Sequential Detection of Target Maneuvers∗
Jifeng Ru  Vesselin P. Jilkov  X. Rong Li  A. Bashi
Department of Electrical Engineering
University of New Orleans, New Orleans, LA 70148, USA
{jru, vjilkov, xli }@uno.edu, (504) 280-7416, 3950 (fax)

Abstract – This paper addresses target maneuver onset detection based on sequential statistical tests. Cumulative sums (CUSUM) type and Shiryayev sequential probability ratio (SSPRT) tests are developed by using a likelihood marginalization technique to cope with the difficulty that the target maneuver accelerations are unknown. The approach essentially utilizes a priori information about the maneuver accelerations in typical tracking engagements and thus allows to improve detection performance as compared with traditional maneuver detectors. Simulation results are presented that demonstrate the capabilities of the maneuver detectors developed.

Keywords: Target Tracking, Maneuver Detection, Hypothesis Testing, SPRT, CUSUM

1 Introduction

Most maneuver detectors developed for maneuvering target tracking are based on either the chi-square significance test or the generalized likelihood ratio test [5]. Six such maneuver onset detection algorithms have been studied recently in [12] using different scenarios. Another class of statistical tests that can be applied to maneuver detection is the sequential tests for change point detection, as pointed out in [5, 12]. Sequential detection procedures have been successfully applied to fault detection (see, e.g., [8, 15]) but not to maneuver detection, to our knowledge, except for a quickest detector in [16]. Some solutions based on min-max approach have been proposed in [9, 10, 11] with applications for navigation system integrity monitoring. It optimizes the worst case situations with given decision error rates.

For maneuver detection, sequential testing procedures are actually preferable because measurements are available sequentially. Moreover, a sequential test does not need to determine the sample size in advance unlike the non-sequential tests. A sequential test involves a stopping rule and a final decision to achieve a trade-off between sample size and decision accuracy. We consider detecting a target’s maneuver as a binary hypothesis testing problem. Once a target starts maneuvering, it should be detected as quickly as possible under certain constraints such as the rates of false alarms and missed detections. It is well known that for binary simple hypothesis testing, Wald’s sequential probability ratio test (SPRT) is optimal in the sense that it makes the quickest detection under both hypotheses \( H_0, H_1 \) given any fixed false alarm and missed detection probabilities. The SPRT makes the decision by comparing the likelihood ratio with two thresholds, which are obtained to guarantee the specified error probabilities. Recently the Shiryayev sequential probability ratio test (SSPRT) has been applied to the fault detection problem. The SSPRT is optimal in the sense that it provides the quickest detection of a change in a sequence of conditionally independent measurements under the given decision error rates. The SPRT and SSPRT are different in problem formulation. In particular, the SPRT assumes all data relates to one of the two hypotheses so that it simply chooses one of the hypotheses that there is a maneuver or that there is no maneuver. The SSPRT does not make this assumption and tests for an occurrence of a change. One note to emphasize is that both SPRT and SSPRT hold the optimality only when there is no unknown parameter, which is often not the case for maneuver detection where maneuver magnitude is usually unknown. In order to apply these sequential tests, modifications have to be made.

This paper presents two target maneuver onset detectors based on sequential tests. Cumulative Sums (CUSUM) type and SSPRT tests are developed by using a likelihood marginalization technique to cope with the difficulty that target maneuver accelerations are unknown. The proposed approach essentially utilizes a priori information about the maneuver accelerations in typical tracking engagements and thus allows to improve the detection performance, especially for normal accelerations, as compared with two widely used maneuver detectors.

The rest of the paper is organized as follows. In Sec. 2, the problem of maneuvering target detection is formulated as binary composite hypothesis testing. In Sec. 3, sequential detection procedures based on a repeated-SPRT and SSPRT are formulated and discussed. In Sec. 4, test statistics are derived by means of likelihood marginalization using two typical prior models of maneuvers. The performance of the proposed maneuver detectors is evaluated by simulations and compared with that of two widely used detectors in Sec. 5. Conclusions are provided in Sec. 6.
2 Problem Formulation

The target-measurement model is given by

\[ x_{k+1} = F_k x_k + u_k + w_k \]
\[ z_k = H_k x_k + v_k, \quad k = 1, 2, \ldots \]

where \( x_k \) is the target state, \( u_k \) is the maneuver control input, and \( z_k \) is the measurement. \( w_k \sim \mathcal{N}(0,Q_k) \) and \( v_k \sim \mathcal{N}(0,R_k) \) are independent process and measurement noises, respectively, and the initial state \( x_0 \sim \mathcal{N}(0,P_0) \) is independent of \( w_k \) and \( v_k \).

It is assumed that \( u_k = 0 \) when the target is not maneuvering and \( u_k \neq 0 \) when the target is maneuvering. If the target begins a maneuver at an unknown time \( n \leq k \) then

\[ (u_k) = \{ \ldots, 0, \ldots, 0, u_n, u_{n+1}, \ldots, u_k \} \]

In general, it is not necessary for \( u \) to remain constant during the maneuver. The focus of the maneuver onset detection is to decide on a maneuver and estimate the onset time \( n \), which can be formulated as a binary hypothesis testing problem:

\[ H_0 : \quad u_m = 0 \quad \text{for} \quad m = 1, \ldots, k \]
\[ H_1 : \quad \left\{ \begin{array}{l}
  u_m = 0 \quad \text{for} \quad m = 1, \ldots, n - 1 \\
  u_m \neq 0 \quad \text{for} \quad m = n, \ldots, k
\end{array} \right. \]

where \( u \) and \( n \) are unknown parameters, and \( n \) is referred to as the change point. This is known as change point detection in the statistical literature.

Since both \( u \) and \( n \) are usually unknown in practice, hypothesis \( H_1 \) is clearly composite. This makes the maneuver detection difficult since in general there is no existing optimal non-Bayesian solution for composite hypothesis testing problems.

3 Sequential Detection of Maneuvers

3.1 Repeated SPRT-based Detector

Before proposing sequential test procedures for target maneuver detection, we briefly present the SPRT for binary simple hypothesis testing. Let

\[ L^k = \log \frac{f(z^k|H_1)}{f(z^k|H_0)} = \sum_{\kappa=1}^{k} \log \frac{f(z_{\kappa}|H_1, z^{\kappa-1})}{f(z_{\kappa}|H_0, z^{\kappa-1})} \]

denote the log-likelihood ratio of two simple hypotheses \( H_1 \) and \( H_0 \) based on the measurements up to \( k \), where \( z^k \triangleq (z_1, \ldots, z_k) \), \( f(z^k|H_1) \) and \( f(z^k|H_0, z^{\kappa-1}) \) (\( \kappa = 0, 1 \)) are the joint and marginal likelihoods of the hypotheses, respectively. Further, let \( A \) and \( B \) be two thresholds such that \( P(\sim H_1^*|H_0) \leq \alpha, P(\sim H_0^*|H_1) \leq \beta \) (\( 0 < \alpha, \beta < 1 \)), where “\( H_i^* \)” stands for “deciding on \( H_i \)”. Then the SPRT decision rule is

\[ \begin{align*}
  &\text{Accept } H_1 \quad \text{if } L^k \geq \log B \\
  &\text{Accept } H_0 \quad \text{if } L^k \leq \log A \\
  &\text{Continue } (k \mapsto k+1) \quad \text{otherwise}
\end{align*} \]

Clearly, the standard SPRT presented above cannot be applied directly to maneuver onset detection as formulated by (3). We modify this test to address several issues next.

First, the SPRT assumes that measurements are independent. However, measurements are correlated in the target tracking problem (1)-(2). In this case the above test still works provided the sequence \( (l_k) \) of marginal likelihood ratios is independent, where

\[ l_k = \frac{f(z_k|H_1, z^{k-1})}{f(z_k|H_0, z^{k-1})} \]

Fortunately, this is approximately the case since the measurement residual sequence is approximately Gaussian distributed and weakly coupled under some conditions [4]. Thus measurement residuals, instead of measurements, should be used to compute likelihood ratios.

Second, the SPRT only decides which hypothesis is to accept. Once \( H_0 \) or \( H_1 \) is accepted the test will terminate. However, this is not the goal of maneuver onset detection. In fact, we want to know “when \( H_1 \) occurs”. Therefore, the test should continue to the next cycle with more measurements if \( H_0 \) is deemed true, that is, we should keep restarting (resetting) the SPRT as long as \( H_0 \) is accepted. The standard CUSUM algorithm was first proposed by Page for classical change detection problems. It guarantees the quickest decision given the decision error rate for simple hypotheses, i.e., if \( u \) is known. Page’s test can be interpreted as a repeated SPRT with the lower threshold \( \log A \) equal to 0 and the upper threshold equal to \( \lambda \) determined by error probabilities. The key idea is to restart the SPRT algorithm once \( H_0 \) is accepted, which makes it naturally fit our problem. The CUSUM algorithm [1] can be written in a recursive manner:

\[ L^k = \max \left\{ L^{k-1} + \log \frac{f(z_k|H_1, z^{k-1})}{f(z_k|H_0, z^{k-1})}, 0 \right\}, \quad L^0 = 0 \]

and the decision rule is

1) Accept \( H_1 \) (declare maneuver) if \( L^k \geq \lambda \). Then stopping time \( \hat{n} = \min \{k : L^k \geq \lambda\} \) is the time maneuver is detected.

2) Continue the test \( (k \mapsto k + 1) \) if \( L^k < \lambda \).

Compared with the standard SPRT, it is shown that two tests differ only in the value of the lower thresholds: SPRT uses \( A \) determined by two error probabilities while CUSUM uses 0 which was also formally proven for its optimality.

Third, the SPRT is optimal only for simple hypothesis testing, which is obviously not the case for maneuver detection since the maneuver input \( u \) is usually unknown. This makes the marginal likelihood ratio in the test statistics unknown. There are two common approaches to solving this problem. In the first one, the generalized likelihood ratio (GLR) test, the unknown \( u \) (treated as nonrandom) is replaced with the maximum likelihood estimate \( \hat{u} \) (see [5]). The second technique is a Bayesian approach, where \( u \) is
treated as random and the likelihood \( f(\hat{z}_k|H_1) \) is determined by an appropriate marginalization—averaging over all possible values of the unknown \( u \) [2]. Specifically,

\[
f (\hat{z}_k|H_1, z^{k-1}) = E \left[ f (\hat{z}_k|H_1 (u), z^{k-1}) \right] = \int f (\hat{z}_k|H_1 (u), z^{k-1}) f (u) du \quad (7)
\]

where \( f (u) \) is the a priori probability density function (PDF) of \( u \).

Clearly, this approach provides a good means of utilizing the a priori information if it is available. A major difficulty for its application is how to evaluate the integral in (7) which is rarely possible analytically. In Sec. 4 we will use typical prior distributions of target accelerations and propose a fairly general way to approximately evaluate this expectation integral based on a Gaussian sum approximation of the prior PDFs.

### 3.2 SSPRT-based detector

The SSPRT focuses on the detection in a series of conditionally independent measurements by noting the change in the probability density function of the measurement. Good results have recently been reported for fault detection using SSPRT [8, 15], however we are not aware of any work that addresses the adaptation and effectiveness of SSPRT for maneuver detection.

The SSPRT minimizes an expected cost at each time step. This cost includes the measurement cost and the cost due to a terminal decision error by false alarm or miss detection. The decision function can be written in terms of a likelihood ratio or posterior probability, which are equivalent once a prior is known. Since a Bayesian framework is used, it is necessary to define prior information. This includes the prior probabilities of \( H_i \) and the transition probabilities of \( H_0 \) to \( H_1 \) from \( k-1 \) to \( k \). When the maneuver onset detection is formulated as binary hypothesis testing, the transition probabilities degenerate into a single number, assumed to be time-invariant for simplicity.

The decision rule of the SSPRT is obtained by defining the posterior probability ratio \( P_k \). Let \( p_k = P \{ n \leq k | z^k \} \) denote the posterior probability that a maneuver starts at an unknown time \( n \) at or before time \( k \) given the available measurements \( z^k \), and

\[
P_k \triangleq \frac{p_k}{1-p_k}, \quad P_T \triangleq \frac{p_T}{1-p_T} \quad (8)
\]

where \( P_T \) is a preset threshold. Then the SSPRT detector either declares a maneuver if \( P_k \geq P_T \) or takes another measurement if \( 0 \leq P_k < P_T \). The choice of \( p_T \) is related to the desirable decision error rate. More details and examples are discussed in [8, 15].

Clearly, calculating the posterior probability is the key to SSPRT. Fortunately, \( p_k \) can be calculated recursively. Let \( \phi_k \) denote the prior probability of hypothesis \( H_1 \) being true \( (i = 0, 1), \pi \) the transition probability from \( H_0 \) to \( H_1 \), and \( \phi_k^1 \triangleq P \{ n \leq k | z^k \} \). The recursive form of the posterior probability of \( H_i \) is given by

\[
p_k^1 = \frac{\phi_{k-1}^1 f (\hat{z}_k|H_1, z^{k-1})}{\sum_{i=0}^{\pi} \phi_{k-1}^i f (\hat{z}_k|H_i, z^{k-1})}, \quad p_k^0 = 1 - p_k^1 \quad (9)
\]

\[
\phi_{k-1}^0 = \pi (1 - \phi_{k-1}^1), \quad \phi_{k-1}^1 = 1 - \phi_{k-1}^0 \quad (10)
\]

Then, the test statistic of the SSPRT is

\[
P_k \triangleq \frac{p_k^1}{p_k^0} = \frac{f(\hat{z}_k|H_1, z^{k-1}) P_{k-1} + \pi}{f(\hat{z}_k|H_0, z^{k-1})}/1 - \pi, \quad P_0 \triangleq \frac{p_0^1}{p_0^0} \quad (11)
\]

and the SSPRT becomes

1) Accept \( H_1 \) (declare maneuver) if \( P_k \geq P_T \)

2) Continue the test \( (k \rightarrow k + 1) \) if \( P_k < P_T \).

Note that no reset mechanism is necessary for the SSPRT due to its nature of the problem formulation that determines when a disruption \( H_1 \) is true. Its recursive nature and the problem formulation make it well suited to maneuver detection.

### 4 Maneuver Detection Algorithms

#### 4.1 Maneuver model

A 2D target maneuver acceleration vector can be decomposed along two directions: tangential (along the velocity vector) and normal (orthogonal to the velocity vector). Such a target maneuver motion can be described as

\[
x_{k+1} = F x_k + \Gamma(x_k) a_k + w_k \quad (12)
\]

where the state in the Cartesian coordinates is \( x = [x, y, \dot{x}, \dot{y}]' \), and the acceleration is \( a = [a_t, a_n]' \) with tangential and normal components \( a_t \) and \( a_n \), respectively. The system matrices are

\[
F = \text{diag} \{ F_2, F_2 \}, \quad \Gamma(x) = \text{G} \text{Y}(x), \quad \text{G} = \text{diag} \{ G_2, G_2 \}
\]

\[
F_2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}, \quad \text{Y}(x) = \begin{bmatrix} c(x) & -s(x) \\ s(x) & c(x) \end{bmatrix}
\]

where \( \text{Y}(x) \) is the rotation matrix with

\[
c(x) = \cos \phi = \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \quad s(x) = \sin \phi = \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}
\]

that maps \( a_k \) to the Cartesian coordinates, and \( \phi = \arctan(\dot{x}/\dot{y}) \) is the target heading angle. A target maneuver is described in this system model through the control input \( u_k = \Gamma(x_k)a_k \). Different models of the acceleration \( a_k \) can be used [6] depending on the maneuver capabilities of the targets of interest in the tracking application. To be more specific we develop the maneuver detectors for manned maneuvering aircraft in mind.

The normal acceleration is induced by the lift forces and is usually the dominant one during maneuver. Its direction is determined by the target aspect angle and its magnitude
Appendix A) as development of the maneuver detector, can be easily derived (see nominal some mined a posteriori

Fig. 1: Asymmetric PDFs of normal acceleration. can be modeled as a colored random process with an asymmetrical distribution. It was proposed in [3] that the normal acceleration $a_n(t) = \alpha + \beta e^{\gamma t}$ where $\alpha, \beta, \gamma$ are design parameters, depending on the particular target type and $b(t)$ is a zero-mean first-order Gauss-Markov process. The marginal probability density function, essential for the further development of the maneuver detector, can be easily derived (see Appendix A) as

$$f(a_n) = \frac{1}{\sqrt{2\pi} |\gamma(a_n - \alpha)|} e^{-\frac{1}{2\gamma}(\ln \frac{a_n - \alpha}{\gamma})^2} \quad (13)$$

For three typical choices of $\alpha, \beta, \gamma$, this highly asymmetrical density is shown in Fig. 1, including one with $(\alpha, \beta, \gamma) = (8, -4, 0.5)$ that is considered typical of modern piloted aircraft in evasive maneuvers. This model is more accurate than the usual symmetrical models [6] (e.g., the Singer model) at the cost of designing these parameters, which requires knowledge of the target type, obtained either a priori or a posteriori (e.g., with the help of an image-sensor-aided target classification).

The tangential acceleration is determined by the thrust-minus-drag force. Various random process models for its magnitude are discussed and analyzed in detail in [6]. Most important for our maneuver detection approach is the choice of marginal PDF of the process. Purely a priori symmetrical models (such as the ternary-uniform density in the Singer model) or a posteriori, adaptive (such as the conditional Rayleigh density in the mean-adaptive “current” model) are possible candidates. In this paper we use Gaussian marginal PDF model

$$f(a_t) = \mathcal{N}(a_t; \bar{a}_t, \sigma^2_t) \triangleq \frac{1}{\sqrt{2\pi}} e^{-\frac{(a_t - \bar{a}_t)^2}{2\sigma^2_t}} \quad (14)$$

with parameters $\bar{a}_t, \sigma^2_t$ either specified a priori (e.g., with some nominal values for the targets of interest) or determined a posteriori through filter-based estimates $\bar{a}_t = \bar{a}_t(k|k-1), \sigma^2_t = \hat{\sigma}^2_t(k|k-1)$. Both cases were investigated. It should be noted that the Gaussian sum approximation technique used to cope with the non-Gaussian $f(a_n)$ in the next section can be also applied to $f(a_t)$ if a non-Gaussian model is adopted.

Under the assumption of independence between tangential and normal maneuvers, the PDF of the total acceleration vector is $f(a) = f(a_t, a_n) = f(a_t)f(a_n)$.

For the particular implementation of the maneuver detectors, in the sequel it is assumed also that direct position measurements of the target are available, i.e., $H = \text{diag}\{[1 \ 0], [1 \ 0]\}$ in (2).

4.2 Test statistics

As discussed in Sec. 3, the key to computing test statistics of the detectors is to obtain likelihood functions $f(\hat{z}_k|H_i, z^{k-1})$ under each hypothesis ($i = 0, 1$).

Denote by $\hat{z}_k$ and $S_k$ the measurement residual and its covariance, respectively, which are provided at time $k$ by a nonmaneuver Kalman filter (i.e., $u_\kappa = 0, \kappa = 0, 1, \ldots, k - 1$) for the system (1)-(2).

The marginal likelihood of $H_0$ is then (under the linear Gaussian assumptions)

$$f(\hat{z}_k|H_0, z^{k-1}) = \mathcal{N}(\hat{z}_k; 0, S_k) = \frac{1}{|2\pi S_k|^\frac{1}{2}} e^{-\frac{1}{2} \hat{z}_k^T S_k^{-1} \hat{z}_k} \quad (15)$$

To obtain the likelihood $f(\hat{z}_k|H_1, z^{k-1})$ of $H_1$, we implement the marginalization approach (7) discussed in Sec. 3 with the acceleration PDF models (13)-(14). Specifically, we write $f(\hat{z}_k|H_1, z^{k-1}) = f(\hat{z}_k|H_1, \hat{x}_{k-1}k-1)$. Then, according to (7) and (12) we have

$$f(\hat{z}|H_1, \hat{x}) = E[f(\hat{z}|H_1, \hat{x}, a)]$$

$$= \int \mathcal{N}(\hat{z}; H\Gamma(\hat{x})a, S)f(a)da$$

$$= \int \int \mathcal{N}(\hat{z}; H\Gamma(\hat{x})(a_t, a_n), S)f(a_t)f(a_n)da_t da_n \quad (16)$$

where $f(a_t)$ and $f(a_n)$ are given by (13) and (14), respectively.

However, due to the complex prior density $f(a_n)$, the integral in (16) has no analytical solution available, which makes the implementation of the proposed detectors highly complicated. To overcome this difficulty we propose to employ the Gaussian sum approximation techniques (see, e.g., [14]). That is, we use

$$f(a_n) \approx \sum_{i=1}^{N} \lambda_i \mathcal{N}(a_n; \bar{a}^{(i)}_n, \sigma^{(i)}_n) \quad (17)$$

where $N$ is the number of Gaussian components, $\lambda_i$ are their weights ($0 < \lambda_i < 1$, $\sum_{i=1}^{N} \lambda_i = 1$) and $\bar{a}^{(i)}_n$, $\sigma^{(i)}_n$ are their means and variances, respectively. The determination of these parameters is a part of the design and does not require on-line approximation. For our implementation of the maneuver detectors we obtained a fairly accurate approximation of the asymmetric $f(a_n)$ of (13) by a sum of only two components, which is illustrated in Fig. 2. We used standard Matlab functions for nonlinear multidimensional optimization to obtain a locally best fit. Other expectation maximization (EM) based mixture estimation techniques can also be used. Higher accuracy can be obtained by using more components of the sum. In general,

\footnote{We drop the time index to simplify the notation.}
The standard 2D curvilinear-motion model

5.1 Ground-truth model

generated by a curvilinear-motion model. All results are

Four scenarios were designed to compare different aspects

5 Simulation

where

\[ \int \int \int \mathcal{N}(\hat{z}; H \Gamma(\hat{x})[a_t, a_n] + S) \mathcal{N}(a_t; \bar{a}_t, \sigma_t^2) \mathcal{N}(a_n; \bar{a}_n, \sigma_n^2) \, da_t \, da_n \] (19)

As derived in the Appendix B, \( I_i(\hat{z}; \hat{x}) \) is explicitly deter-

\[ I_i(\hat{z}; \hat{x}) = \int \int \mathcal{N}(\hat{z}; H \Gamma(\hat{x})[a_t, a_n] + S) \mathcal{N}(a_t; \bar{a}_t, \sigma_t^2) \mathcal{N}(a_n; \bar{a}_n, \sigma_n^2) \, da_t \, da_n \] (19)

where \( \bar{a}_t = [\bar{a}_t, \bar{a}_n, \bar{a}_n(t)] \) and \( \Lambda(t) = \text{diag} \{ \sigma_t^2, \sigma_n^2 \} \).

5 Simulation

Four scenarios were designed to compare different aspects

of the detection algorithms using ground truth trajectories

generated by a curvilinear-motion model. All results are

averages over 100 Monte Carlo runs.

5.1 Ground-truth model

The standard 2D curvilinear-motion model

\[ \begin{align*}
\dot{x}(t) &= v(t) \cos(\phi(t)) + w_x(t) \\
\dot{y}(t) &= v(t) \sin(\phi(t)) + w_y(t) \\
\dot{v}(t) &= a_t(t) + w_v(t) \\
\dot{\phi}(t) &= a_n(t)/v(t) + w_{\phi}(t)
\end{align*} \] (21-24)

was used to generate the ground truth trajectories, where

\( x, y, v, \phi \) denote the target position, speed and heading. This model is fairly general since it accounts for along- and cross-track accelerations.

\[ x_0 \sim \mathcal{N}(\bar{x}_0, \sigma_x^2), \quad \bar{x}_0 = 60\text{km}, \quad \sigma_x = 5\text{km} \]
\[ y_0 \sim \mathcal{N}(\bar{y}_0, \sigma_y^2), \quad \bar{y}_0 = 90\text{km}, \quad \sigma_y = 5\text{km} \]
\[ v_0 \sim \mathcal{N}(\bar{v}_0, \sigma_v^2), \quad \bar{v}_0 = 300\text{m/s}, \quad \sigma_v = 5\text{m/s} \]
\[ \phi_0 \sim \mathcal{N}(\bar{\phi}_0, \sigma_{\phi_0}), \quad \bar{\phi}_0 = 45\text{deg}, \quad \sigma_{\phi_0} = 1\text{deg} \]

with

\[ \sigma_{w_x} = 5m, \sigma_{w_y} = 5m, \sigma_{w_v} = 1\text{m/s}, \sigma_{w_{\phi}} = 0.01\text{deg} \]

The initial states used for each scenario were generated as

The scenarios included deterministic and random ones.

The target track length starts from \( k = 30 \) and continued for another 70sec. The sampling time \( T = 1s. \)

(DN) Deterministic normal acceleration scenario. The
target makes a normal maneuver during the period \( k = [80, 100] \) with a magnitude of \( 20\text{ m/s}^2 \).

(DT) Deterministic tangential acceleration scenario. The
target makes a tangential maneuver during the period \( k = [80, 100] \) with a magnitude of \( 20\text{ m/s}^2 \).

(RN) Random normal acceleration scenario. In order to
test detectors on multiple maneuvering scenarios simultane-
ously, a random maneuver scenario was developed, where
the magnitude of the normal acceleration was fixed

during \( k = [80, 100] \) but random over runs: \( a_n \sim f(a_n) \) as
given by (13) with \( (\alpha, \beta, \gamma) = (4, -2, 0.5) \).

5.2 Simulation results

There were some parameters to be designed. The threshold
for each detector was determined by simulations with
given false alarm rate \( P_{fa} = 1\% \). Two point differencing
was used to initialize the Kalman filter in state estimation.
The corresponding noise covariances were \( Q = (0.5)^2 I \) and \( R = (50)^2 I \).

For the SSPRT detector, the prior probability for \( H_1 \) be-
ing true was 0.01 and transition probability \( \pi \) from \( H_0 \)
to \( H_1 \) was 0.005. In addition, prior densities for testing
normal accelerations were used by the CUSUM and
SSPRT detectors: \( f(a_t) = \mathcal{N}(a_t; 0, (0.5g)^2) \) and \( f(a_n) \approx
0.44\mathcal{N}(a_n; 0.65g, (1.3g)^2) + 0.56\mathcal{N}(a_n; 2.2g, (0.6g)^2) \)
which were determined by local fit. Such design is obvi-
ously better to detect normal maneuvers since prior \( f(a_t) \)
will not affect normal accelerations much. It is possible
that we design a model with prior densities particularly
good for tangential accelerations. In this paper, we de-
sign \( f(a_t) = \mathcal{N}(a_t; 0, (3.5g)^2) \) and \( f(a_n) \) with zero mean
and very small variance to test tangential accelerations. We
could have different design if we had better prior infor-
mation for the tangential accelerations. For targets with both
maneuvers, naturally we consider combination with two or
more models, which is under our study [13].

The CUSUM and SSPRT represent the two proposed de-
tectors. The MR represents a detector based on measure-
ment residuals. The IE represents one based on input esti-

mation technique. Details for MR and IE can be found in
[5, 12]. The size of data window used in IE and MR de-
tectors was 5. The performance of four detectors was com-
pared in terms of average onset detection delay \( (\bar{n} - n) \),
receiver operating characteristics (ROC) curves and com-
putational load.
5.2.1 Average onset detection delay

The average onset detection delay under four scenarios is shown in Table I. It is clear that SSPRT detectors have smaller detection delay in both scenarios with normal accelerations as well as tangential acceleration. For the cases of tangential acceleration, the performance of the CUSUM and SSPRT detectors should be improved if we have the proper prior knowledge for tangential accelerations.

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<thead>
<tr>
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<th>DN</th>
<th>RN</th>
<th>DT</th>
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<tbody>
<tr>
<td>CUSUM</td>
<td>5.33</td>
<td>5.57</td>
<td>5.75</td>
</tr>
<tr>
<td>SSPRT</td>
<td>4.68</td>
<td>4.69</td>
<td>5.10</td>
</tr>
<tr>
<td>MR</td>
<td>5.19</td>
<td>5.12</td>
<td>5.15</td>
</tr>
<tr>
<td>IE</td>
<td>5.04</td>
<td>5.03</td>
<td>5.11</td>
</tr>
</tbody>
</table>

Furthermore, the SSPRT detector also provides the posterior probability of a maneuver. The posterior hypothesis probability for scenario DN is shown in Fig. 3. We note that posterior probability of $H_1$ quickly increases after the change time. Similar results were observed for scenario DT.

5.2.2 ROC curves

The ROC curves for deterministic normal acceleration scenarios were generated by computing the $P_d$ at the time of interest with different $P_{fa}$ using 100 Monte Carlo runs. The ROC curves for each maneuver onset detector at $k = 83$ and $k = 84$ for scenario DN are given in Fig. 4 and Fig. 5, respectively. It is apparent from the curves that SSPRT has the best performance for tested scenarios. The CUSUM detector outperforms IE and MR detectors for scenario DN with $P_{fa} > 0.2$. Note that IE performed better as time goes due to the improved accuracy of the input estimate as more maneuver data become available. For the scenario DT, shown in Fig. 6 for ROC curve at $k = 83$, IE and SSPRT have compatible performance when $P_{fa}$ is really small. Clearly, the ROC curves verify what we observed for maneuver onset detection delay.

5.2.3 Computational complexity

The computational complexity of different algorithms were compared by the ratio of the CPU processing time per iteration in Table II. All other algorithms were compared with MR detector which is the simplest. It shows that the proposed detection algorithms have a much less computational load than the IE-based algorithms.

<table>
<thead>
<tr>
<th></th>
<th>CPU(s)</th>
<th>MR</th>
<th>CUSUM</th>
<th>SSPRT</th>
<th>IE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2.97</td>
<td>3.27</td>
<td>7.61</td>
<td></td>
</tr>
</tbody>
</table>

6 Conclusions

In this paper, two sequential detectors for maneuver onset have been developed and evaluated over various scenarios designed for different aspects of maneuvers. The proposed detection algorithms are based on sequential likelihood ratio test procedures, CUSUM and SSPRT, and use properly marginalized likelihoods to cope with the unknown magnitude of maneuver accelerations – a key difficulty in maneuver onset detection. Both CUSUM and SSPRT detection
algorithms developed are explicit, recursive and general. Utilizing prior distributions of the accelerations is an essential task involved in the application of the proposed scheme. Performance evaluation and comparison have demonstrated that the proposed detectors are more effective than two popular detectors when appropriate prior distributions of accelerations are known.

**Appendix A: Proof of (13)**

According to (19) the PDF $f_Y(y)$ of $Y = g(X)$ can be determined from the PDF $f_X(x)$ of $X$ by

$$f_Y(y) = \sum_i \frac{f_X(x_i)}{|g'(x_i)|}$$

where $g'(x) = \frac{df}{dx}g(x)$ and $x_i$’s are the real roots of the equation $y = g(x)$ in terms of $y: y = g(x_i)$.

Let $g(b) = \alpha + \beta e^{\gamma b}$. Then $g'(b) = \beta \gamma e^{\gamma b}$. The root of $a_n = \alpha + \beta e^{\gamma b}$ is $b_1 = \frac{1}{\gamma} \ln \frac{a_n - \alpha}{\beta \gamma}$. Thus we have (13)

$$f(a_n) = \frac{f(b_1)}{|g'(b_1)|} = \frac{N(\frac{1}{\gamma} \ln \frac{a_n - \alpha}{\beta \gamma}; 0, 1)}{|\gamma(a_n - \alpha)|} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\gamma^2}(\ln \frac{a_n - \alpha}{\gamma \beta})^2}$$

**Appendix B: Proof of (20)**

According to (19)

$$I_t(\tilde{z}, \hat{z}) = \int \frac{1}{\sqrt{2\pi S}} e^{-\frac{1}{2}(\tilde{z} - H\Gamma a)'S^{-1}(\tilde{z} - H\Gamma a)} f(a)da$$

$$= \int \frac{1}{\sqrt{2\pi S}} e^{-\frac{1}{2}(\tilde{z} - H\Gamma a)'S^{-1}(\tilde{z} - H\Gamma a)} \frac{1}{\sqrt{2\pi \Lambda}} e^{-\frac{1}{2}(a - \bar{a})' \Lambda^{-1}(a - \bar{a})} da$$

Let $m = H\Gamma a$, $\Lambda_m = H\Gamma \Lambda (H\Gamma)'$, and $\bar{m} = H\Gamma \bar{a}$. Then $dm = |H\Gamma| da$, and

$$I_t(\tilde{z}, \hat{z}) = \frac{1}{\sqrt{[(2\pi)^2S\Lambda_m]}} \int e^{-\frac{1}{2}(\tilde{z} - \bar{a})'S^{-1}(\tilde{z} - \bar{a}) + (m - \bar{m})' \Lambda_m^{-1}(m - \bar{m})} dm$$

Define

$$C_1 \triangleq S^{-1} + \Lambda_m^{-1}, C_2 \triangleq S^{-1} \tilde{z} + \Lambda_m^{-1} \bar{m}, C_3 \triangleq \tilde{z}'S^{-1} \tilde{z} + m'\Lambda_m^{-1} m, D \triangleq m'C_1 m - 2m'C_2 + C_3$$

and let $t = C_1^{1/2}(m - C_1^{-1/2}C_2)$, $dt = C_1^{-1/2}dm$. Then

$$I_t(\tilde{z}, \hat{z}) = \frac{1}{\sqrt{2\pi SC_1\Lambda_m}} \int e^{-\frac{1}{2}t't} \frac{1}{\sqrt{2\pi}} \int e^{-\frac{1}{2}t't} dt$$

It can be directly verified that

$$c_3 - C_2' C_1^{-1} C_2 = (\tilde{z} - \bar{m})'(S + \Lambda_m)^{-1}(\tilde{z} - \bar{m})$$

$$SC_1 \Lambda_m = S + \Lambda_m$$

Thus

$$I_t(\tilde{z}, \hat{z}) = \frac{1}{\sqrt{|2\pi SC_1\Lambda_m|}} e^{-\frac{1}{2}t'(c_3 - C_2' C_1^{-1} C_2)}$$

$$= N(\tilde{m}, S + \Lambda_m)$$

**References**


