Limits of Linear Multitarget Tracking

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Abstract—Multitarget tracking in clutter has two levels of complexity. One is caused by the exponential increase of number of measurement histories in time, and the other is caused by complexity in allocating measurements to tracks in each scan, which is also exponential in the number of tracks and the number of measurements involved. Linear Multitarget tracking is a Bayesian method for multi target tracking which dispenses with measurement to track allocation completely. This results in complexity which is linear in the number of tracks and the number of measurements. Linear Multitarget tracking is a recent development, with published results in a limited environment of just two targets in heavy clutter. This paper presents a simulation study which investigates the limits of Linear Multitarget tracking, both in the number of targets and the clutter measurement density. False track discrimination and target retention statistics are presented and compared with other single target and multi target tracking algorithms.

I. INTRODUCTION

Data association target tracking algorithms deal with situations where there are measurements of uncertain origin. In many radar and sonar applications measurements (detections) originate from both targets and non-targets, i.e. from various objects such as terrain, clouds etc., as well as from thermal noise. Unwanted measurements are usually referred to as clutter. In addition, the target measurements are present at each measurement scan with only a certain probability of detection. In a multi target situation, the measurements may have originated from one of several targets whose number, existence and trajectory is generally unknown.

Automatic tracking in this environment initiates and maintains tracks using both target and clutter measurements. If a track follows a target, we call it a true track otherwise we call it a false track. To discriminate between true and false tracks, a track quality measure is necessary, for example the probability of target existence [1], [2] or the probability of target detectability [3]. When a track is considered to be a true track, it is confirmed and presented to the operator or to the next level of processing. When a track is considered to be a false track, it is terminated. We call this process “False Track Discrimination”.

Single scan target tracking algorithms compress the measurement histories into one track state estimate, and multi scan target tracking algorithms keep measurement histories separate over last number of scans. For reasons of clarity and simplicity, this study is applied to single target single scan algorithms based on Integrated PDA (IPDA) [1]. IPDA algorithm extends Probabilistic Data Association (PDA) [4] by integrating the probability of target existence to target trajectory state estimation.

Single target tracking (STT) algorithms ignore the possibility that a measurement may have originated from a target not being followed by the current track, for example IPDA and related algorithms [1], [5], [6], GPB1-PDA [7], [8] and IMM-PDA [3]. Multi-target tracking (MTT) algorithms allow for the possibility that measurements may have arisen from the targets being followed by other tracks. Optimum all-neighbours MTT forms all possible joint measurement-to-track assignment hypotheses and recursively calculates their a posteriori probabilities, for example JPDA [9], [10], IJPDA [11], JIPDA [12] and MHT [13], [14], [15]. The number of possible measurement assignments grows exponentially with the number of tracks and the number of measurements. Thus, they have a high ratio of peak to average use of computational resources, and very often in difficult tracking situations (large number of targets and/or dense clutter) they will exceed the available computational resources. This situation is alleviated somewhat by applying the optimal MTT on the set of confirmed tracks only, but the computational complexity still remains a problem.

A number of attempts have been made to create multi target tracking filters which do not suffer from exorbitant computational requirements. One of the first was “Simple JPDA” [16], which replaces JPDA calculation with ad-hoc approximations. It turns out that this breaks down when applied to relatively small (four) number of targets [17]. The recently published Linear Joint method, LJIPDA [18], LJITS [2] and Multitarget Linear Convertor (MLC) [19] algorithms achieve all-neighbours multi-target capabilities by splitting the measurements according to the a priori [2], [18] or a posteriori [19] probabilities of measurement origin. These algorithms have a linear number of operations in the number of tracks and the number of measurements involved.

Linear Multitarget method is also an all-neighbour Bayesian MTT approach with a linear number of operations in the number of tracks and the number of measurements. It dispenses with the measurement to track allocation entirely.

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instead it treats interfering tracks as additional sources of special clutter. When updating one track, Linear Multitarget procedure modifies the clutter density with the foreign target measurement density, after which the single target tracker is applied. This paper tests the limits of Linear Multitarget procedure when applied to IPDA [1], to obtain LMIPDA. Thus Linear Multitarget tracking can be applied in complex situations with a number of tracks and measurements well beyond the capabilities of exponential multi target trackers.

Existing Linear Multitarget publications demonstrate the procedure in a very limited environment with only two targets with crossing trajectories. This publication tests the procedure in a more demanding environment, with up to fifty targets with crossing trajectories, in substantial clutter.

An overview of Linear Multitarget method is presented in Section II. Section III describes simulation experiments used to verify Linear Multitarget method and compare it to IPDA and JIPDA when tracking multiple targets in clutter.

II. LINEAR MULTITARGET METHOD

Linear Multitarget method is based on the notion that, when updating one track, (possible) detections of targets followed by other tracks are unwanted measurements or clutter, with some specific properties. Thus, when updating track \( \tau \) with measurements \( z_k \), clutter density is modulated with a priori measurement densities of other tracks and the other tracks are subsequently ignored.

IPDA, JIPDA and LMIPDA are based on the probability of target existence paradigm. Track state at time \( k \) is defined with a discrete event \( \chi_k \) which denotes target existence and a continuous variable \( x_k \) which denotes track trajectory estimate at time \( k \). Track estimate probability density function is conditional on the target existence event:

\[
P(\chi_k, x_k) = P(\chi_k | x_k) P(x_k | \chi_k) . \tag{1}
\]

Track state evolves between scans as a Markov process and is updated at each scan with measurements. For reasons of space, only the update of the probability of target existence is described below.

Measurement of each target is present with a probability of detection \( P_D \), and if present, it is corrupted with a measurement noise of covariance \( R \). Let \( z_k \) denote the set of measurements of cardinality \( m_k \) received at scan \( k \) and let \( z_{k,i} \) denote the \( i \)th measurement of \( z_k \), with \( Z^k = z_k \bigcup Z^{k-1} \) denoting the set of sets of measurements up to and including scan \( k \). Each measurement \( z_{k,i} \) consists of the kinematic (position) measurement component \( z^c_{k,i} \) and feature component \( z^f_{k,i} \) (e.g. amplitude as in [20], [21]). Target \( \tau \) and clutter probability density functions of feature component of measurement \( z_{k,i} \) are denoted by \( p^\tau_{k,i} \) and \( p^0_{k,i} \) respectively. When a measurement feature is not available or is not being modelled, assume \( p^f_{k,i} = p^c_{k,i} \). Let \( p^0_{k,i} \) denote a priori density of kinematic component of clutter measurements, and denote with \( p^\tau_{k,i} \) the a priori target measurement coordinate probability density function of track \( \tau \), given that the measurement is selected [4] with selection probability denoted by \( P_W \). Total clutter measurement density and track \( \tau \) pdf at measurement \( z_{k,i} \) are given by

\[
\begin{align*}
\rho_i &= \rho^\tau_{k,i} p^0_{k,i} \\
\rho^\tau_i &= \frac{ p^\tau_{k,i} }{ \sum_{j=1}^{m_k} \rho^\tau_j } \\
\rho_i &= \frac{ p^\tau_{k,i} }{ \sum_{j=1}^{m_k} \rho^\tau_j } \\
\end{align*} \tag{2}
\]

respectively.

The first step of the LM approach is to calculate a priori data association probabilities for each track, under the assumption that the selected measurements may have originated from the target or from clutter only. Under this assumption, the a priori probability that measurement \( i \) is the detection of target \( \tau \) is approximated by

\[
P^\tau_i = P_D^\tau P_W^\tau P \left( \chi^k_{k-1} | Z^k \right) \frac{ p^\tau_i }{ \sum_{j=1}^{m_k} p^\tau_j } . \tag{4}
\]

The a priori scatter measurement density of measurement \( z_{k,i} \), when updating track \( \tau \) is given by

\[
\tilde{\rho}^\tau_i = \rho_i + \sum_{\sigma=1, \sigma \neq \tau}^{T} \rho^\sigma_i \frac{ P^\sigma }{ 1 - P^\sigma } \tag{5}
\]

with \( T \) denoting the total number of tracks. Using \( \tilde{\rho}^\tau_i \) instead of \( \rho_i \) when updating track \( \tau \) with measurement \( z_{k,i} \), and otherwise ignoring other tracks, Linear Multi target tracker is obtained.

When tracking targets in clutter using single target tracker IPDA, a posteriori probability of target existence is given by [1], [6]:

\[
P \left( \chi^k_{k-1} | Z^k \right) = \frac{ (1 - \delta^k) P \left( \chi^k_{k-1} | Z^{k-1} \right) }{ 1 - \delta^k P \left( \chi^k_{k-1} | Z^{k-1} \right) } \tag{6}
\]

where

\[
\delta^k = P_D^\tau P_W^\tau \left( 1 - \sum_{i=1}^{m_k} \frac{ p^\tau_i }{ \rho_i } \right) \tag{7}
\]

is related to track \( \tau \) measurement likelihood ratio \( \Lambda^\tau_k \):

\[
\Lambda^\tau_k = 1 - \delta^k . \tag{8}
\]

When tracking targets in clutter using LMIPDA, a posteriori probability of target existence is still given by eq. 6, with:

\[
\delta^k = P_D^\tau P_W^\tau \left( 1 - \sum_{i=1}^{m_k} \frac{ p^\tau_i }{ \tilde{\rho}^\tau_i } \right) \tag{9}
\]

The expression for other data LMIPDA association probabilities are also obtained from IPDA association probabilities, by simply using \( \tilde{\rho}^\tau_i \) instead of \( \rho_i \).

The LMIPDA is easily extended to track multiple maneuvering targets in clutter, in the manner of [22].
III. SIMULATION STUDY

The purpose of the simulation study is to explore limits of Linear Multitarget tracking procedure. Specifically, performance limits with respect to a large number of targets and significant clutter are investigated. Where applicable, these results are compared to performance of equivalent single target tracking algorithm and “optimal” multi target tracking algorithm.

Linear multitarget procedure is applied to single scan single target tracking algorithm, IPDA to obtain LMIPDA [23], [22] multi target tracking algorithm. Single target and “optimal” multi target tracker used for comparison are IPDA [1], [6] and JIPDA [12] respectively. In the experiments labeled “JIPDA”, IPDA was applied to the set of non–confirmed tracks, and JIPDA was applied to the set of confirmed tracks. In spite of this, JIPDA experiments were limited to five targets only, due to the exponential computational requirements of JIPDA. In the experiments labelled “IPDA” and “LMIPDA”, the corresponding algorithm was applied to all tracks.

Each simulation experiment consists of 1000 runs, and each run consists of 40 scans. Two scan differencing track initiation [24] procedure is applied. All measurements in each two consecutive scans are used to initialize tentative tracks, provided certain restrictions are observed. One restriction is the maximum attainable target speed of 40$m$/s, and the other is that if both measurements are selected by the same existing track, they are not used to initiate new tracks. Thus both true tracks and false tracks are created in every scan of every simulation run. In addition, true tracks may become false tracks when they “lose” their targets, and false tracks may become true tracks when they start following a target. Probability of target existence, which is updated recursively in all algorithms simulated, is used as the track quality measure for the purpose of the false track discrimination.

When the probability of target existence rises above the conformation threshold, the tentative track becomes a confirmed track. When the probability of target existence falls below the termination threshold, the track is terminated. Additionally, at the end of each run all true tracks are terminated, and the false tracks were retained, to attain a stable field of false tracks. The confirmation thresholds are adjusted for each algorithm and each experiment separately, in order to obtain approximately equal number of confirmed false track scans, which was in the vicinity of 200 for each experiment (or approximately one in 200 scans). Once the confirmed false tracks are equalized, the true track statistics comparisons become meaningful. To put this number in perspective, 57 false tracks are initiated on the average in each scan of the fifteen targets experiments, and 123 false tracks are initiated on the average in each scan of the fifty targets experiments. The same set of simulated measurement data is applied to all algorithms.

Non–parametric versions of the algorithms are employed. The performance measures used to compare the algorithms are:

- target retention statistics of confirmed true tracks,
- false track discrimination, and
- average root mean square tracking error for selected experiments.

The target retention statistics were obtained by noting the identity of confirmed true track (if any) following each of the targets at scan 14. These identities are checked again at scan 35, and the following statistics are accumulated for each experiment:

- total number of cases (nCases) of target being followed by a confirmed track at scan 14,
- total number of tracks still following the original target (nOK) at scan 35,
- total number of tracks which end up following different targets (nSwitched) at scan 35,
- total number of tracks which did not make it to the scan 35 (nLost), and
- total number of tracks lost by merging (nMerged) with confirmed tracks which at scan 14 followed some other target.

False track discrimination statistics is presented with a curve showing the total number of targets which were followed by a confirmed true tracks, over the simulation run time. Thus, the value of “number of targets” = “number of simulation runs" denotes 100% probability that a target will be followed by a confirmed track.

A two dimensional surveillance situation is considered. The area under surveillance is 1000$m$ long and 1000$m$ wide, for the experiments involving five and fifteen targets, and 1400$m$ long and 1400$m$ wide for the fifty targets experiments. Clutter measurements satisfy a Poisson distribution, with uniform clutter measurements density of $10^{-4}$ measurements/scan/m².

Sensor is linear in Cartesian coordinates, and at each scan detects targets with the probability of detection for each target equal to 0.9. The sensor introduces measurement errors with the error covariance matrix of $25I_2$, where $I_2$ is the two dimensional identity matrix.

Number of targets is also a simulation parameter. Targets are initially positioned at the edges of a circle with center at (500,500) and radius 450, for the five and fifteen target experiments, and with center at (700,700) and radius 650 for the fifty targets experiments. Each target moves with the uniform speed towards the center, which they should reach in 20 scans. A random component is added to the speed vector of each target, thus at scan 20 the variance of the distance between each target and the center of the circle will be double sensor measurement error noise covariance matrix. In the five target scenario, the targets trajectories intersect at an angle of $10^\circ$. There are two fifteen target scenarios. In one, the targets trajectories intersect at $10^\circ$, and in the other the angular separation is $15^\circ$. Fifty targets initial positions are uniform at the edge of the circle; their angular separation is $7.2^\circ$. Trajectories in 3 consecutive simulation runs for the cases of five, fifteen (for the $10^\circ$ separation) and fifty targets are shown in Figures 1, 2 and 3 respectively. Due to the time constraints, the fifty target simulation experiments had only 250 runs.
The target retention statistics are summarized in Table I for five target experiments, in Table II for fifteen targets with $10^\circ$ angular separation, in Table III for fifteen targets with $15^\circ$ angular separation, and in Table IV for fifty targets experiments.

Figure 4 shows the number of confirmed true tracks over simulation run time, accumulated across runs, for the five targets experiment, Figure 5 and Figure 6 show the number of confirmed true tracks over time for the fifteen targets experiments, with the angular separation of $10^\circ$ and $15^\circ$ respectively, and Figure 7 shows the number of confirmed

### Table I

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<th>LMIPDA</th>
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### Table II

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### Table IV

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true tracks over time for the fifty targets experiment. False track discrimination properties of LM(IPDA) appear not to depend on the number of targets involved; it is affected by the separation angle between targets. In the five target experiment (Figure 4) LMIPDA appears to work better than JIPDA. It should be remembered, though, that JIPDA is applied to the set of confirmed tracks only, whereas LMIPDA is applied to all tracks.

Figures 8, 9, 10 and 11 show the average root mean square errors of confirmed true tracks over simulation run time, for the case of five targets, fifteen targets with the angular separation of 10°, fifteen targets with the angular separation of 15°, and for the case of fifty targets respectively. Fifty targets LMIPDA experiment took 22,860 seconds to execute, while the fifty targets IPDA experiment took 22,623 seconds to execute.

These experiments show that Linear Multitarget, although adding minimal relative penalty in terms of both computational resources and additional logic complexity, adds multitarget
capabilities to single target tracking algorithm, in this case IPDA. These capabilities are retained when tracking a large number of targets with crossing trajectories in substantial clutter.

IV. CONCLUSION

Linear Multitarget method greatly simplifies multitarget tracking. Not only is tracking more efficient, being linear in the number of tracks and the number of measurements involved, it is also much easier to understand and to implement. However, previous publications involved simulations with a very limited number of targets (two).

This publication presents a simulation study which simulates Linear Multitarget tracking in a demanding environment, in which “optimal” multitarget tracking methods cannot be applied for the reasons of computational resources. Whereas JIPDA limits appear to be between 5 and 10 targets scenario, LMIPDA was applied to the 50 target scenarios. These experiments show that LMIPDA is remarkably resilient to increases of the number of targets to very high levels. The additional computational resources needed relative to IPDA are minimal and relatively constant, and the performance in the terms of track retention and false track discrimination appear not to deteriorate with the increasing number of targets.

REFERENCES