Fusion of Active Phased Array Radars within a Quick Reaction time using Multidimensional Data Association

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Abstract This paper presents several approaches to realize the data association for modern multisensor data fusion systems. Therefore the requirements of data association algorithms in real multisensor data fusion systems, as applied in air traffic control or in air defence systems are discussed. Especially, the effect and possible integration methods, of the new electronically scanned array radar (ESA) are illustrated. Further, the presented algorithms are optimised to satisfy quick reaction requirements while delivering a highly reliable result (fused track).

Keywords: Tracking, filtering, estimation, Lagrange relaxation, data association, convex analysis, non-differentiable optimisation, linear programming.

1 Introduction

Today’s multisensor data fusion systems have to integrate very diverse sensor suites. The sensors may be primary and secondary radar, electro-optical sensors like TV or infrared (IR), laser-sensors, acoustic sensors, for example sonar, or special electronic support measurement sensors (ESM). Active sensors emit own signals, that are reflected by the targets of interest, while passive sensors only receive signals emitted by the targets themselves. They differ also in the type of generated information: active sensors use the time differences to estimate the distance of the target and doppler measurements to calculate the range rate while passive sensors determine only the direction to the target. The sensors may be distributed over different locations or platforms. However, a network is also an essential component of such a multisensor data fusion system. It is necessary in such systems, to share information like sensor measurements or commands, which control the sensors. Applications of such systems are found both in the civil and defence areas: air traffic control, naval systems like the aegis system or the new German F124 anti air warfare frigates or ground based distributed air defence systems. We will investigate the data association components of these multisensor data fusion systems, i.e. those parts, which have to find the correct correlation between confirmed tracks representing targets and observations found by the sensor suite.

1.1 Electronically vs Mechanically Scanned Antenna

Mechanically scanned antenna (MSA) like classical radar or IR TWS – track while scan – systems receive observation through a continuously rotating antenna. The set of observations collected through a full turn of the antenna is called a scan. Therefore one assumes that a scan satisfies that:

- Each measurement of a scan belongs at most to one target and
- Each target is reported with at most one measurement of the scan.

Through the fixed antenna rotation rate the position of the antenna is predetermined and therefore the region investigated by the sensor is always known.

Another kind of sensor is the electronically scanned array (ESA) radar or agile beam radar. This type of sensor is able to perform sampling by directing the radar beam...
instantaneously in any direction. Through this beam control capability, the ESA can improve on the MSA sensors, because it allows adaptive tracking facilities. The requests about the target positions may be dependent on the target behaviour (manoeuvres, speed) and even a potential new detection can be confirmed through several beams. The newest class of such agile beam radar is the active phased array radar. This radar type is used as a multifunction sensor, which supports simultaneously different modes. For example it may respond to requests like

- Horizon search: Search on the horizon for new targets,
- Limited volume search: Surveillance of a specified volume of the space,
- Cued search: Confirmation and tracking of targets, found by additional sensors, like long range surveillance radar, ECM, IR or laser,
- Target Designation: Support existing tracks
- Weapon support: missile guidance and kill assessment: Illumination of multiple missiles during mid-course, as well as terminal, guidance.

Such sophisticated ESA must be considered as complex subsystems. Besides the actual antenna, the sensor needs a sensor management unit, which controls a dwell scheduler based on the results of a sensor internal tracking system and external requests.

1.2 The requirements for Data Association

The modern types of sensors, the targets, which are to be observed, and the underlying networks, used to share the selected information determine the architecture and requirements for the data association used in a modern multisensor data fusion system. As shown, some sensors like ESAs can not operate without their own sensor management and sensor internal tracking system. One might expect that such sensors deliver only those measurements which are associated with sensor internal tracks. Other MSA like sensors might report only unassociated measurements without any sensor internal track (for example IFF interrogators). The reports of course, might be different between active and passive sensor types, i.e. some sensors detect positions in the 3 dimensional space including doppler information while other types detect only directions. Different sensor management units and even the network produce time
delays and as a consequence, out of sequence reports (measurements). On the other side, dense target situations, high speed objects (tactical ballistic missiles), low detection targets (seaskimmers), manoeuvring fighters or dense target situations should be taken into account. These lead to requirements for short reaction times and the application of time consuming advanced data association and filtering methods on the other side. Therefore the following requirements have to be satisfied by the data association algorithm:

- Functional requirements
  - Handle different ESAs and MSAs
  - Accept associated measurements with sensor tracks and unassociated measurements
- Timing requirements:
  - Handle out of sequence measurements
  - Operate in short time intervals, i.e. no significant increase of time between sensor report and fusion result,
- Quality requirements:
  - High performance data association and
  - Filtering

2 Data Association Techniques

There are several approaches to solving the data association problem, i.e. to find the correct correlation between tracks representing targets and observations found by the sensor suite. A first classification of such algorithms is between hard decision and statistical approaches. Hard decision methods are related to the idea, of collecting the observations in the different scans mentioned in the introduction and making a one to one relationship between the observations of a single scan and the established tracks. One divides the hard decision methods by the number (dimension) of scans, which are taken into account. On the other side there is a class of algorithms, which renounce establishing such a fixed relation. These algorithms allow the association of a track with multiple measurements. Therefore the measurements are incorporated in a single track with some weighting. This paper focuses on multidimensional data association techniques and their realisation in real systems.

2.1 Multidimensional Data Association

In the language of data fusion and tracking, the definition of scan is translated into

- Each measurement of a scan is associated with at most one track and
- Each track is associated with at most one observation.

The aim is to find an optimal relation between observations and tracks, that satisfies the above constraints. In a more mathematical language, this takes the following form:

$$\min_{\chi_{i_1-\ldots-i_s}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \ldots \sum_{i_s=0}^{n_s} c_{i_1-\ldots-i_s} \chi_{i_1-\ldots-i_s}$$

subject to the following constraints:

$$\sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{i_s=0}^{n_s} \chi_{i_1-i_2-\ldots-i_s} = 1, \quad i_1 = 1 \ldots, n_1$$

$$\sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{i_s=0}^{n_s} \chi_{i_1-i_2-\ldots-i_s} = 1, \quad i_2 = 1 \ldots, n_2$$

...  

$$\sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{i_s=0}^{n_s} \chi_{i_1-i_2-\ldots-i_s} = 1, \quad i_s = 1 \ldots, n_s$$

and $\chi_{i_1-\ldots-i_s} \in \{0,1\}$.

In this terminology $\chi_{i_1-\ldots-i_s} = 1$ means, that track number $i_s$ is continued with measurement number $i_2$ in the 1st, with number $i_3$ in the 2nd, ... and with number $i_s$ in the last, the (s-1)st scan. Measurement 0 stands for a false measurement and track 0 for the “dummy” track (i.e. new track). The $c_{i_1-\ldots-i_s}$ are the negative logarithms of the association probabilities:

$$L_{i_1-\ldots-i_s} = \prod_{s=2}^{S} (1 - P_D)^{\delta_{i_1,0}} \left( \frac{P_D}{\rho_F} \frac{N(z_{i_s,i_1}; H, X_{i_1|i_1-1}, \Sigma) \prod_{s=2}^{S} \delta_{i_s,0}}{\delta_{i_1,0}} \right)$$

where $\rho_F$ is the density of false alarm, $P_D$ the detection probability of the target-originated measurements and $z_{i_s,i_1}$ is the $i_s$-th measurement of the s-th scan, $H$ the transformation from the state-space into the measurement-space, $X_{i_1|i_1-1}$ the propagated state and $\Sigma$ the innovation-covariance-matrix.
The solution of this optimisation problem depends on the dimension $S$. The classical approaches to addressing the 2 dimensional optimisation problem are the classical Hungarian methods or its modification, the Munkres algorithms [19,20]. Newer Algorithms are the Jonker-Volgenant-Castanon Algorithm (JVC) [16, 22], the auction technique [2, 4, 16, 21] or the modified signature method [16, 23]. Overcoming the Problem in dimensions higher than 2 is much more difficult. While the 2 dimensional Munkres algorithm is of complexity $O(n^2)$, $n = \min(n_1, n_2)$, $m = \max(n_1, n_2)$, the $S$ dimensional assignment problem is NP-hard, i.e. the computational cost of an optimal solution can grow at an exponential rate.

2.1.1 Lagrange relaxation

One method to handle the optimisation problem uses the suboptimal Lagrangian relaxation technique [2, 3, 5, 6, 16]. In the first step, one uses the last $S-2$ set of constraints and Lagrange multipliers $u^k_i$, $i_k \in \{1, \ldots, n_k\}$, $k = 3, \ldots, S$ to relax the $S$-dimensional problem into a 2-dimensional one:

$$
\phi((u^k_i)_{i_k \in \{0, \ldots, n_k\}, k=1, \ldots, S}) = \min_{\chi_{i_k-1 \ldots -S}} \sum_{i_k=0}^{n_k} \sum_{i_k=0}^{n_k} c_{i_k, \chi_{i_k}} \chi_{i_k-1 \ldots -S}
$$

subject to the constraints

$$
\sum_{i_k=0}^{n_k} \chi_{i_k} = 1, \quad i_1 = 1, \ldots, n_1
$$

$$
\sum_{i_k=0}^{n_k} \chi_{i_k} = 1, \quad i_2 = 1, \ldots, n_1
$$

and $\chi_{i_1, \ldots, i_S} \in \{0,1\}$.

One then searches the Lagrange Multipliers, which maximise the function $\phi((u^k_i)_{i_k \in \{0, \ldots, n_k\}, k=1, \ldots, S})$, i.e.

$$
\arg \max_{(u^k_i)_{i_k \in \{0, \ldots, n_k\}, k=1, \ldots, S}} \phi((u^k_i)_{i_k \in \{0, \ldots, n_k\}, k=1, \ldots, S})
$$

It can be shown, that $\phi((u^k_i)_{i_k \in \{0, \ldots, n_k\}, k=1, \ldots, S})$ is a piecewise affine, concave and continuous function. Therefore this problem belongs to the area of convex analysis and non differential optimisation theory. The mathematical theory is able to solve this problem via advanced bundle methods or subgradient methods [7, 8, 9, 13, 14, 15]. Having determined a solution for the Lagrange Multipliers, one finds a paring of $i_1$ and $i_2$, which satisfies the relaxed two dimensional assignment problem. One can fix this paring to formulate a $S-1$ dimensional “recovery assignment” problem. The technique continues by iteration and finally stops with a 2 dimensional “recovery assignment” problem. The quality of the solution is verified by special “gap”-calculations.

2.1.2 Linear Programming Methods

A different approach [10] is to use modern linear programming techniques. So the Homogeneous Self-Dual Interior Point Methods [11, 12] is suitable for application to the optimisation problem. In contrast to the Lagrange Technique, this Method does not find a solution, which necessarily satisfies the constraint $\chi_{i_1 \ldots, i_S} \in \{0,1\}$. The advantage is, that this method is not suboptimal, like the Lagrange relaxation. The lack of $\chi_{i_1 \ldots, i_S}$ being not an integer, can be solved by interpreting the $\chi_{i_1 \ldots, i_S}$ as pseudo-probabilities. This interpretation allows one to update tracks in a manner similar to the Joint Probabilistic Data Association Formula (JPDA).

2.2 Multidimensional Data Association for ESA

The above data association requires the collection of observations into scans. Of course it is easy to fulfill this assumption for a MSA. Whenever the antenna has finished a full rotation, a new scan is provided. If the sensor suite contains additional ESAs, the problem is much more complicated.

2.2.1 Pseudo scans & central level fusion

The pseudo scan approach assumes that the sensor delivers only plots without any pre-tracking information (sensor tracks) and therefore realizes central level fusion.

![Figure 5. pseudo-scan partition.](image)
To be able to apply the multidimensional data association techniques, one has to generalize the definition of the scan, which is the pseudo-scan [17]. A pseudo-scan is a maximal collection of successive beams into non-overlapping sectors. If the beams pointed consecutively into the sectors a, b, c, a, d, a, b, e (fig. 5) we get the pseudo-scan partition \{a\}, \{b, c\}, \{a, d\}, \{a\} and \{b, e\}.

2.2.2 Tracklets & Hybrid Fusion

One easily detects the problem of the pseudo-scan method in the central fusion approach: A short time interval may be split into a high number of pseudo-scans. This becomes a problem, when some targets are updated with a very high update frequency, while other targets are tracked with a slow frequency. Further if one has to consider targets of high speed (several Mach), the targets may not remain in the same segment or at least in a neighbouring segment, even when the update frequency is high. Therefore, to provide the pseudo-scan partition, one has to verify a possible overlap with neighbouring segments, up to a possible high degree. Further to get a sufficient time depth, the dimension of the multidimensional association method should contain several measurements of each target. So it may happen, that one has to increase the dimension of the multidimensional data association up to high dimensions. If the ESA sensor also provides the association information found by a local sensor tracker before, a hybrid fusion approach is applicable. Under this assumption, one use a partition into disjoint time-intervals and collects the report for every sensor track over these disjoint time intervals into so called tracklets. The association is now performed between these tracklets and the established “fused tracks”, generated by the data fusion. Therefore let the indices \(i_k \) determine a reference onto this sensor tracklet of the \(k\)-th time interval instead of single associated measurements. The coefficients \(c_{i_k-i_s}\) must be updated in the following manner.

\[
L_{i_k-i_s} = \prod_{x=2}^{S} \left( \prod_{k=1}^{l_i} \left( \frac{P_D}{(1-P_D)P_F} N(z_{x,i_k,k}; H_{x,i_k,k}y_{i_k-1}, S) \right)^{-1} \delta_{i_k,i_s} \right)
\]

\[
c_{i_k-i_s} = -\ln L_{i_k-i_s}
\]

Here \(z_{x,i_k,k}\) is the \(k\)-th measurement of the \(i\)-th sensor tracklet in the \(s\)-th time interval and \(l_i\) the cardinality of this tracklet. Now one proceeds as explained in 2.1, using tracklets instead of single observations. The disadvantage of this method is of course that the data association does not use every measurement individually and independently. The advantage is, that the time depth can be increased without increasing the necessary dimension of the data association

2.3 Multisensor Environment and Data Association

Considering multiple sensors instead of a single sensor leads to time-depth conflicts, especially for asynchronous sensors with time-varying sampling intervals. To reach a maximal-time depth the scheme in [1, 17] gives a practical solution. One introduces a frame list for every sensor, which contains the latest scans. Whenever the frame list reaches it maximal length (i.e. \(S-1\)) a \(S\) dimensional data association between the system tracks and the scans (tracklets/pseudo-scans) of the frame-list is applied (fig. 6). The oldest scan is exchanged by the updated system tracks (fig.6, last picture) and the scans contained in the frame list of investigation are moved one position to left. Therefore the frame-list is similar to a conventional sliding window approach. But if we continue in this way, the system tracks will be updated through the scans of different sensors (sensor-depth) without destroying the time-depth.

3 Confirmed & Tentative Phase

To meet both timing and quality requirements, on split the data fusion into 3 phases: A confirmed and two tentative phases (fig. 7).

3.1 Confirmed phase

The confirmed phase is the pure \(S\)-dimensional data association already described. It is the most time consuming part of the whole processing. Together with the 1st tentative phase it runs as a kind of background process.
Whenever the confirmed phase is finished, all closed scans are incorporated with exception of the last S-2 closed scans. The result is a confirmed fused track, which will never change in the following processing. For ESAs one uses pseudo-scans (resp. tracklet-scans) instead of normal scans.

### 3.2 1\textsuperscript{st} Tentative phase

The 1\textsuperscript{st} tentative phase incorporates the remaining closed scans. The best method for establishing the association of the remaining closed scans of the individual sensors, is to apply a sequence of multidimensional data associations of decreasing dimension cyclical between closed scans in the different frames analogous to the maximal time depth algorithm \cite{1, 17}. I.e. one starts with S-1 dimensional data associations (fig. 8, first and second pictures) and continuing as long as frames with S-2 closed scans are still available. When no S-2 frame is available one continues with S-2 dimensional data associations (fig. 8 third picture) and finalize with 2 dimensional data associations. When the 1\textsuperscript{st} tentative phase is finished, all closed scans are incorporated into the association (fig. 8 last picture). Of course this scheme may result in a high amount of out of sequence observations, which hampers the weight calculations. Therefore an alternative is to use the result of the SD data association performed in the confirmed phase (fig. 6) and remember the association result over all scans (fig. 9). This minimizes the occurrence of out of sequence problems. The results of the 1\textsuperscript{st} tentative phase are the 1\textsuperscript{st} tentative fused tracks. In contrast to the confirmed fused tracks these parts are allowed to change later.

### 3.3 2\textsuperscript{nd} Tentative Phase

This phase considers the scans (resp. pseudo-scans/tracklet-scans) which are still open and realise the quick reaction time. It uses the 1\textsuperscript{st} tentative fused tracks and the observations itself as input. The incoming observations are associated with the 1\textsuperscript{st} tentative tracks with a nearest neighbour approach. This can almost be done in realtime. In the case of hybrid fusion there is also another method. One remembers the sensor track numbers fused with the 1\textsuperscript{st} tentative tracks and keeps this relation for incoming tracks. Of course this may cause track coalescence for inconsistent sensor tracks. To avoid this, one should check the incoming observations between different sensors for inconsistency.

### 4 Complexity and Example

The complexity of the algorithms depends on the calculation of the weight matrices of the confirmed phase.
This consumes more than 95% of the CPU power in the simulation. Therefore the complexities of the algorithms in this paper are determined by the filter characteristics, i.e. dimension of the state and measurement space, resp. the number of models, if an IMM is used. Hence the implementation of the ESA within the central fusion (pseudo scans) has the problem that a decomposition of the observations set, within a time period $T_j$, into pseudo-scans of length $l_1,...,l_m$ will cost a factor $\prod_{i=1}^{m} (l_i + 1)$ in the calculation of the weight matrix (and an factor $n$ in the dimensionality of the optimisation problem), where the decomposition into tracklets has a factor equal or lower than $\sum_{i=1}^{m} l_i + 1$. If the ESAs include internal sensor tracker, the proposed hybrid algorithm is therefore much faster than the central approach, but of course coarser. The following example illustrates the hybrid version (tracklets of length 3) of the above algorithms. The 1st tentative update was performed with method 2 and the 2nd tentative update uses the sensor track relations. The ESA was simulated by producing updates within random frequency for each of the targets. Note that the sensor trackers are also contradicting.

5 Conclusion

The realisation of data association within real multisensor data fusion systems was discussed. It was shown how to effectively integrate electronically scanned array antennas such as active phased array radar. This was done using multidimensional data association via Lagrange relaxation techniques and the Homogeneous Self-Dual Interior Point Methods. The result is algorithms which satisfy both time-depth and sensor-depth. A further tentative update phase ensures quick reaction processing. Therefore the algorithm meets the requirements for data association as applied in air traffic control and air defence.

References


