Tracking Separating Targets with a Monopulse Radar: Idealized Resolution

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Abstract – The tracking of separating objects with a monopulse radar is a particularly challenging problem in that the presence of merged measurements result in significant delays in the initiation of tracks on the new objects. Furthermore, the unknown inflation in the variance and the bias of the angle-of-arrival estimates result in extra tracks and hinders track continuity. This paper presents an algorithm for tracking separating objects with a monopulse radar. The algorithm combines a Neyman-Pearson hypothesis test for detecting merged measurements, a method of parsing merged measurements into two angle-of-arrival estimates, and a track initiation procedure that takes advantage of the parsed measurements. The new algorithm was implemented in a sophisticated computer simulation environment to evaluate the performance improvements provided by the new algorithm. The output of the simulation environment is used to illustrate the challenges associated with tracking separating objects with a monopulse radar.

Keywords: Unresolved Objects, Target Tracking, Data Association, Radar Signal Processing, Monopulse.

1 Introduction

Monopulse is defined as the comparison of two squinted beams or simultaneous lobes in radar terminology. It is commonly used in radar systems to determine the angle-of-arrival (AOA) of a target. The AOA is estimated by comparing either the phase or the amplitudes of return signals from two beams that are squinted about boresight, normally in azimuth or elevation. A problem arises when the returns from two or more closely-spaced targets interfere, which means the return signals are not resolved in the frequency and time domains. In the case of merged measurements, one detection occurs rather than the expected two, and the AOA estimate indicated by the monopulse ratio for that detection may be outside the angular extent of the two targets [1]. In other words, the measurement is biased relative to the predicted measurement of either target, and the variance of the measurement is inflated relative to that for a single resolved target. Merged measurements often result in errors in the track filtering and data association algorithms that are severe enough to cause tracks to be lost and false tracks started. The tracking of separating objects is a particularly challenging problem in that the presence of merged measurements result in significant delays in the initiation of tracks on new objects. Furthermore, the unrecognized inflation in the variance and the bias of the AOA estimates result in the formation of extra tracks and hinders track continuity. Using an artificially low measurement variance when the presence of merged measurements is detected prevents the measurements from associating with existing tracks, which gives rise to undesirable extra tracks. The extra tracks in effect steal measurements causing a delay in track initiation on the new object.

In [2], Asseo studied the effects of rejecting measurements that exceeded a threshold test that indicated target multiplicity (i.e., merged measurements). Asseo found that threshold tests using the magnitude of the quadrature or sum channels removed large errors in the AOA estimates. This technique resulted in AOA estimates that tended to be closer to the centroid of the two targets, which was shown to improve average accuracy. While disregarding measurement data when merged measurements are detected was shown to improve tracking performance in [2], disregarding measurement data when merged measurements are detected delays track initiation on new objects, which in the case of separating ballistic missiles could be the track on the re-entry vehicle (RV). The focus of this paper is minimizing the time at which the two objects
are resolved (i.e., the track initiation time on the second object).

Testing for the detection of unresolved objects has been extensively covered in the literature [3]. The test used for this paper is a conditional generalized likelihood ratio test (GLRT) for the detection of the presence of unresolved Rayleigh targets with a Neyman-Pearson algorithm. The Neyman-Pearson detection algorithm uses both the in-phase and quadrature portions of the monopulse ratio and requires no a priori knowledge of the SNR or AOA of either target. A GLRT method for detecting merged measurements was developed in [5] using a test statistic based on the log likelihood ratio, and was shown to have the same detection performance of the test used in this paper.

Rather than computing a single estimate for the AOA of the centroid of two unresolved targets, [3] provides an approach that parses the single estimate for two fully unresolved Rayleigh targets into two separate estimates about the centroid. Two objects are considered to be fully unresolved if the echoed pulses from both objects essentially overlap in time so that all of the echoed energy is captured in common samples of the output of the matched filter. This approach is used in this paper to parse measurements once a merged measurement detection threshold is exceeded.

The tracking of closely-spaced and possibly unresolved targets can be viewed as three distinct cases. The first case involves the tracking of targets that are initially resolved into separate tracks and the targets become unresolved later in the scenario. The second case involves targets that are initially unresolved, while the third case involves a single object that splits into multiple objects. The third case is most relevant to tracking for ballistic missile defense, which involves separations of attitude control module (ACM)/RV from the booster tank and separation of the RV from the ACM. The third case is the focus of this paper.

For this paper, radar resolution is modeled as idealized in that all of the energy for each target is detected in a single resolution cell. Thus, two targets are either completely resolved or the measurements are fully merged. This simplifies the problem because return waves do not straddle range bins. If bin straddling occurs, it is necessary to centroid measurements before the merged measurement detection algorithm. The case of non-ideal resolution will be the focus of future work by the authors.

A brief background is given on monopulse radar equations and statistics in the next section. Sections 3 and 4 give an overview of the algorithm for detecting merged measurements and the algorithm for parsing merged measurements respectively. Section 5 gives the tracking methodology using the two algorithms. Simulation results are given in Section 6, and concluding remarks are given in Section 7.

2 Monopulse Radar Equations and Statistics

In an amplitude comparison monopulse radar system, a pulse of energy is transmitted directly at the predicted position of the target, and the target echo is received with two beams that are squinted relative to the predicted position of the target. (Note that two beams are required for each angular coordinate.) Traditionally, the AOA of a target is estimated with the in-phase part (i.e., the real part) of the monopulse ratio, which is formed by dividing the difference of the two received signals by their sum. The in-phase and quadrature parts of the sum and difference signals for two unresolved Rayleigh targets can be written as

\[
\begin{align*}
    s_I &= \alpha_1 \cos \phi_1 + \alpha_2 \cos \phi_2 + n_{SI} \\
    s_Q &= \alpha_1 \sin \phi_1 + \alpha_2 \sin \phi_2 + n_{SQ} \\
    d_I &= \alpha_1 \eta_1 \cos \phi_1 + \alpha_2 \eta_2 \cos \phi_2 + n_{dI} \\
    d_Q &= \alpha_1 \eta_1 \sin \phi_1 + \alpha_2 \eta_2 \sin \phi_2 + n_{dQ}
\end{align*}
\]

where \(\alpha_i\) is the Rayleigh distributed amplitude of target \(i\), \(\phi_i\) is the phase of the received signal from target \(i\), \(\eta_i\) is the direction-of-arrival (DOA) for target \(i\), and

\[
\begin{align*}
    n_{SI} &\sim N(0, \sigma_{SI}^2) & n_{SQ} &\sim N(0, \sigma_{SQ}^2) \\
    n_{dI} &\sim N(0, \sigma_{dI}^2) & n_{dQ} &\sim N(0, \sigma_{dQ}^2)
\end{align*}
\]

The notation \(N(\bar{x}, \sigma^2)\) denotes a Gaussian distribution with mean \(\bar{x}\) and variance \(\sigma^2\). For this paper, the errors \(n_{SI}, n_{SQ}, n_{dI}\), and \(n_{dQ}\), are assumed to be independent.

Letting \(A\) and \(\psi\) denote the measured amplitude and phase of the sum signal gives

\[
    s_I = A \cos \psi \quad s_Q = A \sin \psi
\]

Then the observed SNR will be defined as

\[
    \mathcal{R}_o = \frac{A^2}{2\sigma^2} \tag{8}
\]

Since \(\alpha_1\) and \(\alpha_2\) are Rayleigh distributed and \(\phi_1\) and \(\phi_2\) are uniformly distributed on \((-\pi, \pi]\), \(s_I\) and \(s_Q\) are Gaussian random variables. Applying the transformation of random variables in (7) to the PDF of \(s_I\) and \(s_Q\) gives the PDF of the observed SNR as

\[
    f(\mathcal{R}_o | \mathcal{R}_R) = \frac{1}{\mathcal{R}_R + 1} \exp\left[-\frac{\mathcal{R}_o}{\mathcal{R}_R + 1}\right], \quad \mathcal{R}_o \geq 0 \tag{9}
\]
where $\Re_R$ is the SNR parameter of the Rayleigh signal given by

$$
\Re_R = \frac{E[\alpha_1^2]}{2\sigma_S^2} + \frac{E[\alpha_2^2]}{2\sigma_S^2} = \Re_{R1} + \Re_{R2} \tag{10}
$$

where $E[\cdot]$ denotes the expected value and $\Re_{R1}$ and $\Re_{R2}$ are the SNR for Rayleigh targets 1 and 2, respectively. Since the relative RCS of the targets is assumed to be known, let $\Re_{R2} = \lambda \Re_{R1}$ for $\lambda > 1$. Then $\Re_R = (1 + \lambda)\Re_{R1}$. For $N$ subpulses at distinct frequencies (i.e., independent), the Maximum Likelihood (ML) estimate of $\Re_R$ is given by

$$
\hat{\Re}_R = Y_N - 1, \quad Y_N = \frac{1}{N} \sum_{k=1}^{N} \Re_{ok} \tag{11}
$$

where $\Re_{ok}$ denotes the observed SNR for subpulse $p$. Then $\hat{\Re}_R$ is an unbiased, efficient estimator of $\Re_R$ with variance given by $\text{var}(\hat{\Re}_R) = (\Re_R + 1)^2/N$.

Denoting $s = s_1 + js_2$ and $d = d_1 + jd_2$, the in-phase and quadrature parts of the monopulse ratio are given by

$$
y_I = \Re \left( \frac{d}{s} \right) = \frac{s_1d_1 + s_2d_2}{s_1^2 + s_2^2} \tag{12}
$$

$$
y_Q = \Im \left( \frac{d}{s} \right) = \frac{s_1d_2 - s_2d_1}{s_1^2 + s_2^2} \tag{13}
$$

where $y_I$ and $y_Q$ are conditionally independent, Gaussian random variables with a common variance. The mean of $y_I$ is a “power” weighted average of the DOAs of the two targets, while the mean of $y_Q$ is zero.

### 3 Detecting Unresolved Objects

In [6] a GLRT test is derived for detection of target multiplicity (i.e., merged measurements). The GLRT is a hypothesis test with the null hypothesis, $H_0$, equal to the case of resolved objects, and $H_1$ equal to the case of unresolved measurements.

The PDFs of $y_I$ and $y_Q$ for a single resolved target are given in [6] as

$$
f(y_I|H_1, \Re_o, \Phi) = N \left( \frac{\Re_{R1}}{\Re_{R1} + 1} \eta_1, \frac{p}{2\Re_o} \right) \tag{14}
$$

$$
f(y_Q|H_0, \Re_o, \Phi) = N(0, \frac{p}{2\Re_o}) \tag{15}
$$

where

$$
p = \left[ \frac{\sigma_2^2}{\sigma_s^2} + \frac{\Re_{R1} \eta_1^2}{\Re_{R1} + 1} \right], \tag{16}
$$

and $\Phi$ denotes the parameter set

$$
\Phi = \{\Re_{R1}, \eta_1, \sigma_s, \sigma_d\}, \tag{17}
$$

where the $\Re_{R1}$ term is the SNR of the target, and $\eta_1$ is the DOA for that target.

Since $y_I$ is a conditional Gaussian random variable under $H_0$ or $H_1$, the ML estimate of $\hat{y}_I$, which is the conditional mean of $y_I$ under $H_0$ or $H_1$, is given for $N$ independent pulses by

$$
\hat{y}_I = \left[ \sum_{n=1}^{N} \Re_{on} \right]^{-1} \sum_{k=1}^{N} \Re_{ok} y_{1k} \tag{18}
$$

where $\Re_{ok}$ and $y_{1k}$ denote the observed SNR and in-phase monopulse ratio for subpulse $k$ and $Y_N$ is given by (11). Since the $y_{1k}$ are Gaussian random variables, $\hat{y}_I$ is the minimum variance estimate of $\hat{y}_I$ and it is a Gaussian random variable with variance given by

$$
\sigma_{\hat{y}_I}^2 = \frac{p}{2} \left( \sum_{k=1}^{N} \Re_{ok} \right)^{-1} = \frac{p}{2NY_N} \tag{19}
$$

The variance of the monopulse ratio of pulse $k$ under hypothesis $H_0$ is

$$
\sigma_{\hat{y}_I}^2 = \frac{\hat{p}}{2\Re_{ok}} \tag{20}
$$

$$
\hat{p} = \frac{\sigma_2^2}{\sigma_s^2} \left( 1 + \frac{1}{\Re_{R1}} \hat{y}_I^2 \right) \tag{21}
$$

The GLRT test statistic, $T_N$, is defined as

$$
T_N = X_N^2 R_N X_N \hat{p}^{-1} \tag{22}
$$

where

$$
X_N = [y_I - \hat{y}_I \cdots y_{IN} - \hat{y}_I \cdots y_{QN}]' \tag{23}
$$

$$
R_N = 2 \text{ diag}[\Re_{o1} \cdots \Re_{oN} \cdots \Re_{o1} \cdots \Re_{oN}] \tag{24}
$$

The $y_{1k}$ and $y_{Qk}$ are the in-phase and quadrature monopulse ratios for pulse $k$, and $\Re_{ok}$ is the observed SNR for pulse $k$. The $T_N$ is chi-square distributed with $2N - 1$ degrees of freedom, so the detection rule, $\delta$, for merged measurements is given as

$$
\delta = \begin{cases} 
H_0, & T_N \leq \hat{C}^2 \\
H_1, & T_N > \hat{C}^2 
\end{cases} \tag{25}
$$

where $\hat{C}^2$ represents the detection threshold of a chi-square distributed random variable with $2N - 1$ degrees of freedom and a given probability of detection. Chi-square threshold values for several detection probabilities and degrees of freedom are given in [7].
4 Parsing Merged Measurements

The PDFs of \( y_t \) and \( y_{d} \) under \( H_1 \), for two unresolved Rayleigh targets, are given as

\[
\begin{align*}
    f(y_t|H_1, \eta_1, \eta_2) &= N(q, 2\mathbb{R}_o) \quad (26) \\
    f(y_{d}|H_1, \eta_1, \eta_2) &= N(0, 2\mathbb{R}_o) \quad (27)
\end{align*}
\]

where

\[
q = \frac{\sigma_d^2}{\sigma_s^2} + \frac{\mathbb{R}_1\eta_1^2 + \mathbb{R}_2\eta_2^2 + \mathbb{R}_1\mathbb{R}_2(\eta_1 - \eta_2)^2}{\mathbb{R}_1 + \mathbb{R}_2 + 1},
\]

and \( \Phi \) denotes the parameter set

\[
\Phi = \{ \mathbb{R}_1, \mathbb{R}_2, \eta_1, \eta_2, \sigma_s, \sigma_d \}. \quad (29)
\]

The \( \mathbb{R}_1 \) and \( \mathbb{R}_2 \) terms are the SNRs of the two targets, while \( \eta_1 \) and \( \eta_2 \) are the DOAs for the targets. The larger errors caused by the combined SNR of the unresolved targets occur because the targets produce a value of \( \mathbb{R}_o \), that is small relative to \( \mathbb{R}_1 \mathbb{R}_2(\mathbb{R}_1 + \mathbb{R}_2 + 1)^{-1} \).

Since the \( y_{tk} \) are Gaussian random variables, \( \hat{y}_t \) is the minimum variance estimate of \( y_t \), and it is a Gaussian random variable with variance given by

\[
\sigma_{\hat{y}_t}^2 = \frac{q}{2} \left( \sum_{k=1}^{N} \mathbb{R}_{mk} \right)^{-1} = \frac{q}{2NY_N}. \quad (30)
\]

The DOA estimator for two unresolved targets is derived in [3], where the first step in parsing a merged measurement is to calculate an estimate of \( q \), given in (28), with

\[
\hat{q} = \frac{X_N N X_N}{2N-1} \quad (31)
\]

where \( X_N \) is defined in (23), \( R_N \) is defined in (24), and \( N \) is the number of subpulses. Then, the DOAs, \( \eta_i \), for the two unresolved targets are given by

\[
\begin{align*}
    \hat{\eta}_1 &= \hat{y}_t + \sqrt{\mathbb{R}_1\mathbb{R}_2\hat{q}} \quad (32) \\
    \hat{\eta}_2 &= \hat{y}_t - \sqrt{\mathbb{R}_1\mathbb{R}_2\hat{q}} \quad (33)
\end{align*}
\]

where

\[
\hat{q} = \begin{cases} 
    0, & \text{if } \hat{q} \geq \frac{\sigma_d^2}{\sigma_s^2} \\
    \frac{4\sigma_d^2 \mathbb{R}_1 \mathbb{R}_2}{(\mathbb{R}_1 + \mathbb{R}_2)^2}, & \text{if } \hat{q} \geq \frac{\sigma_d^2}{\sigma_s^2} \\
    \hat{q} - \frac{\sigma_d^2}{\sigma_s^2}, & \text{otherwise}
\end{cases} \quad (34)
\]

\( \mathbb{R}_i \) is given in (11), and \( \eta_{bw} \) denotes the one-way, half power point on the antenna gain pattern of the sum channel. \( \eta_{bw} \) is approximated as \( \frac{e_{bw}}{2} \), where \( k_{bw} \) is the monopulse error slope.

5 Tracking Methodology

The merged measurement detection and measurement parsing algorithms are placed at the output of the signal processor. Tracker feedback is used to associate detections to existing tracks in range only using a 2D assignment algorithm. If a detection associates to an existing track, then a GLRT test is performed for target multiplicity. A buffer of the test statistic, \( T_N \), is the number of subpulses. Then, the DOAs, \( \eta_1 \) and \( \eta_2 \), are Gaussian random variables, \( \hat{y}_t \) is the minimum variance estimate of \( \hat{q} \), given in (31), where the first step in parsing a merged measurement is to calculate an estimate of \( q \), given in (28), with

\[
\hat{q} = \frac{X_N N X_N}{2N-1} \quad (31)
\]

where \( X_N \) is defined in (23), \( R_N \) is defined in (24), and \( N \) is the number of subpulses. Then, the DOAs, \( \eta_i \), for the two unresolved targets are given by

\[
\begin{align*}
    \hat{\eta}_1 &= \hat{y}_t + \sqrt{\mathbb{R}_1\mathbb{R}_2\hat{q}} \quad (32) \\
    \hat{\eta}_2 &= \hat{y}_t - \sqrt{\mathbb{R}_1\mathbb{R}_2\hat{q}} \quad (33)
\end{align*}
\]

where

\[
\hat{q} = \begin{cases} 
    0, & \text{if } \hat{q} \geq \frac{\sigma_d^2}{\sigma_s^2} \\
    \frac{4\sigma_d^2 \mathbb{R}_1 \mathbb{R}_2}{(\mathbb{R}_1 + \mathbb{R}_2)^2}, & \text{if } \hat{q} \geq \frac{\sigma_d^2}{\sigma_s^2} \\
    \hat{q} - \frac{\sigma_d^2}{\sigma_s^2}, & \text{otherwise}
\end{cases} \quad (34)
\]

\( \mathbb{R}_i \) is given in (11), and \( \eta_{bw} \) denotes the one-way, half power point on the antenna gain pattern of the sum channel. \( \eta_{bw} \) is approximated as \( \frac{e_{bw}}{2} \), where \( k_{bw} \) is the monopulse error slope.

6 Simulation Results

The simulations included a ground X-band radar and a ballistic missile with a 600 km range as shown in Figure 1. The missile’s full stack separated into an RV and a booster tank at 96.1 seconds. The radar cross section (RCS) of the missile’s full stack, RV, and booster tank are all equal (i.e., constant RCS). The radar’s broadside is pointed directly at the separation event, and the sensor resolution is modeled as idealized.

Simulations were run with no merged measurement detection or measurement parsing (i.e., a baseline case). Figure 2 shows the GLRT test statistic for track 1 on Monte Carlo run 1 for this case. The chi-square threshold used in all cases is based on a 0.01 probability of false alarm. The plot shows test exceedances just after separation, and that the second track is initiated at 127.9 seconds. Therefore, it takes more than
RV separates from booster tank at 96.1 s.

Ground X-band radar

600 km Ballistic Missile Trajectory

Launch

Impact

Figure 1: Scenario geometry used for simulations.

Second track initiated at 127.9 s.

Figure 2: GLRT Test Statistic for Az and El without Measurement Parsing.

Figure 3: Measurement Counts without Measurement Parsing.

Object resolved at 127.9 s.

Figure 4: Tracker Completeness Metric without Measurement Parsing.

Figure 4 shows the tracker completeness metric for 100 Monte Carlo experiments with no merged measurement detection or measurement parsing. As expected, the average tracker completeness metric drops to 0.5 just after separation, and the tracker does not regain full completeness of 1 until 130 seconds.

The redundant track ratio is defined as the ratio of total number of tracks to the total number of valid reportable objects. Figure 5 shows the redundant track ratio averaged over 100 Monte Carlo runs for the case with no measurement parsing (i.e., the baseline). Redundant tracks are present due to the oscillation between resolved and unresolved measurements.

The GLRT test statistic for track 1 of Monte Carlo run 1 for the case with merged measurement detection and measurement parsing is shown in Figure 6.

30 seconds to resolve the objects in this case.

A count of the measurements at the output of the signal processor sent to the tracker as a function of time is shown in Figure 3. The plot shows that from 96.1 seconds to approximately 116 seconds the RV and the booster tank are unresolved. From approximately 116 seconds to 133 seconds the objects are oscillating between resolved and unresolved, and beyond 133 seconds the objects are fully resolved. Measurement counts greater than 2 indicate the presence of a false alarm.

Tracker completeness is defined as the ratio of the total number of tracks held at a given time to the total number of valid reportable objects at that time. In other words, a tracker completeness of 1 is perfect, because we have one track for every trackable object.
The second track is initiated at 101.5 seconds. The plots show that the GLRT test is exceeded well after the second track is initiated. Again, this is due to the oscillation between resolved and unresolved measurements.

The measurement counts in Figure 7, show that from 96.1 seconds to 101.5 seconds the objects are unresolved, from 101.5 seconds to 123 seconds the measurement count oscillates between resolved and unresolved objects, and beyond 123 seconds the objects are resolved. In this case, it is shown that measurement parsing provides resolved measurements to the tracker earlier than the baseline case, but it also provides more false alarm measurements to the tracker after the objects are fully resolved.

The tracker completeness metric for several Monte
Carlo experiments is shown in Figure 8 for this case. When averaged over 100 Monte Carlo experiments, the tracker regains completeness quicker than the baseline case, with a completeness of 0.99 regained by 119 seconds. In Figure 9, the redundant track ratio for the case with measurement parsing shows a significant increase in redundant tracks, which could be reduced by delaying the tentative status on the track.

7 Conclusions

This paper presents a method for tracking separating targets with a monopulse radar under idealized resolution. The algorithm presented combines a Neyman-Pearson hypothesis test for detecting merged measurements, a method of parsing merged measurements into two AOA estimates, and a track initiation procedure that takes advantage of the parsed measurements. The tracking of separating objects is a particularly challenging problem in that the presence of merged measurements results in significant delays in the initiation of tracks on the new objects, which in the case of separating ballistic missiles could be the track on the re-entry vehicle (RV). The results from simulations show that over 100 Monte Carlo experiments, an earlier track initiation time for the second object is achieved. In this paper, the average track initiation time on the second object was reduced by 11 seconds. However, the algorithms do result in a higher false alarm rate after the objects are fully resolved, which in this case caused redundant tracks. Mitigating the redundant tracks and considering non-ideal resolution using the technique of [8] will be the focus of future work by the authors.

References


