A New Approach to Grid Adaption of AGIMM Algorithm

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Abstract — Grid adaption is one of several approaches for model-set adaption in variable structure multiple model(VSMM) algorithm. Compared with other approaches, it is unique one with ability to produce new model-set that are not predefined and model parameter can be any value within continuous mode space. In order to improve estimation performance of system mode transition stage and steady-state stage, a new approach to grid adaption is presented in this paper. In contrast to existed grid adaption methods, grid adaption is achieved through using both model posterior probability of last scan and model prediction probability of current scan and thus information of posterior expected-mode of previous scan and predicted expected-mode of current scan are both utilized. Performance of proposed method is evaluated via two simulated example of maneuvering target tracking problem. Simulation results indicate that performance of the AGIMM algorithm, in which our new grid adaption approach is adopted, actually outperform those of algorithms in which only one kind probability is applied.

Keywords: Grid adaption, variable structure, IMM, target tracking, maneuvering target.

1 Introduction

The multiple model (MM) approach have come into vogue in hybrid state estimation field and interesting multiple model (IMM) algorithm is thought as the most cost-effective one for maneuvering target tracking. The early MM algorithms, including static MM, GPBn and IMM, have a fixed structure since fixed model-set is used all the while, however the set of possible value of unknown system parameters is continuous. A small, fixed structure model-set is not enough to accurately cover continuous variable system parameter, on the other hand use of more models does not guarantee better performance [1]. To circumvent this problem, MM algorithms with variable structure model-set are developed by Li and his team [1, 3, 4, 6, 7, 8, 9] and has been got several successful application, including nonstationary noise identification and ground target tracking with constraint [2].

VSMM algorithms have a two-level hierarchical structure corresponding respectively model-set sequence and model sequence. Optimal implementation of VSMM algorithms which have need to consider all possible model-set sequences and model sequences is computationally infeasible, so in almost all practical VSMM algorithms only single model-set sequence which can be obtained through model-set adaption is taken into account. Model-set adaption, which involves determination of multiple candidate model-sets and selection of best model-set from candidate sets, is considered as prevailing approach for VSMM in next few years [1, 6]. In [1], multiple model Markov switching system is described as a stochastic digraph, referred as supporting digraph, in which system modes and possible mode switching are regarded as vertices and edges of a graph, respectively. Three different schemes of graph-theoretic formulation about model-set adaption are also presented, including active digraph(AD), digraph switching(DS) and adaptive grid(AG). Existed model-set adaption approaches fall into three categories, including model-group switching (MGS) [6, 8], likely-model set (LMS) [3, 7] and grid adaption (GA) [5, 6]. They are the embodiments of idea of AD, DS and AG schemes, respectively.

MGS selects the new model group among a number of predefined model groups by hard decision while LMS identify each model in \( M_{k-1} \) as unlikely, principal or significant and obtain \( M_k \) by discarding all unlikely models in \( M_{k-1} \) and activating all neighbors of any principal model of \( M_{k-1} \), here \( M_k \) denotes model-set of the \( k \)th scan.

Compared with MGS and LMS approaches, GA is
unique one for its ability to produce new model-set, which are not predefined and model parameter can be any value within continuous mode space, in real time. Grid quantizes mode space unevenly and adaptively. A coarse grid is set initially, and then adjustment of grid spacing is made according to measurement and prior information. VSMM estimator with grid adaption is usually referred as AGIMM algorithm. As shown in [3], it is suggested to employ the expected mode as the center of an adaptive grid. In [5], for maneuvering target tracking problem, two VSMM algorithm with different model-set adaption approaches, SGIMM and AGIMM, have been designed, performance of the two algorithms are also accessed. Recently, an expected-mode augmentation (EMA) VSMM algorithm is proposed in [9], basic idea of the algorithm is that model-set is augmented by an expected-mode as $M_k = E_k \cup (M_{k-1} - E_{k-1})$ at every time step, here $E_k$ represent set of model match expected-mode of the $k$th scan. In our opinion, it generalizes idea of GA approach.

In [5], center grid of current scan is calculated as weighted combination of model parameters of model-set of last scan while weight coefficient is either model posterior probability of last scan or model prediction probability of current scan. In order to improve estimation performance of both mode transition stage and steady-state stage, a new approach to grid adaption is presented in this paper. In contrast to existed grid adaption methods, grid adaption is achieved through using both model posterior probability of last scan and model prediction probability of current scan and thus information of posterior expected-mode of previous scan and predicted expected-mode of current scan are both utilized.

The remaining part of this paper is organized as follows. Section 2 summarizes briefly basic idea of the AGIMM algorithm designed by Jlikov. In section 3, a new approach to grid adaption of AGIMM algorithm is presented. In section 4, simulation result showing the comparison of estimation performance using new algorithm vs existed algorithm are presented. Finally, conclusions are given section 5.

2 AGIMM algorithm and discussion

2.1 Summary of AGIMM

In [5], a design of AGIMM algorithm for tracking maneuvering target in 2D coordinate turn system is given. CT model with known turn rate is chosen as target dynamic model, system transition matrix is

$$F(\omega) = \begin{bmatrix} 1 & \frac{\sin \omega T}{\omega} & 0 & -\frac{1-\sin \omega T}{\omega} \\ 0 & \cos \omega T & 0 & -\frac{\sin \omega T}{\omega} \\ 0 & \frac{1-\cos \omega T}{\omega} & 1 & \frac{\sin \omega T}{\omega} \\ 0 & \sin \omega T & 0 & \cos \omega T \end{bmatrix}.$$  

where $\omega$ denotes turn rate, $T$ is sample interval. In the design, the real turn rate of target is taken as unknown mode parameter and is assumed to be with in a predefined continuous range $[-\omega_{max}, \omega_{max}]$. At each time step, algorithm model-set consist of, left, center and right, three different models,

$$M_k = \{\omega_L(k), \omega_C(k), \omega_R(k)\}$$

where $\omega_L(k), \omega_C(k), \omega_R(k) \in [-\omega_{max}, \omega_{max}]$. Mode Markov switching probability matrix

$$\Pi = \begin{bmatrix} \pi_{LL} & \pi_{LC} & \pi_{LR} \\ \pi_{CL} & \pi_{CC} & \pi_{CR} \\ \pi_{RL} & \pi_{RC} & \pi_{RR} \end{bmatrix}.$$  

Initially, the AGIMM is started with a uninformational setting of grid,

$$D_0 = \{\omega_L(0) = \omega_{max}, \omega_C(0) = 0, \omega_R(0) = -\omega_{max}\}.$$  

At each time step of processing, grid adaption is achieved by two step hard decision logic. Model probability play a key role in making decisions since they carry the most amount of information about which model is in effect.

Step 1. Center Grid Readjustment.

$$\omega_C(k + 1) = \mu_L(k)\omega_L(k) + \mu_C(k)\omega_C(k) + \mu_R(k)\omega_R(k)$$  

where $\{\mu_L(k), \mu_C(k), \mu_R(k)\}$ are model posterior probability of $k$th scan obtained by IMM algorithm.

Step 2. Grid Distance Readjustment.

Case 2.1 : $\mu_C(k) = \max\{\mu_L(k), \mu_C(k), \mu_R(k)\}$

$$\omega_L(k + 1) = \begin{cases} \omega_C(k + 1) + \lambda_L(k)/2 & \text{if } \mu_L(k) < t_1 \\ \omega_C(k + 1) + \lambda_L(k) & \text{otherwise} \end{cases}$$

$$\omega_R(k + 1) = \begin{cases} \omega_C(k + 1) - \lambda_R(k)/2 & \text{if } \mu_R(k) < t_1 \\ \omega_C(k + 1) - \lambda_R(k) & \text{otherwise} \end{cases}$$

where $\lambda_L(k) = \max\{\omega_L(k) - \omega_C(k), \delta_\omega\}$, $\lambda_R(k) = \max\{\omega_C(k) - \omega_R(k), \delta_\omega\}$, $t_1 < 0.1$ is threshold for detecting unlikely model. $\delta_\omega$ is a predefined minimal grid distance.

Case 2.2 : $\mu_L(k) = \max\{\mu_L(k), \mu_C(k), \mu_R(k)\}$

$$\omega_L(k + 1) = \begin{cases} \omega_C(k + 1) + 2\lambda_L(k) & \text{if } \mu_L(k) > t_2 \\ \omega_C(k + 1) + \lambda_L(k) & \text{otherwise} \end{cases}$$

$$\omega_R(k + 1) = \omega_C(k + 1) - \lambda_R(k)$$

where $t_2 > 0.9$ is threshold for detecting significant
model.
Case 2.3: \( \mu_R(k) = \max\{\mu_L(k), \mu_C(k), \mu_R(k)\} \)
\[
\omega_L(k+1) = \omega_C(k+1) + \lambda_L(k)
\]
\[
\omega_R(k+1) = \begin{cases} 
\omega_C(k+1) - 2\lambda_R(k) & \text{if } \mu_R(k) > t_2 \\
\omega_C(k+1) - \lambda_R(k) & \text{otherwise.}
\end{cases}
\]

Once grid adaption has been achieved, system state transition matrix \( F \) of each model must be re-computed according to newly obtained turn rate parameters. The remaining processing of AGIMM is just like that of IMM.

2.2 Discussion

For above grid adaption decision logic, our attention is focused on the step for center grid readjustment, namely (1). It is actually a weighted combination of grid parameters of model-set adopted in last scan, and posterior model probability is used as weight coefficient. Thus, in this approach, center grid of current scan is assumed to be expected-mode of last scan.

Generally, through grid center tuning, what we really want to know is center grid of model-set of current scan, however what obtained from such calculation is not center grid of current scan but a posterior expected mode of last scan. When system keeps the same mode, the latter will gradually converge to the former. Moreover, during this stage, utilizing information of latter will be helpful for accelerating the process that center grid of current scan approaches true mode of system. However as system mode switches, it is evident that the the center grid of current scan is different from posterior expected-mode of last scan. If the latter is taken as the former during transition stage of system mode, it will lead to quite large peak estimation errors.

In [9], an other method of readjusting center grid of current scan is provided, in which grid parameters of last scan are weighted by its prediction probability and combined.

\[
\omega_C(k+1) = \mu_L(k+1|k)\omega_L(k) + \mu_C(k+1|k)\omega_C(k) + \mu_R(k+1|k)\omega_R(k)
\]  
(2)

where \( \mu(k+1|k) \) denotes model prediction probability.

\[
\hat{\mu}_i(k+1|k) = \sum_{j \in M(k)} \pi_{ij}\hat{\mu}_j(k)
\]

Note that model prediction probability contains information about possibility of mode switching, as grid parameter calculated according to (2), referred as prediction expected-mode, is assumed to be center grid of current scan, the problem of huge peak error will be mitigated to some extent.

On the other hand, time length of system mode transition is in general far less than that of system mode keeps unchanged. Just as said above, (1) still provides valuable information for grid adaption during periods of which system mode keeps on.

According to above, there is a dilemma about which one, posterior expected-mode of last scan or prediction expected-mode of current scan, should be used as center grid of new model-set.

3 A New Approach to Grid Adaption of AGIMM Algorithm

In order to get out of above dilemma and also to reduce both dynamic peak error and steady state error of estimation, a new approach of grid adaption is proposed in this section. In our approach, the above two center grid adaption methods and corresponding model-sets are involved, but only one model-set participates in filtering and output combination. The details of AGIMM algorithm that adopted new approach to grid adaption are given as follows.

1. Start of time-step \( k + 1 \): we have \( \hat{x}_i(k), P_i(k), \mu_i(k) \) and \( \omega_i(k), i \in M(k) \).
2. Obtain model-set \( M_1(k+1) = \{\omega_L^1(k+1), \omega_C^1(k+1), \omega_R^1(k+1)\} \) using model posterior probability as does in section 2.1.
3. Calculating model prediction probability \( \hat{\mu}_i(k+1|k) \).
4. Obtain prediction expected-mode \( \omega_C^2(k+1) \) according to (2), and assume it to be center grid of model-set \( M_2(k+1) \).
5. \( \omega_L^2(k+1) \) and \( \omega_R^2(k+1) \) of \( M_2(k+1) \) are obtained using decision logic as same as step 2 of section 2.1, except that \( \mu_i(k) \) and \( \omega_C(k+1) \) are replaced by \( \hat{\mu}_i(k+1|k) \) and \( \omega_C^2(k+1) \), respectively.
6. Input interaction.
7. Measurement prediction. \( x_i^0(k) \) and \( P_i^0(k) \) are obtained as IMM does.
8. Model likelihood.

\[
L_i(k+1) = \frac{1}{(2\pi(S_i(k+1)))^{1/2}} \exp\left\{-1/2\nu_i^T(k+1)S_i^{-1}(k+1)\nu_i(k+1)\right\}
\]

9. Marginal likelihoods of model-set, \( M_1 = M_1(k+1), M_2(k+1) \):

\[
L_{M_1} = \sum_{i \in M_1} L_i(k+1)\hat{\mu}_i(k+1|k)
\]

(4)

if \( L_{M_1}/L_{M_2} > t_3 \), then terminate model-set \( M_2(k+1) \) and let \( M(k+1) = M_1(k+1), \) else terminate model-set \( M_1(k+1) \) and let \( M(k+1) = M_2(k+1) \).
10. For each model of \( M(k+1) \), perform filtering and
4 Simulation Results

In this section, performances of standard three model IMM algorithm, AGIMM algorithm in which only model posterior probabilities are used for grid adaption (AGIMM-Pos), AGIMM algorithm in which only model prediction probabilities are used for grid adaption (AGIMM-Pred) and AGIMM algorithm presented in section 3(AGIMM-Com) are evaluated via Monte Carlo simulations over two representative maneuvering target tracking scenarios. The measures of performance examined include position RMSE, speed RMSE and the average CPU run-time.

In the simulation, the target-measurement model is

\[ x(k + 1) = Fx(k) + Gw(k) \]
\[ z(k + 1) = Hx(k + 1) + v(k + 1) \]

where \( x = (x, \dot{x}, y, \dot{y})' \) denotes target states, \( w \sim N(0, Q) \) is process noise, \( z = (z_x, z_y)' \), \( v \sim N(0, R) \) is measurement noise sequence. \( F \) is just like that given in subsection 2.1, and

\[
G = \begin{bmatrix}
T^2/2 & 0 \\
T & 0 \\
0 & T^2/2 \\
0 & T
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

The transition probability matrix \( \pi \) is also the same as that of [5]. \( R = 2500I, Q = 0.01 \) and sample interval \( T \) is 1s. Initial value of turn rate is set to \( \omega_{\text{max}} = 5.6^\circ/s \) (3g normal acceleration).

The first scenario is just the same as trajectory 1 of section IV in [5]. The second scenario includes a nonmaneuvering flight during scan 1 to 60; a left turn lasting from scan 61 to 95 with turn rate \( 3.74^\circ/s \) (2g normal acceleration); a right turn with turn rate \( -1.87^\circ/s \) (1g normal acceleration) from scan 96 to 180, and a nonmaneuvering flight during scan 181 to 220 scan. The target initial state is \([20000m, 50000m, 150m/s, -260m/s]\). Figure 1 and figure 2 present two typical scenario.

The value of parameter \( t_1, t_2 \) of AGIMM-Pos and AGIMM-Pred are just like those in section III of [5], and the parameter \( t_3 \) of the third algorithm is assumed to be 1. The RMS errors of two scenarios, which are obtained based on 100 Monte Carlo runs, are given by figures 3 through 6.

From these figures, it is not difficult to find that peak estimation errors of algorithm proposed, compared with those of the AGIMM-Pos algorithm, are indeed reduced and at same time its steady state estimation errors are smaller than those of the AGIMM-Pred algorithm. Percent of average RMSE improvements of AGIMM-Com algorithm over IMM, AGIMM-Pos and AGIMM-Pred algorithms are given in Table.1.

Note that improvements on speed RMSE are more evident than those on position RMSE. Thus, AGIMM-Com algorithm integrates effectively the advantages of AGIMM-Pos and AGIMM-Pred and discards their shortcomings.

The computational load of above four algorithms are accessed through CPU runtime of single cycle of algorithms. The simulation program are run in a PC with PIII 966MHz CPU and 256M byte RAM. The CPU runtime for IMM, AGIMM-Pos, AGIMM-Pred and AGIMM-Com are 4.2ms, 4.8ms, 4.8ms and 6.1ms, respectively. Since computing speed increases daily, a little addition of computational load of AGIMM-Com algorithm is acceptable completely.

<table>
<thead>
<tr>
<th></th>
<th>IMM</th>
<th>AGIMM-Pos</th>
<th>AGIMM-Pred</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>9.0</td>
<td>11.3</td>
<td>9.6</td>
</tr>
<tr>
<td>Speed</td>
<td>25.4</td>
<td>22.0</td>
<td>28.6</td>
</tr>
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</table>

Table 1: Percent of Average RMSE improvement of AGIMM-Com over IMM, AGIMM-Pos, AGIMM-Pred.

5 Conclusions

In this paper, we present a new approach to grid adaption for AGIMM algorithm. In contrast to existed grid adaption scheme, in our new method, both model posterior probability of last scan and model prediction probability of current scan are used as weight coefficients to product two different model-sets. According to likelihood ratio of candidate model-sets, one more likely model-set is chosen as effective one for filtering and output combination of current scan. Simulation results indicate that performance of AGIMM-Com algorithm, in which our new grid adaption approach is adopted, actually outperform those of algorithms in which only one kind probability is applied to grid adaption.

References

Figure 1: Test trajectory 1

Figure 2: Test trajectory 2

Figure 3: Position RMSE of trajectory 1

Figure 4: Speed RMSE of trajectory 1
Figure 5: Position RMSE of trajectory 2

Figure 6: Speed RMSE of trajectory 2


