phi \) is the case then it follows that \( \phi \) must be True, that is, \( \phi \) occurred. The axiom (4) is the axiom of positive introspection and tells us that if an agent \( i \) knows that \( \phi \) is the case then it follows that agent \( i \) knows that he knows \( \phi \). This axiom could be used to model a certain hierarchy among a group of agents where only some agents possess a capacity of introspection. The axiom (5) is the axiom of negative introspection and tells us that if an agent \( i \) does not know whether \( \phi \) is the case then it follows that agent \( i \) knows that he does not know \( \phi \). This axiom follows directly from axiom (4) and is used to model the knowledge of ignorance. Axioms (4) and (5) are often used in the same systems, since they are quite complementary. The axiom (D) is the axiom of consistency and tells us that if an agent \( i \) believes that \( \phi \) is the case then it follows that agent \( i \) does not believe that \( \phi \) is not the case.

Different combinations of the previous five axioms plus two inference rules lead to different systems of modal logics. While systems K, T, S4 and S5 are designed for dealing with knowledge, systems KD45 and KD45 are more appropriate for belief. In these systems, the operator \( K \) for knowledge is often replaced by the operator \( B \) for belief.

The knowledge structure \( S_K \) can easily be extended to the multiagent case leading to \( S_{K_i} = (S, \pi, R_1, \ldots, R_n) \), where \( R_i \)
is the accessibility relation for agent \( i, i = 1, \ldots, n \). Then, different agents can have different properties on the same structure, and formulas such as \( K_i \neg K_j \phi \) (“agent \( j \) knows that agent \( j \) does not know \( \phi \)”) can be evaluated.

Furthermore, considering a group of \( n \) agents say \( G = \{A_1, \ldots, A_n\} \), the epistemic knowledge operators \( E_G, C_G \) and \( D_G \) can be considered. \( E_G \phi \) reads “Everybody in \( G \) knows \( \phi \).” \( C_G \phi \) reads “\( \phi \) is common knowledge in \( G \)” and \( D_G \) reads “\( G \) has distributed knowledge about \( \phi \).” Hence, by closing off the set of formulas under the negation, conjunction and the epistemic knowledge operators \( K_1, \ldots, K_n, E_G, C_G \) and \( D_G \), we are now able to deal with knowledge (and belief), common knowledge (\( C_G \)) and distributed knowledge (\( D_G \)). Notice that these operators can provide a mean to extend the concept of SAW to the ones of common awareness, distributed awareness, “everybody is” awareness. See [24] for more details on the definition of these operators.

Common knowledge is the most difficult concept to implement because it lies on a recursive definition. To overcome this problem and in order to prevent the problem of omniscience, Fagin and Halpern proposed in 1988 the sieve models [25], introducing a function that acts as a sieve. Instead of introducing nonstandard world or situations, sieve models introduce a segregation between formulas that can be known or believed and others that cannot. The sieve function \( A \) indicates in fact if the agent is aware of a given formula in a given situation. Being aware amounts to knowing or believing the formula in question. A sieve model is a quadruple structure \( S \) of the form \( \langle S, \pi, R, A \rangle \) where \( S \) is a non-empty set (the set of possible worlds); \( \pi \) is a truth assignment to the atoms per possible world; \( R \) is a serial, transitive and Euclidian belief accessibility relation; \( A : S \rightarrow \wp(\cal L) \) is the awareness function, assigning per state. This language contains the modal operator \( \square \) plus an awareness operator \( A \), interpreted on \( S \) given a state \( s \in S \) as

\[
(S, s) \models \square \phi \text{ iff } \phi \in A(s) \text{ and } (S, t) \models \phi \text{ for all } t \\
\text{such that } R(s, t) \\
(S, t) \models A \phi \text{ iff } \phi \in A(s)
\]

The conditions on the above mentioned awareness function \( A(s) \) state in fact that the agent is aware of the consistency axiom (D), and of the positive (4) and negative (5) introspection axioms. This model is thus built upon a KD45 modal logic.

For the purpose of Situation Analysis in the sense generally accepted by the High Level Data Fusion community, this model is extremely interesting. In fact it could be used as a starting point for the formalization of a logic-based approach to Situation Awareness and Situation. From a practical point of view, one still has to determine what is the extension of this domain of awareness, but it does not seem to be a critical aspect of this model and thus seems to be manageable from the outside (i.e. by putting a human in-the-loop, using theoretical limits of sensors, etc.). Fagin and Halpern proposed a way to incorporate time in the sieve model. The introduction of a temporal component allows one to reason about awareness in terms of acquiring and forgetting information. A temporal function can also model the awareness of an agent (or group of agents) by allowing the application of a certain deduction rule at every time step, depending on the current knowledge or historical records.

B. Probabilities

Probabilities are perhaps the oldest mean to represent and quantify uncertainty. With a subjective interpretation probability theory is even able to deal with propositional attitudes such as belief. Although probabilities cannot model all types of uncertainty like fuzziness for example (see [15] for a discussion on the subject), they remain an unavoidable framework to quantify and deal with uncertainty.

However, like most quantitative approaches, probability theory lacks a clear semantics that can serve as a basis for reasoning. One of the first steps toward a logic of probability is the work of Nilsson, who proposed in [20] a structure for probabilistic logic based on the notion of possible world. Fagin and Halpern [18] extend Nilsson’s structure and propose a definition for a probability structure as a tuple \( S = \langle S, \pi, R, \mu \rangle \) where \( S \), the set of all possible worlds, \( \pi \) is a \( \sigma \)-algebra of subsets of \( S \) (the set of measurable subsets of \( S \)), \( \mu \) is a probability measure on \( S \), and \( \pi \) is a mapping \( \pi : S \rightarrow \Phi \rightarrow \{ \text{TRUE; FALSE} \} \) is a truth assignment to the atoms per possible world, characterizing for each \( \phi \in \Phi \) the set of possible worlds \( A_\phi = \{ s \in S \text{ in which } \phi \text{ is TRUE} \} \). Such a structure allows then to measure and reason about the different states of the world.

In SA however, besides reasoning about the probability of certain events, it is also fundamental to be able to reason on the knowledge of the agents. In [21], those authors propose a structure to reason on both knowledge and probabilities. Indeed, they define a Kripke structure for knowledge and probability for \( n \) agents to be a tuple \( S_{KP} = \langle S, \pi, R, \mu \rangle \) where \( S \), the set of all possible worlds, \( \pi \) is the truth assignment, \( R \) is the accessibility relation for agent \( i, i = 1, \ldots, n \) and \( \mu \) is a probability assignment which assigns to each agent \( i \) and state \( s \) a probability space \( \mathcal{P}(i, s) = \langle S_i, \chi_i, \mu_i \rangle \), where \( S_i \subseteq S \). “\( \mathcal{P}(i, s) \) describes agent’s \( i \) probabilities on events, given that the state is \( s \)” [21]. In the case of one agent, we simply have \( S_{KP} = \langle S, \pi, R, \mu \rangle \). This general structure allows to evaluate formulas such as \( \mu(\phi) \geq b \) (“the probability of \( \phi \) is greater or equal to \( b \)”) but also \( K_i(\mu_\phi(\phi) \geq b) \) (“agent \( i \) knows that probability of \( \phi \) is greater or equal to \( b \)”) or \( \mu_i(\phi_i) \geq b \) (“probability that agent \( i \) knows that \( \phi \) is greater or equal to \( b \)”).

C. Belief and uncertainty

Probabilities however, constrained by their axiom of additivity \( \mu(\phi \lor \psi) = \mu(\phi) + \mu(\psi) \) if \( \phi \land \psi = \perp \), cannot assign measures to all subsets of possible worlds and are limited in their representation of uncertainty. In particular, nonspecificity cannot be modeled in their framework. Dempster-Shafer theory (DST) [26], [27] has been introduced as an extension of
probability by allowing the representation of supplementary types of uncertainty including nonspecificity. Indeed in the DST, the set of measurable subsets is no longer the σ-algebra \( \chi \) but the power set \( 2^S \) itself, replacing thus the additivity axiom by a subadditivity axiom. Thus the measures associated to the subsets of \( S \) do not need to be related in any sense\(^1\). Fagin and Halpern in \([18]\), define a Dempster-Shafer structure \([18]\) to be a tuple \( DS = (S, \pi, Bel) \), in which \( S \) and \( \pi \) are already defined and \( Bel \) is a belief function in the Shafer’s sense [26], i.e. a mapping \( Bel : 2^S \rightarrow [0,1] \) assigning a measure of belief to each subset of \( S \). Note that the same idea has been explored by Bundy in its incidence calculus theory [19]. Although the two structures (Bundy’s and Fagin and Halpern’s) are not exactly the same, they are very close and their has even been proved to be equivalent under certain conditions [28].

The DS structure can then be extended to include the reasoning on knowledge leading to a tuple \( KB = (S, \pi, R_1, \ldots, R_n, B) \), where \( B \) is a belief assignment for each agent \( i \) and state \( s \) such that \( B(i,s) = (S_{i,s}, 2^{S_{i,s}}, Bel_{i,s}) \), where \( S_{i,s} \subseteq S \).

In conclusion, given a general structure \( KB \), two means are available for representing belief: (1) Considering only the knowledge structure \( K \) and imposing the properties on the agent, leading for example to the system KD45 of modal logic especially design for belief; (2) Extending the probability spaces to non-measurable subsets (\( \chi_{i,s} = 2^{S_{i,s}} \)) on which belief functions are defined.

V. A GENERAL MODEL FOR DISTRIBUTED SYSTEMS

The model of distributed knowledge processing proposed by J. Halpern and Y. Moses in 1990 has been first introduced in [24], but has been and recently in the book Reasoning about knowledge of R. Fagin, J. Halpern, Y. Moses and M. Vardi [29] (FHMV from now on). In this book, the authors “want to use knowledge as a tool for analyzing multiagent systems”. However, another point of view can be adopted. Indeed, in Situation Analysis, knowledge is a state to reach in a multiagent context. Notwithstanding, the result of this general model is to provide a formal framework for representing knowledge, dealing with uncertainty, describing multiagent systems with their behaviors and their actions. We are also interested in the formalization of context, which appears to be an essential notion in SA.

After giving short definitions for the different components of the model and illustrating them on examples and figures (Section V-A), we show in Section V-B how the possible worlds semantics is used as a basis for this general model.

A. The components of the model

1) Agents and environment: An agent is a concrete object that can perform actions, such as a classifier, a robot, or a human agent. Let’s assume a system composed of \( n \) agents \( A_1, \ldots, A_n \). Then a multiagent system can be defined as a collection of interacting agents. The environment can be seen as a special agent.

2) States: The local state \( \ell_i \) of the agent \( i \) is determined by the encapsulation of all the information an agent has access to. \( L_i \) is the set of the possible local states of the agent \( i \).

The state of the environment \( \ell_e \) is defined as the information relevant to the system but not contained in the state of the agents. \( L_e \) is the set of the possible local states of the environment.

The global state \( g \) of a system is given by the pair consisting of the agents’ local states together with the state of the environment. The set of global states is the cross-product \( G = L_e \times L_1 \times \ldots \times L_n \).

3) Time: A run \( r \) over \( G \) is a function from time to global states \( G \). A run is a sequence \( r(0), r(1), \ldots \) over \( G \), that the system goes through across time.

A point is a pair \((r, m)\) consisting of a run \( r \) and a time \( m \). If \( r(m) = (\ell_e, \ell_1, \ldots, \ell_n) \) is the global state at point \((r, m)\), then define \( r_e(m) = \ell_e \) and \( r_i(m) = \ell_i \) for \( i = 1, \ldots, n \) to be respectively the environment local state and the agents local states at point \((r, m)\).

A round \( m \) in run \( r \) is defined to take place between time \( m - 1 \) and time \( m \). These concepts are illustrated on Figure 5 where a single agent is considered.

4) System: A system \( R \) over \( G \) is defined as a set of runs over \( G \). We say that \((r, m)\) is a point in system \( R \) if \( r \in R \) (see Figure 6).

5) Actions: Actions are the cause of changes in the system, and performed by the agents and the environment. Let \( ACT_i \) be the set of actions that can be performed by agent \( i \), and let \( ACT_e \) be the set of actions performed by the environment.

The null action \( \Lambda \) corresponds to the agents or the environment performing no action.

The joint action is a tuple \((a_e, a_1, \ldots, a_n)\) of actions performed by the set of agents and the environment, where \( a_e \) is the action performed by the environment, and \( a_i \) is the action performed by the agent \( i \). A joint action is an element of \( ACT_e \times ACT_1 \times \ldots \times ACT_n \).

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\(^1\)In probability theory, the only way to assign a measure to a non-singleton subset of \( S \) is by referring to an underlying probability distribution on the singletons of \( S \), and applying then the axiom of additivity.
A global state transformer is a mapping from the global states to the global states $T : G \rightarrow G$. With each joint action is associated a global state transformer. Hence, joint actions cause the system to change via $T$; if $g$ is the global state of $\mathcal{R}$ when $(a_e, a_1, \ldots, a_n)$ is being performed, then the state of $\mathcal{R}$ changes for $T(g)$.

A transition function $\tau$ is a mapping from joint actions to global state transformers. Figure 7 illustrates the latter notions.

![Fig. 6. System.](image)

$$G = L_e \times L_1$$

![Fig. 7. Global state transformer and transition function.](image)

6) Protocol: A protocol $P_i$ for agent $i$ is a mapping from the set $L_i$ of local states of the agent $i$ to nonempty sets of actions in $ACT_i$. A protocol $P_e$ for the environment $e$ is a mapping from the set $L_e$ of local states of the environment to nonempty sets of actions in $ACT_e$. A protocol is a function on local states rather than on global states.

A joint protocol $P$ is a tuple $(P_1, P_2, \ldots, P_n)$ consisting of the protocols of each of the agents $i$, $i = 1, \ldots, n$. $P_e$ is the protocol of the environment and is not included in the joint protocol. Rather, the protocol of the environment is supposed to be given and $P$ and $P_e$ can be viewed as the strategies of opposing players. See Figure 8 for an illustration on the link between actions and protocols.

7) Context: A context $\gamma$ is a tuple $(P_e, G_0, \tau, \Psi)$ where $P_e : L_e \rightarrow 2^{ACT_e - \emptyset}$ is a protocol for the environment, $G_0$ is a nonempty subset of the set $G$ of global states, $\tau$ is a transition function and $\Psi$ is an admissibility condition on runs. An admissibility condition $\Psi$ on runs tells us which ones are “acceptable”. Formally, $\Psi$ is a set of runs and $r \in \Psi$ if and only if $r$ satisfies the condition $\Psi$. $G_0$ is the set of initial global states, describing the state of the system at the initiation of the protocol.

Note that the description of the behavior of a system is contextual, i.e., a joint protocol $P$ is always described within a given context $\gamma$.

B. Dealing with knowledge, belief and probabilities

In Section V-A, we showed that the general model of a distributed system proposed by Fagin et al. is a tool for contextual and dynamic multi-agent systems. However, in situation analysis besides these features, we need to deal with mental states or propositional attitudes such as knowledge or belief. As introduced in Section IV, possible worlds semantics appears as a classical mean to knowledge and belief logics. In the following, we detail the bridge between a classical Kripke structure and the general model introduced above.

1) Interpreted system: Let $\Phi$ be a set of primitive propositions, describing basic facts about the system. Formulas are built using the classical operators of propositional logic. The set of formulas is closed off the operators $\neg$ and $\wedge$ (negation and conjunction). Hence, given two formulas $\phi$ and $\psi$, $\neg\phi$, $\phi \wedge \psi$, etc are also formulas.

An interpreted system $\mathcal{I}$ consists of a pair $(\mathcal{R}, \pi)$ where $\mathcal{R}$ is a system over a set $\mathcal{G}$ of global states and $\pi$ is an interpretation for the propositions in $\Phi$ over $\mathcal{G}$, which assigns truth values to the primitive propositions at the global states. Thus, for every $p \in \Phi$ and state $g \in \mathcal{G}$, we have $\pi(g)(p) \in \{\text{TRUE}, \text{FALSE}\}$.

By definition, a system $\mathcal{R}$ is a set of runs $r$ over $\mathcal{G}$. This set of runs over $\mathcal{G}$ defines a binary relation over $\mathcal{G}$, where a link from $g_1$ to $g_2$, two elements of $\mathcal{G}$ means that once the system is in the state $g_1$, it can become into the state $g_2$. In other words, the state $g_2$ is accessible from $g_1$ for the system $\mathcal{R}$. This binary relation is then an accessibility relation $\mathcal{R}$, and the system can be noted $\mathcal{R} = (\mathcal{G}, \mathcal{R})$. 
Hence, an interpreted system $I = (G, R, \pi)$ is simply a Kripke structure, where the possible worlds $s$ are the global states $g$ and $G \equiv S$.

2) A basis for general structures: Now, it is just a question of notation. The discussion of Section IV remains valid, and thus given an interpreted system $I = (G, R, \pi)$ and $\Phi$ a set of primitive propositions we can associate the general Kripke structure for $n$ agents $S_K = (G, \pi, R_1, \ldots, R_n)$, where $R_i$ is the accessibility relation for agent $i, i = 1, \ldots, n$. $R_i$s are then equivalence relations on points. Thus as the authors mention in [30], “the agents’ knowledge is completely defined by their local states”. Moreover, combining knowledge and temporal operators allows to reason about the evolution of the agents’ knowledge in the system [30].

Now the basis is set by associating with each state $s$ a point of $G$, the construction of the general structure for dealing with probability and knowledge $S_KP$ is straightforward, and follows the general structure for belief and knowledge $S_KB$ defined in Sections IV-B and IV-C respectively.

3) Examples of applications: The FHMV model has been successfully applied in particular in robotics and cryptography, in which observations play a crucial role. For example, in [31], a knowledge-level formalization of motion planning is proposed, where observations are described in terms of sensor readings (position, temperature, memory, . . . ).

In order to account for belief revision, in [32] the authors define an observation system as a ranked interpreted system, $(R, \kappa, \pi)$ where $\kappa$ is a ranking function on runs. Observations made by the agents are characterized by formulas in $\Phi$ and the local state of an agent at a given time $m$ is a sequence of $m$ observations.

Also in [33], to solve the problem of the Dining Cryptographers using a model checking approach, the observations are modeled as observation functions $O_i$ for each agent $i$ as a mapping of the set of local states of the environment $L_e$ to some set $O$, such that $O_i(\ell_e)$ is the observation of Agent $i$ in the state $\ell_e$.

VI. CONCLUSION

In this paper we show that the algebraic structure proposed by Fagin, Halpern, Moses and Vardi for the study of distributed systems and knowledge-based programs can be adapted to SA requirements. The FHMV model clearly defines and relates the process-based notions of system, agents’ behavior, environment, local states of agents, observations, global state of a system, run, action, protocol, and high-level context. Based on a relational structure, i.e. Kripke structure, the model provides a basis for reasoning on knowledge and belief with many agents. We have shown that the associated semantics, i.e. the possible worlds semantics used by Dempster-Shafer structures [18] for representing and reasoning on belief and uncertainty could lead to an interesting setting for a SA formalization. From this starting point the basic framework can be extended for the representation of many types of uncertainty [15], [17], [34].

The notion of high-level context defined allows for the specification of differentiated information fusion schemes according to the particular setting in which the SA system operates. For example, for a given context the aggregation operators could be contextually constrained and allow the SA system to adapt to changing agreement among sources, to defensive or offensive means of operation. The same is true for SAW representation.

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